Motion estimation

Digital Visual Effects, Spring 2007

Yung-Yu Chuang

2007/4/10

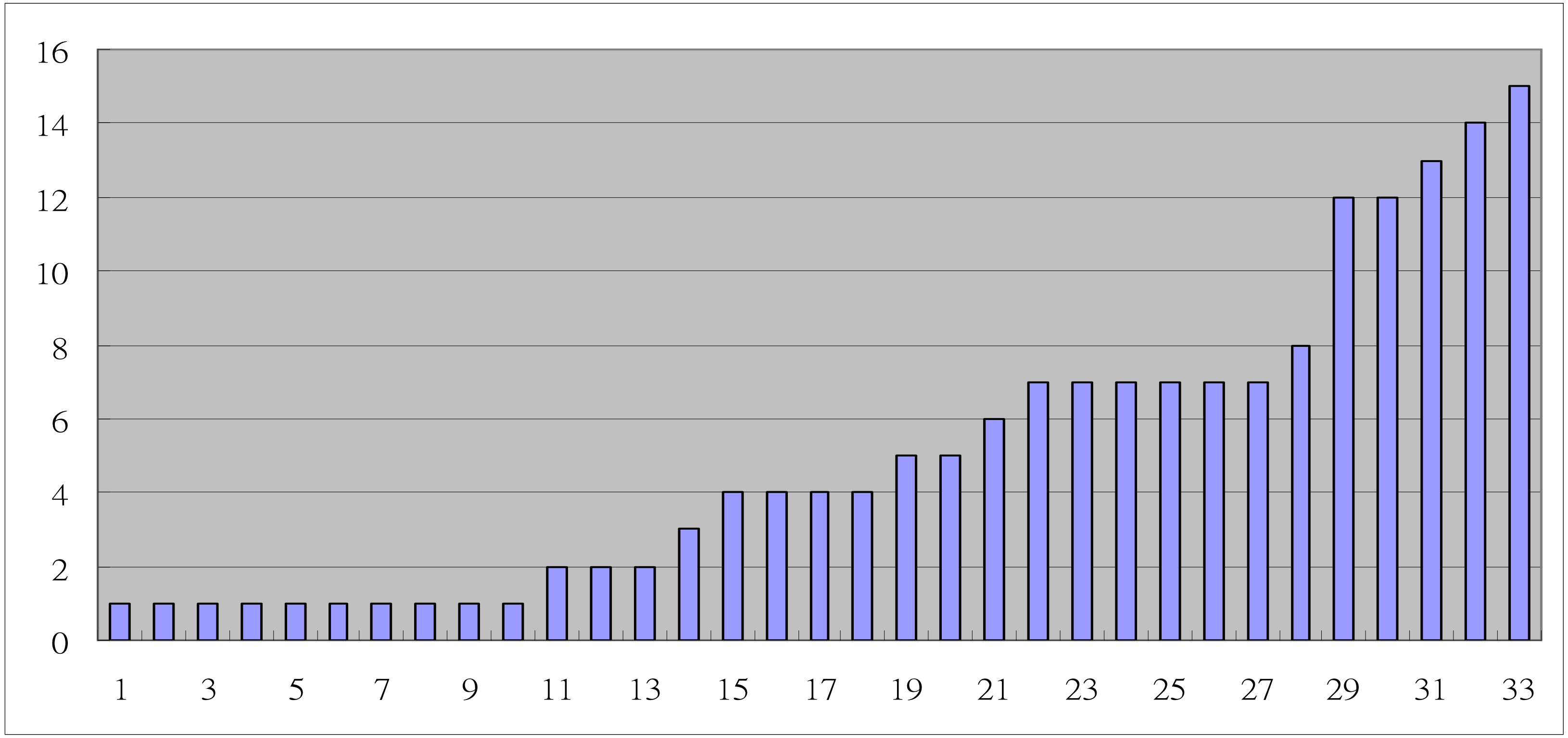
with slides by Michael Black and P. Anandan

Announcement

- The first part of project #2 (feature detection and matching) is due on Sunday, please send your source code and two images showing your results to TAs.


Project #1 artifact voting

- 55 voters; 78.6%
- Total 168 votes
- 33 of 38 artifacts got votes



Honorable mention(12): 蔡宗佑

Honorable mention(12): 李昆霖 施亮宇

Third place (13): 劉恆溫 黃惟婷

Second place (14): 方紀穎 蕭名傑

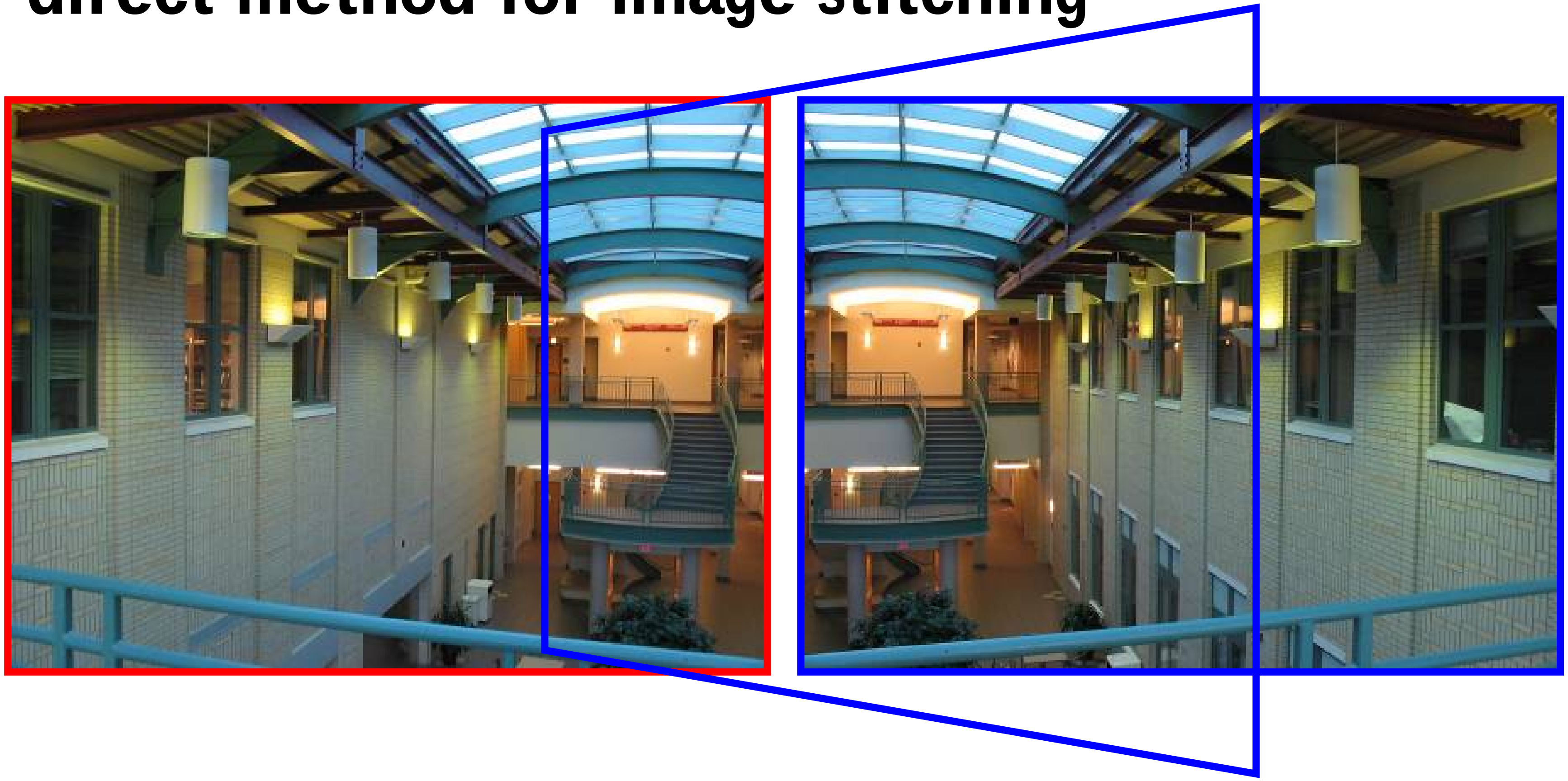
First place (15): 鄭逸廷 陳柏叡

Motion estimation

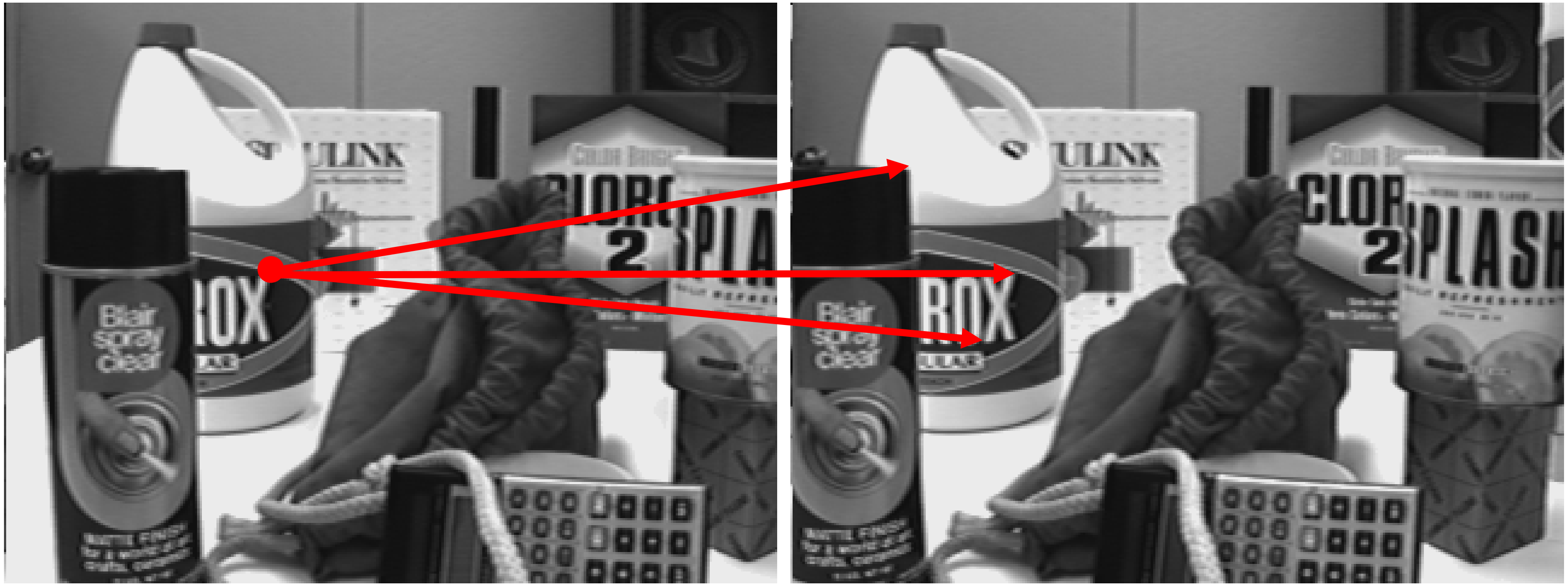
- Parametric motion (image alignment)
- Tracking
- Optical flow

Parametric motion

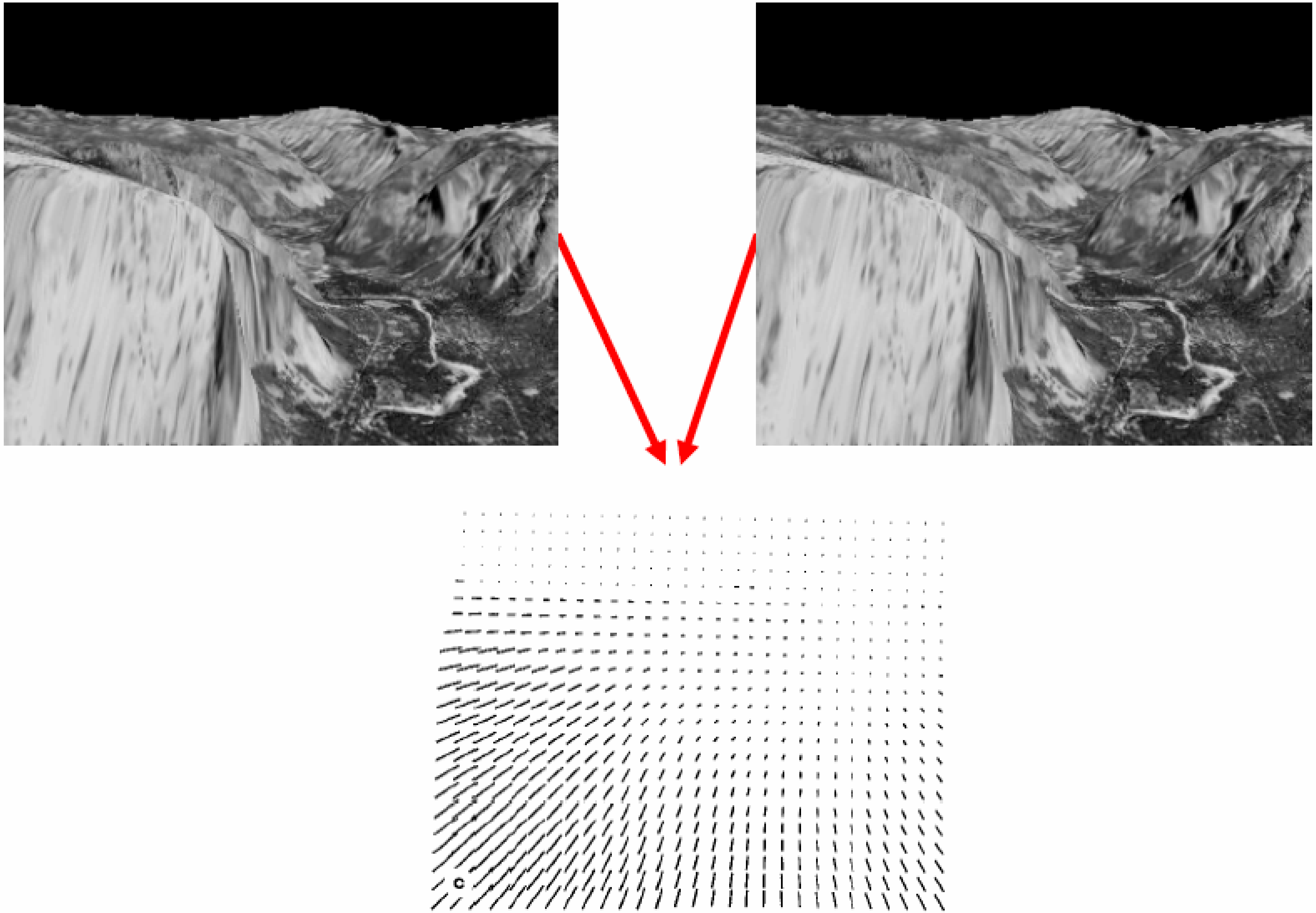
direct method for image stitching



Tracking



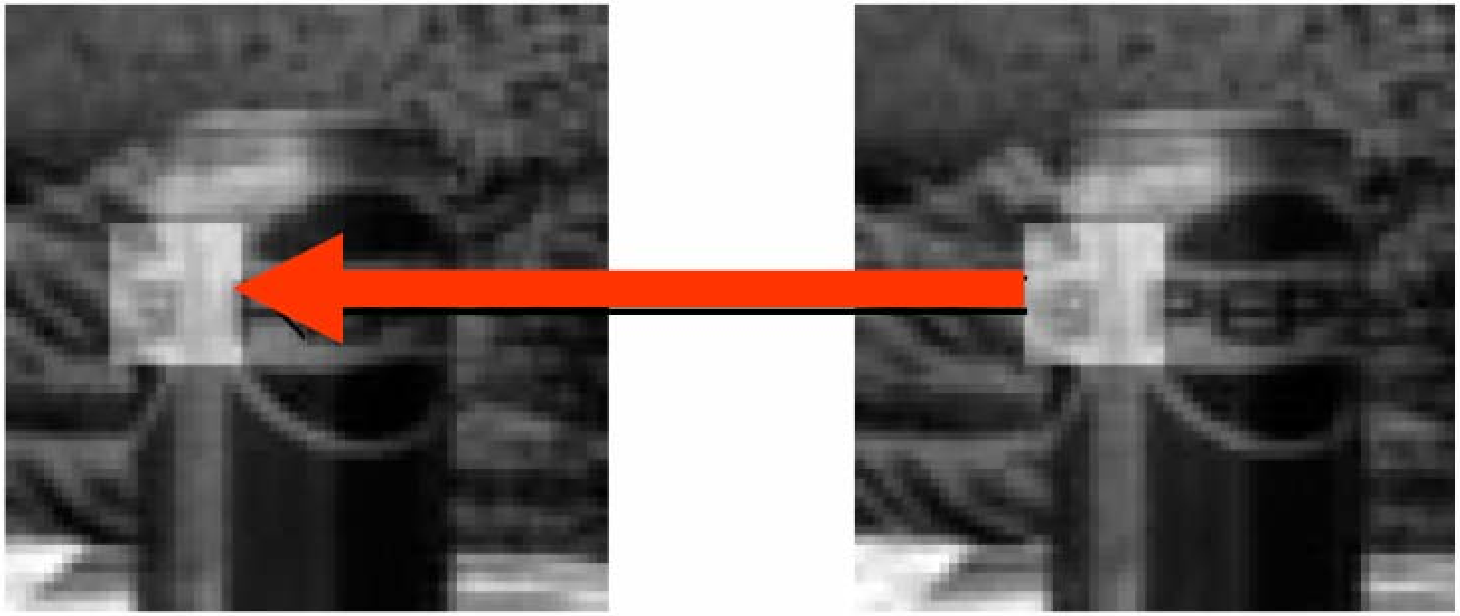
Optical flow



Three assumptions

- Brightness consistency
- Spatial coherence
- Temporal persistence

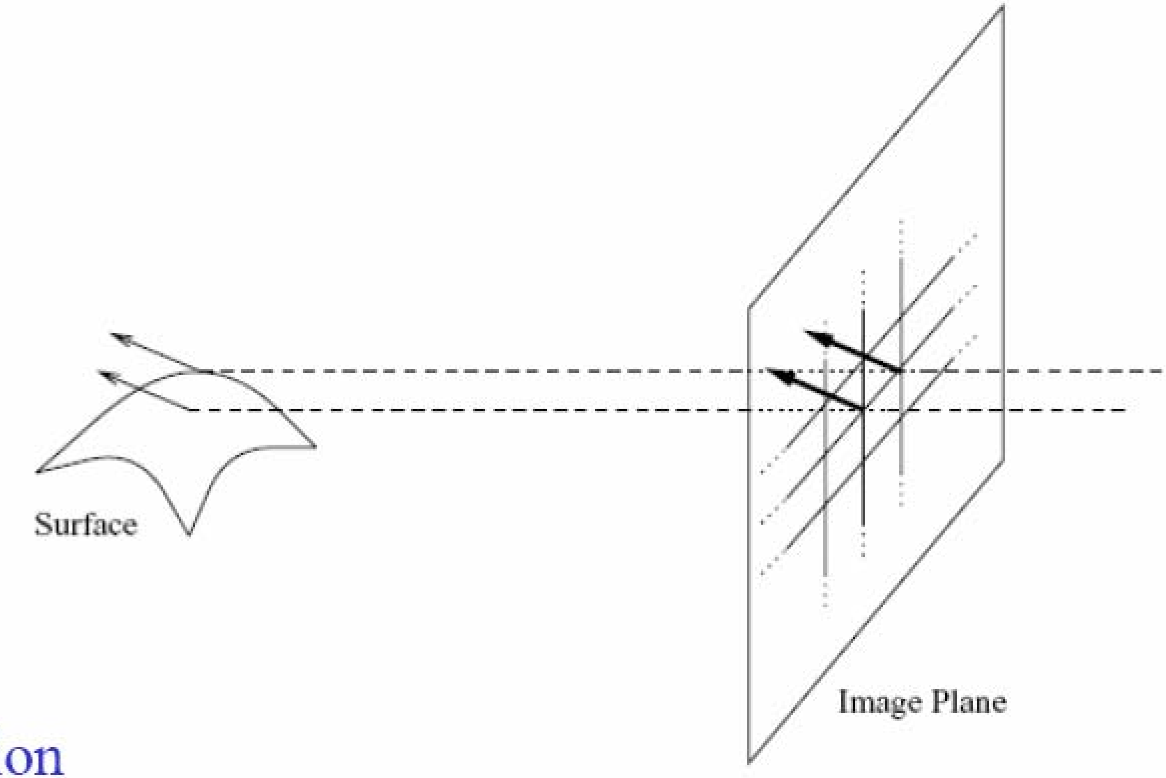
Brightness consistency



Assumption

Image measurements (e.g. brightness) in a small region remain the same although their location may change.

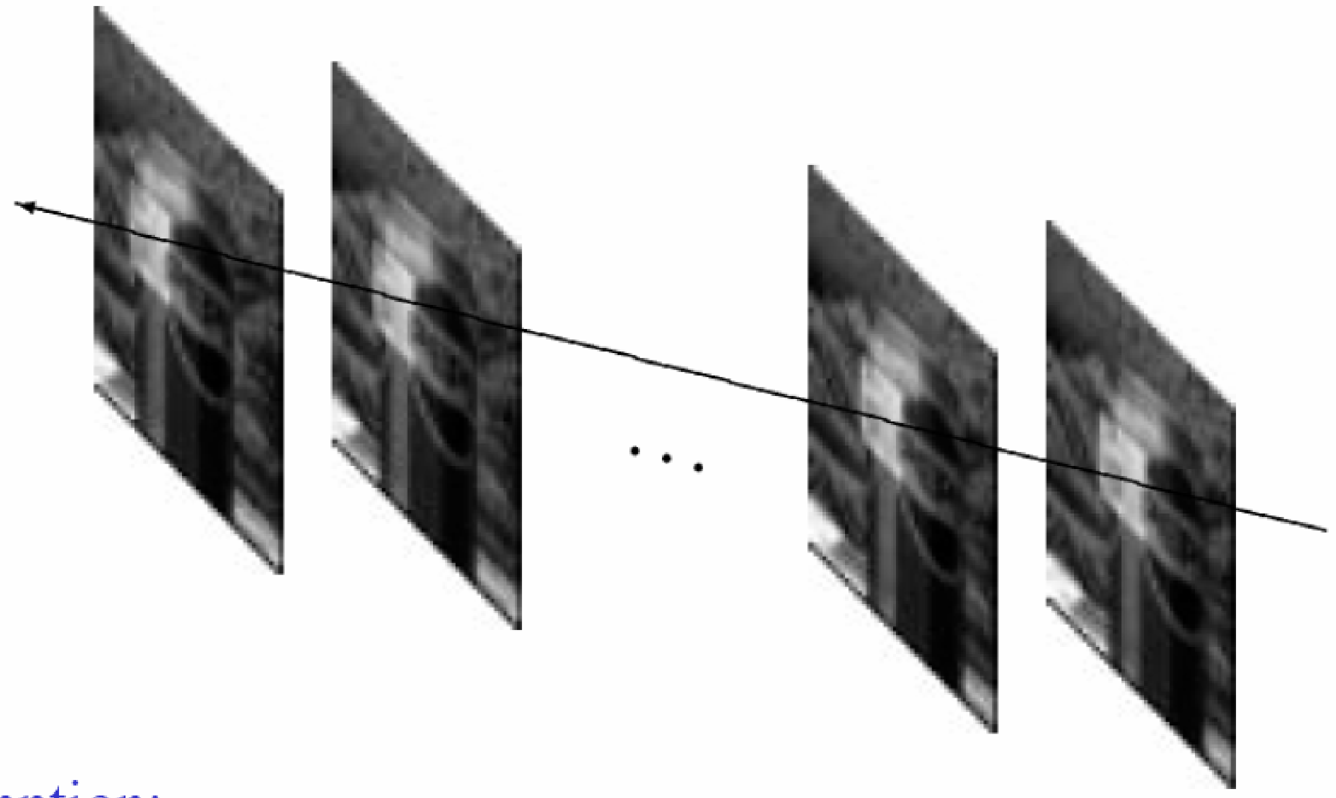
Spatial coherence



Assumption

* Neighboring points in the scene typically belong to the same surface and hence typically have similar motions.
* Since they also project to nearby points in the image, we expect spatial coherence in image flow.

Temporal persistence



Assumption:

The image motion of a surface patch changes gradually over time.

Image registration

Goal: register a template image $J(x)$ and an input image $I(x)$, where $x=(x,y)^T$. (warp I so that it matched J)

Image alignment: $I(x)$ and $J(x)$ are two images

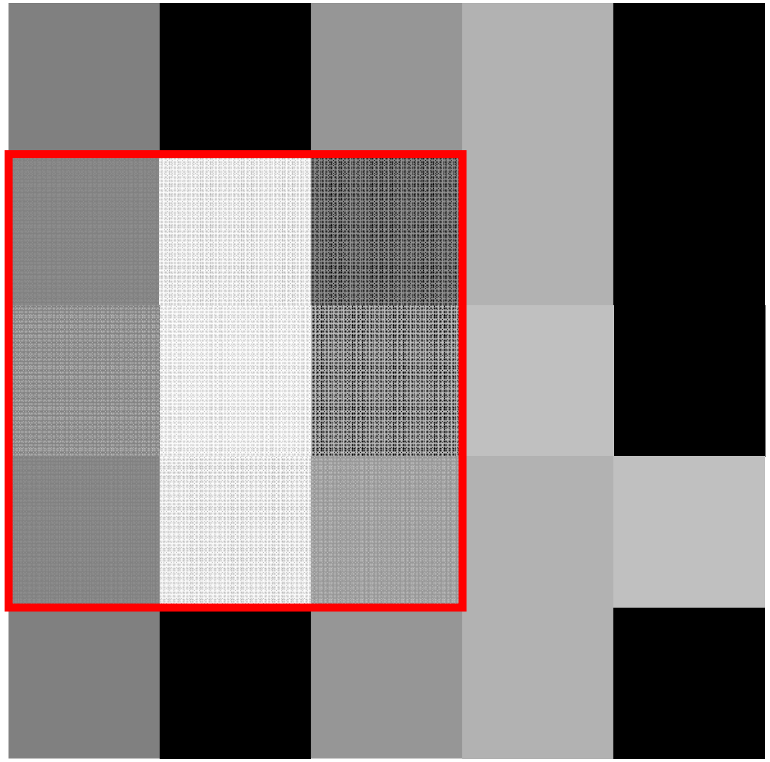
Tracking: $I(x)$ is the image at time t. $J(x)$ is a small patch around the point p in the image at $t+1$.

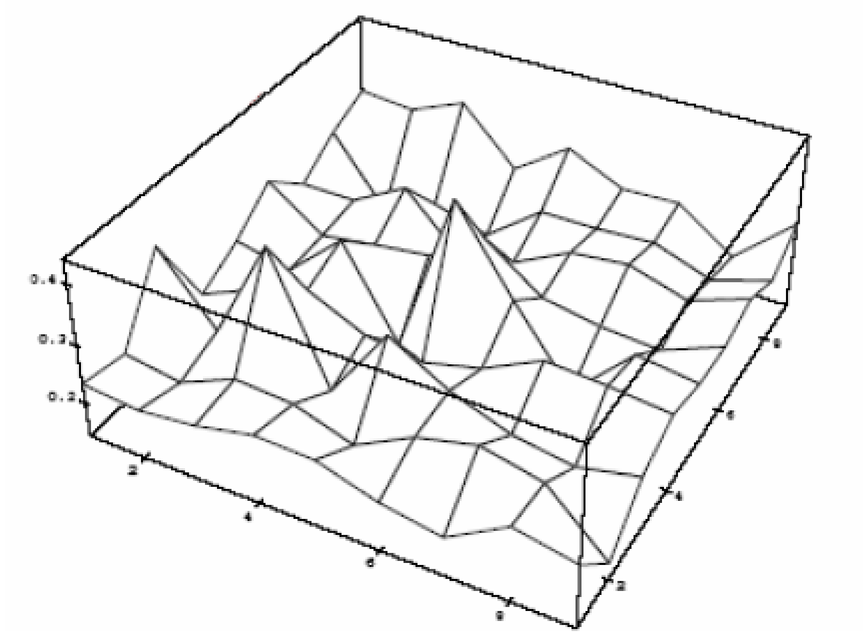
Optical flow: $I(x)$ and $J(x)$ are images of t and $t+1$.

Simple approach (for translation)

- Minimize brightness difference

$$E(u,v)=\sum_{x,y}\left(I(x+u,y+v)-J(x,y)\right)^2$$





Simple SSD algorithm

For each offset (u, v)

compute $E(u,v)$;

Choose (u, v) which minimizes $E(u,v)$;

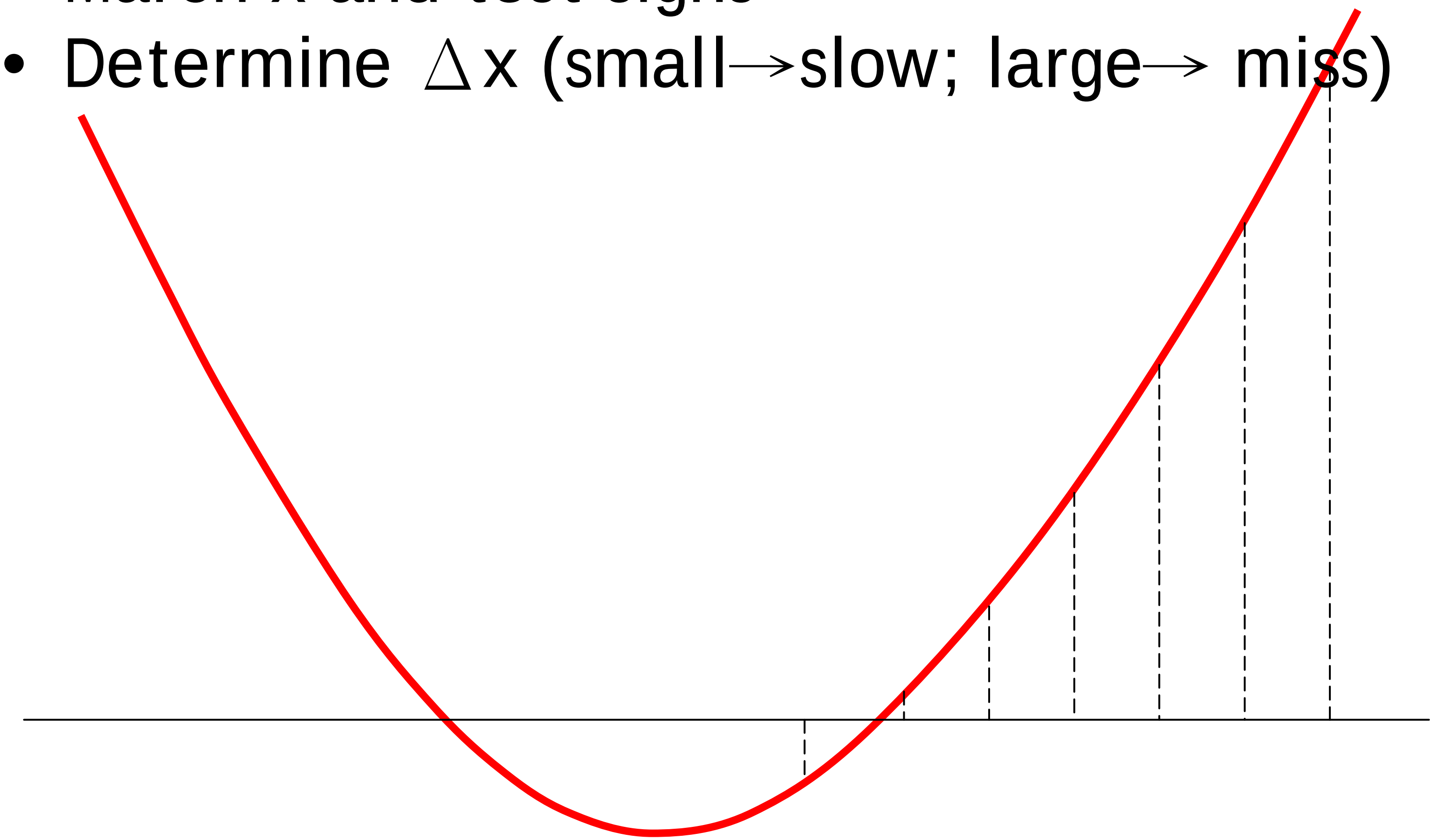
Problems:

- Not efficient
- No sub-pixel accuracy

Lucas-Kanade algorithm

Newton's method

- Root finding for $f(x)=0$
- March x and test signs
- Determine Δx (small$\rightarrow$slow; large$\rightarrow$ miss)



Newton's method

- Root finding for $f(x)=0$

Taylor's expansion:

$$f(x_0+\varepsilon)=f(x_0)+f'(x_0)\varepsilon+\frac{1}{2}f''(x_0)\varepsilon^2+\ldots$$

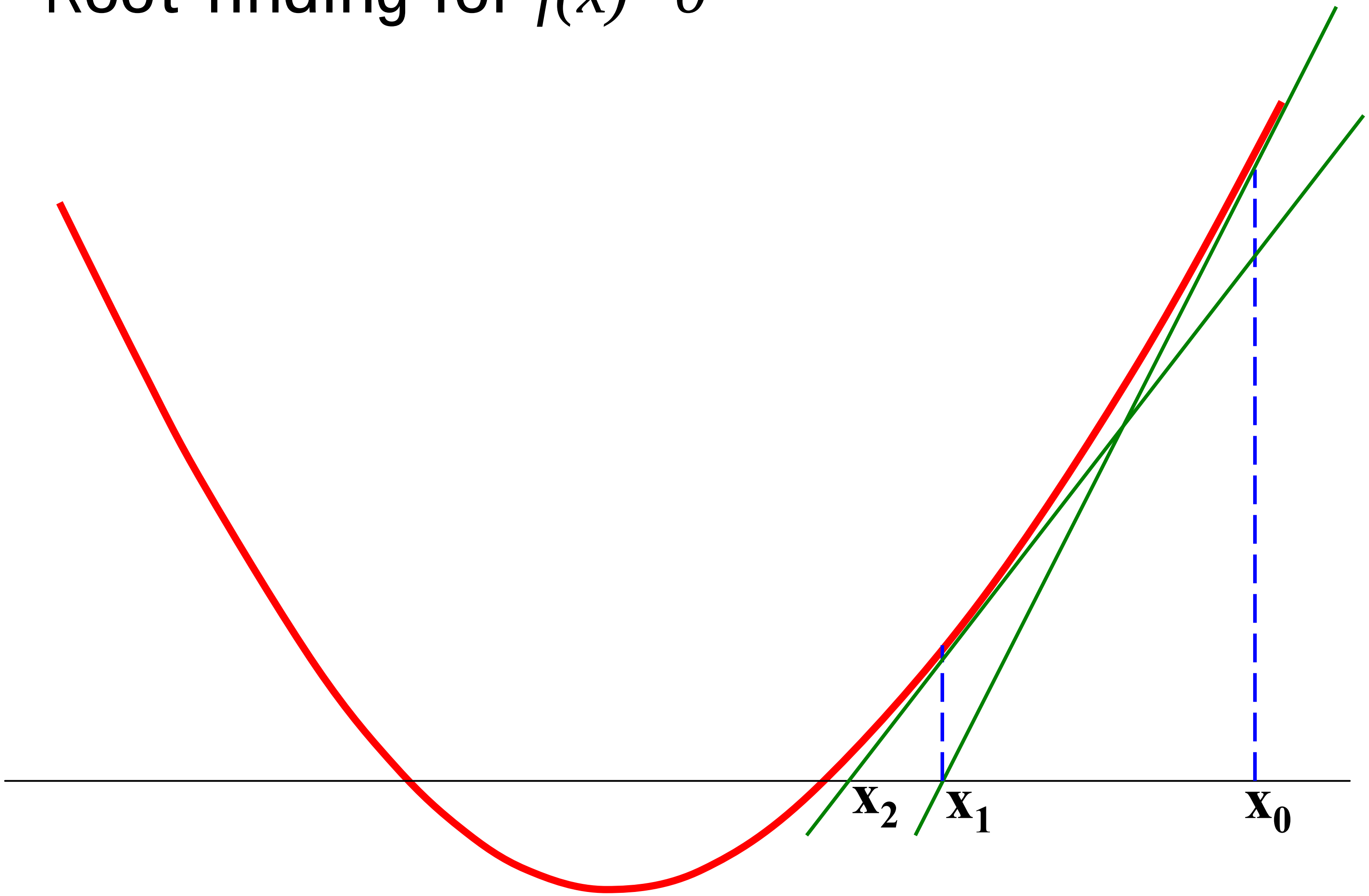
$$f(x_0+\varepsilon)\approx f(x_0)+f'(x_0)\varepsilon$$

$$\varepsilon_n=-\frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$$

Newton's method

- Root finding for $f(x)=0$



Newton's method

pick up $\mathbf{x}=\mathbf{x_0}$

iterate

compute $\Delta x = -\dfrac{f(x)}{f'(x)}$

update $\mathbf{x}$ by $\mathbf{x}+\Delta\,\mathbf{x}$

until converge

Finding root is useful for optimization because

Minimize $g(x) \rightarrow$ find root for $f(x)=g'(x)=0$

Lucas-Kanade algorithm

$$E(u,v) = \sum_{x,y} \left(I(x+u, y+v) - J(x,y) \right)^2$$

$$\boxed{I(x+u, y+v) \approx I(x,y) + uI_x + vI_y}$$

$$= \sum_{x,y} \left(I(x,y) - J(x,y) + uI_x + vI_y \right)^2$$

$$0 = \frac{\partial E}{\partial u} = \sum_{x,y} 2I_x \left(I(x,y) - J(x,y) + uI_x + vI_y \right)$$

$$0 = \frac{\partial E}{\partial v} = \sum_{x,y} 2I_y \left(I(x,y) - J(x,y) + uI_x + vI_y \right)$$

Lucas-Kanade algorithm

$$0=\frac{\partial E}{\partial u}=\sum_{x,y}2I_x\left(I(x,y)-J(x,y)+uI_x+vI_y\right)$$

$$0=\frac{\partial E}{\partial v}=\sum_{x,y}2I_y\left(I(x,y)-J(x,y)+uI_x+vI_y\right)$$

$$\Rightarrow\begin{cases}\sum_{x,y}I_x^2u+\sum_{x,y}I_xI_yv=\sum_{x,y}I_x\left(J(x,y)-I(x,y)\right)\\ \sum_{x,y}I_xI_yu+\sum_{x,y}I_y^2v=\sum_{x,y}I_y\left(J(x,y)-I(x,y)\right)\end{cases}$$

$$\Rightarrow\begin{bmatrix}\sum_{x,y}I_x^2 & \sum_{x,y}I_xI_y\\ \sum_{x,y}I_xI_y & \sum_{x,y}I_y^2\end{bmatrix}\begin{bmatrix}u\\ v\end{bmatrix}=\begin{bmatrix}\sum_{x,y}I_x\left(J(x,y)-I(x,y)\right)\\ \sum_{x,y}I_y\left(J(x,y)-I(x,y)\right)\end{bmatrix}$$

Lucas-Kanade algorithm

DigiVFX

iterate

shift I(x,y) with (u,v)

compute gradient image I_x, I_y

compute error image J(x,y)-I(x,y)

compute Hessian matrix

solve the linear system

(u,v)=(u,v)+(Δu,Δv)

until converge

$$\begin{bmatrix} \sum_{x,y} I_x^2 & \sum_{x,y} I_x I_y \\ \sum_{x,y} I_x I_y & \sum_{x,y} I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \sum_{x,y} I_x \left(J(x,y) - I(x,y) \right) \\ \sum_{x,y} I_y \left(J(x,y) - I(x,y) \right) \end{bmatrix}$$

Parametric model

$$E(u,v)=\sum_{x,y}\big(I(x+u,y+v)-J(x,y)\big)^2$$

$$E(\mathbf{p})=\sum_{\mathbf{x}}\big(I(\mathbf{W}(\mathbf{x};\mathbf{p}))-J(\mathbf{x})\big)^2$$

Our goal is to find **p** to minimize **E(p)**

translation $\mathbf{W}(\mathbf{x};\mathbf{p})=\begin{pmatrix}x+d_x\\ y+d_y\end{pmatrix}, p=(d_x,d_y)^T$

affine $\mathbf{W}(\mathbf{x};\mathbf{p})=\mathbf{A}\mathbf{x}+\mathbf{d}=\begin{pmatrix}1+d_{xx} & d_{xy} & d_x\\ d_{yx} & 1+d_{yy} & d_y\end{pmatrix}\begin{pmatrix}x\\ y\\ 1\end{pmatrix},$

$$p=(d_{xx},d_{xy},d_{yx},d_{yy},d_x,d_y)^T$$

Parametric model

minimize $\sum_{\mathbf{x}} \left(I(\mathbf{W}(\mathbf{x};\mathbf{p}+\Delta\mathbf{p})) - J(\mathbf{x}) \right)^2$

with respect to $\Delta\mathbf{p}$

$$\mathbf{W}(\mathbf{x};\mathbf{p}+\Delta\mathbf{p}) \approx \mathbf{W}(\mathbf{x};\mathbf{p}) + \frac{\partial \mathbf{W}}{\partial \mathbf{p}}\Delta\mathbf{p}$$

$$I(\mathbf{W}(\mathbf{x};\mathbf{p}+\Delta\mathbf{p})) \approx I(\mathbf{W}(\mathbf{x};\mathbf{p}) + \frac{\partial \mathbf{W}}{\partial \mathbf{p}}\Delta\mathbf{p})$$

$$\approx I(\mathbf{W}(\mathbf{x};\mathbf{p})) + \frac{\partial I}{\partial \mathbf{x}}\frac{\partial \mathbf{W}}{\partial \mathbf{p}}\Delta\mathbf{p}$$

$\Rightarrow$ minimize $\sum_{\mathbf{x}} \left(I(\mathbf{W}(\mathbf{x};\mathbf{p})) + \nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}}\Delta\mathbf{p} - J(\mathbf{x}) \right)^2$

Parametric model

warped image

target image

image gradient

$$\sum_{\mathbf{x}} \left(\boxed{I(\mathbf{W}(\mathbf{x};\mathbf{p}))} + \boxed{\nabla I} \boxed{\frac{\partial \mathbf{W}}{\partial \mathbf{p}}} \Delta \mathbf{p} - \boxed{J(\mathbf{x})} \right)^2$$

Jacobian of the warp

$$\frac{\partial \mathbf{W}}{\partial \mathbf{p}} = \begin{pmatrix} \frac{\partial \mathbf{W}_x}{\partial \mathbf{p}} \\ \frac{\partial \mathbf{W}_y}{\partial \mathbf{p}} \end{pmatrix} = \begin{pmatrix} \frac{\partial \mathbf{W}_x}{\partial \mathbf{p}_1} & \frac{\partial \mathbf{W}_x}{\partial \mathbf{p}_2} & \cdots & \frac{\partial \mathbf{W}_x}{\partial \mathbf{p}_n} \\ \frac{\partial \mathbf{W}_y}{\partial \mathbf{p}_1} & \frac{\partial \mathbf{W}_y}{\partial \mathbf{p}_2} & \cdots & \frac{\partial \mathbf{W}_y}{\partial \mathbf{p}_n} \end{pmatrix}$$

Jacobian matrix

- The Jacobian matrix is the matrix of all first-order partial derivatives of a vector-valued function.

$$F(x_1, x_2, \ldots x_n) \qquad F : \mathbf{R}^n \rightarrow \mathbf{R}^m$$

$$= (f_1(x_1, x_2, \ldots x_n), f_2(x_1, x_2, \ldots x_n), \ldots f_m(x_1, x_2, \ldots x_n))$$

$$\begin{array}{c} J_F(x_1, x_2, \ldots x_n) \\ \text{or} \\ \dfrac{\partial(f_1, f_2, \ldots f_m)}{\partial(x_1, x_2, \ldots x_n)} \end{array} = \begin{bmatrix} \dfrac{\partial f_1}{\partial x_1} & \cdots & \dfrac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial f_m}{\partial x_1} & \cdots & \dfrac{\partial f_m}{\partial x_n} \end{bmatrix}$$

$$F(\mathbf{x} + \Delta\mathbf{x}) \approx F(\mathbf{x}) + J_F(\mathbf{x})\Delta\mathbf{x}$$

Jacobian matrix

$$F: \mathbf{R} \times [0, \pi] \times [0, 2\pi] \rightarrow \mathbf{R}^3$$

$$F(r, \phi, \theta) = (t, u, v)$$

$$t = r \sin\phi \cos\theta$$

$$u = r \sin\phi \sin\theta$$

$$v = r \cos\phi$$

$$J_F(r, \phi, \theta) = \begin{bmatrix} \frac{\partial t}{\partial r} & \frac{\partial t}{\partial \phi} & \frac{\partial t}{\partial \theta} \\ \frac{\partial u}{\partial r} & \frac{\partial u}{\partial \phi} & \frac{\partial u}{\partial \theta} \\ \frac{\partial v}{\partial r} & \frac{\partial v}{\partial \phi} & \frac{\partial v}{\partial \theta} \end{bmatrix}$$

$$= \begin{bmatrix} \sin\phi\cos\theta & r\cos\phi\cos\theta & -r\sin\phi\sin\theta \\ \sin\phi\sin\theta & r\cos\phi\sin\theta & r\sin\phi\cos\theta \\ \cos\phi & -r\sin\phi & 0 \end{bmatrix}$$

Parametric model

warped image

target image

image gradient

$$\sum_{\mathbf{x}} \left(\boxed{I(\mathbf{W}(\mathbf{x};\mathbf{p}))} + \boxed{\nabla I} \boxed{\frac{\partial \mathbf{W}}{\partial \mathbf{p}}} \Delta \mathbf{p} - \boxed{J(\mathbf{x})} \right)^2$$

Jacobian of the warp

$$\frac{\partial \mathbf{W}}{\partial \mathbf{p}} = \begin{pmatrix} \frac{\partial \mathbf{W}_x}{\partial \mathbf{p}} \\ \frac{\partial \mathbf{W}_y}{\partial \mathbf{p}} \end{pmatrix} = \begin{pmatrix} \frac{\partial \mathbf{W}_x}{\partial \mathbf{p}_1} & \frac{\partial \mathbf{W}_x}{\partial \mathbf{p}_2} & \cdots & \frac{\partial \mathbf{W}_x}{\partial \mathbf{p}_n} \\ \frac{\partial \mathbf{W}_y}{\partial \mathbf{p}_1} & \frac{\partial \mathbf{W}_y}{\partial \mathbf{p}_2} & \cdots & \frac{\partial \mathbf{W}_y}{\partial \mathbf{p}_n} \end{pmatrix}$$

Jacobian of the warp

$$\frac{\partial \mathbf{W}}{\partial \mathbf{p}} = \begin{pmatrix} \dfrac{\partial \mathbf{W}_x}{\partial \mathbf{p}} \\ \dfrac{\partial \mathbf{W}_y}{\partial \mathbf{p}} \end{pmatrix} = \begin{pmatrix} \dfrac{\partial \mathbf{W}_x}{\partial \mathbf{p}_1} & \dfrac{\partial \mathbf{W}_x}{\partial \mathbf{p}_2} & \cdots & \dfrac{\partial \mathbf{W}_x}{\partial \mathbf{p}_n} \\ \dfrac{\partial \mathbf{W}_y}{\partial \mathbf{p}_1} & \dfrac{\partial \mathbf{W}_y}{\partial \mathbf{p}_2} & \cdots & \dfrac{\partial \mathbf{W}_y}{\partial \mathbf{p}_n} \end{pmatrix}$$

For example, for affine

$$\mathbf{W}(\mathbf{x};\mathbf{p}) = \begin{pmatrix} 1+d_{xx} & d_{xy} & d_x \\ d_{yx} & 1+d_{yy} & d_y \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} (1+d_{xx})x + d_{xy}y + d_x \\ d_{yx}x + (1+d_{yy})y + d_y \end{pmatrix}$$

$$\Rightarrow \quad \frac{\partial \mathbf{W}}{\partial \mathbf{p}} = \begin{pmatrix} x & 0 & y & 0 & 1 & 0 \\ 0 & x & 0 & y & 0 & 1 \end{pmatrix}$$

Parametric model

$$\arg\min_{\Delta\mathbf{p}} \sum_{\mathbf{x}} \left(I(\mathbf{W}(\mathbf{x};\mathbf{p})) + \nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \Delta\mathbf{p} - J(\mathbf{x}) \right)^2$$

$$0 = \sum_{\mathbf{x}} \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^T \left[I(\mathbf{W}(\mathbf{x};\mathbf{p})) + \nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \Delta\mathbf{p} - J(\mathbf{x}) \right]$$

$$\Delta\mathbf{p} = \mathbf{H}^{-1} \sum_{\mathbf{x}} \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^T \left[J(\mathbf{x}) - I(\mathbf{W}(\mathbf{x};\mathbf{p})) \right]$$

Hessian $$\mathbf{H} = \sum_{\mathbf{x}} \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^T \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]$$

Lucas-Kanade algorithm

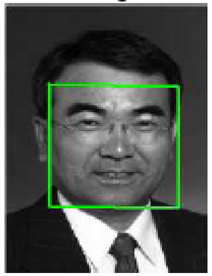
iterate

1) warp I with W(x;p)
2) compute error image J(x,y)-I(W(x,p))
3) compute gradient image ∇I with W(x,p)
4) evaluate Jacobian $\frac{\partial \mathbf{W}}{\partial \mathbf{p}}$ at (x;p)
5) compute $\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}}$
6) compute Hessian
7) compute $\sum_{\mathbf{x}} \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^T \left[J(\mathbf{x}) - I(\mathbf{W}(\mathbf{x};\mathbf{p})) \right]$
8) solve $\Delta\mathbf{p}$
9) update p by p+$\Delta\mathbf{p}$

until converge

$$\Delta\mathbf{p} = \mathbf{H}^{-1} \sum_{\mathbf{x}} \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^T \left[J(\mathbf{x}) - I(\mathbf{W}(\mathbf{x};\mathbf{p})) \right]$$

$T(\mathbf{x})$



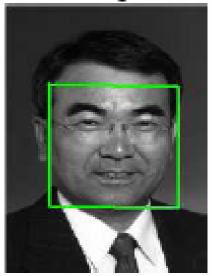
$I(\mathbf{x})$

$$\Delta \mathbf{p} = \mathbf{H}^{-1} \sum_{\mathbf{x}} \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^T \left[J(\mathbf{x}) - I(\mathbf{W}(\mathbf{x};\mathbf{p})) \right]$$

Template

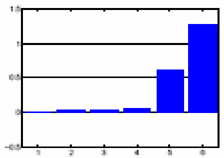
$T(\mathbf{x})$

Image



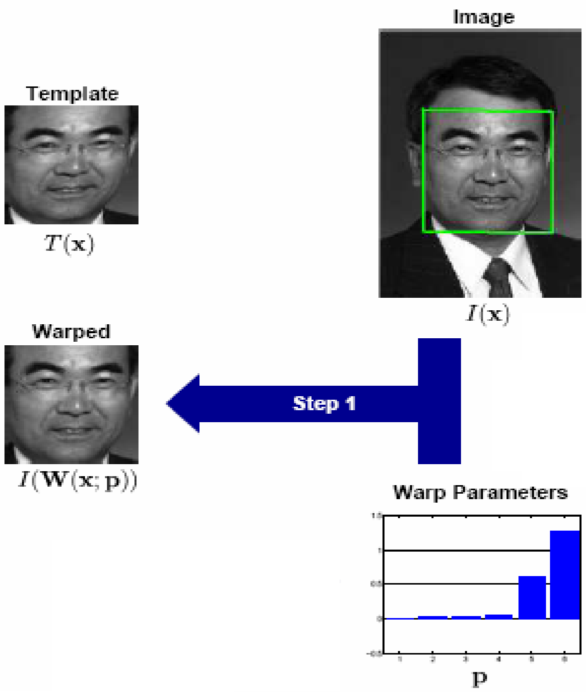
$I(\mathbf{x})$

Warp Parameters

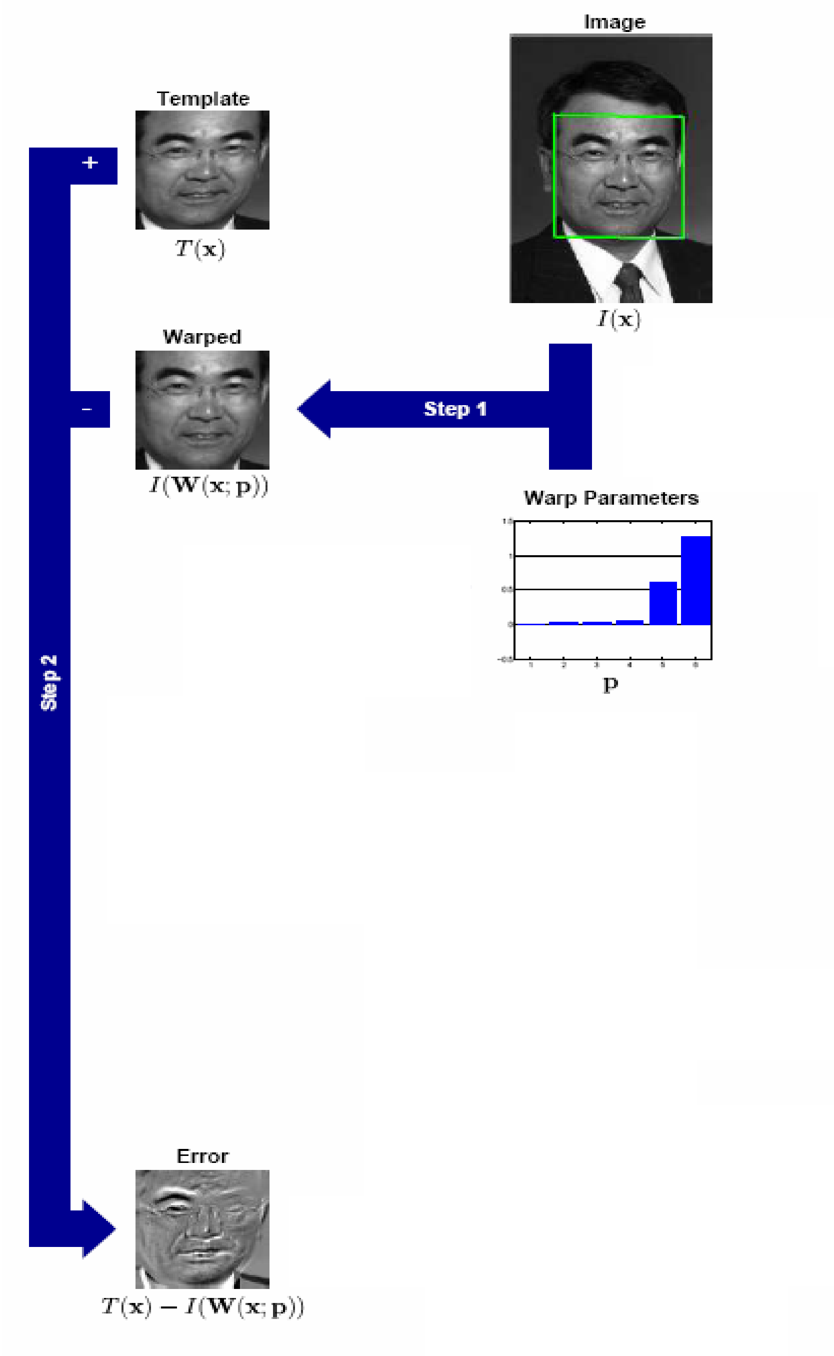


$\mathbf{p}$

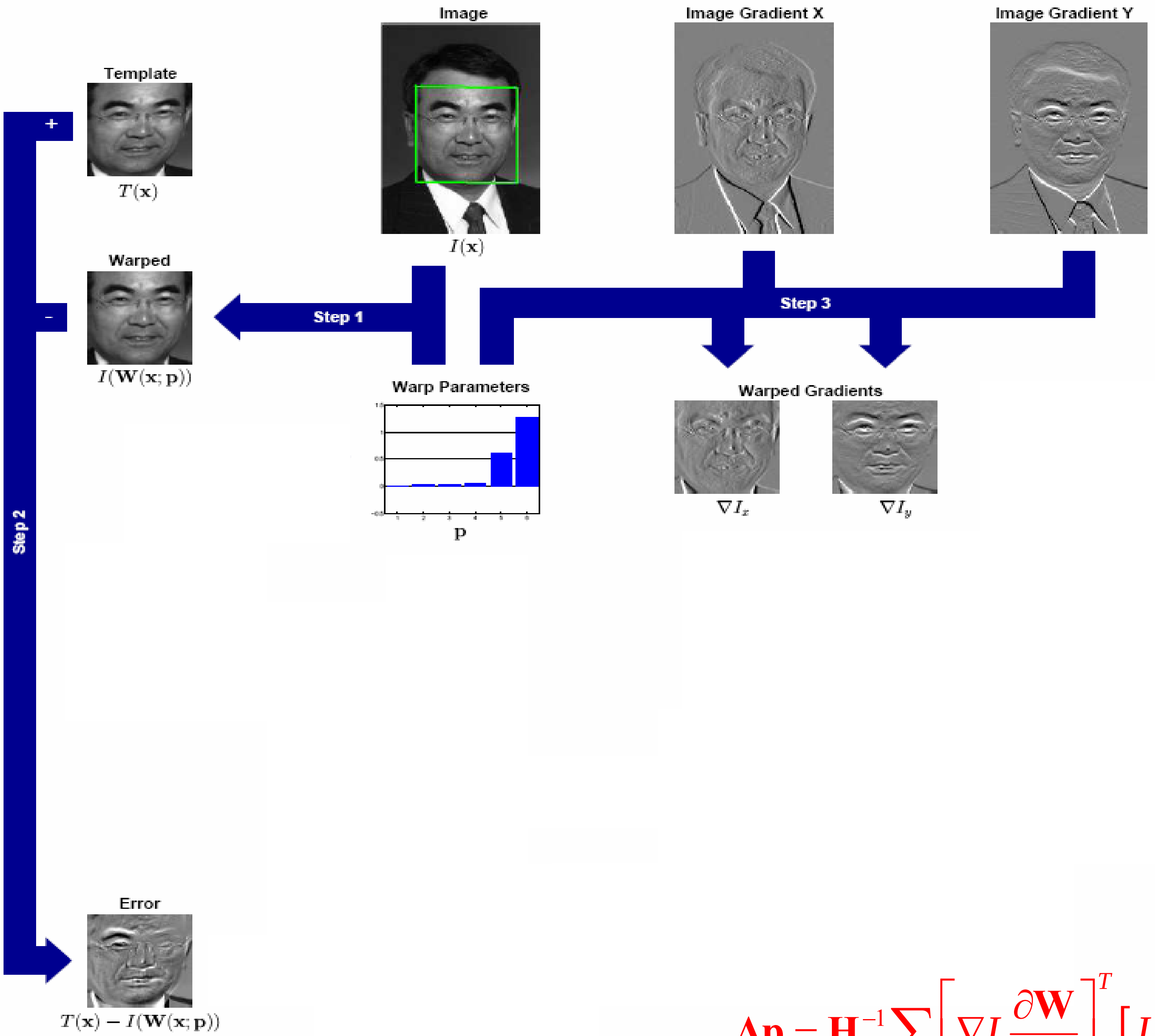
$$\Delta \mathbf{p} = \mathbf{H}^{-1} \sum_{\mathbf{x}} \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^T \left[J(\mathbf{x}) - I(\mathbf{W}(\mathbf{x};\mathbf{p})) \right]$$



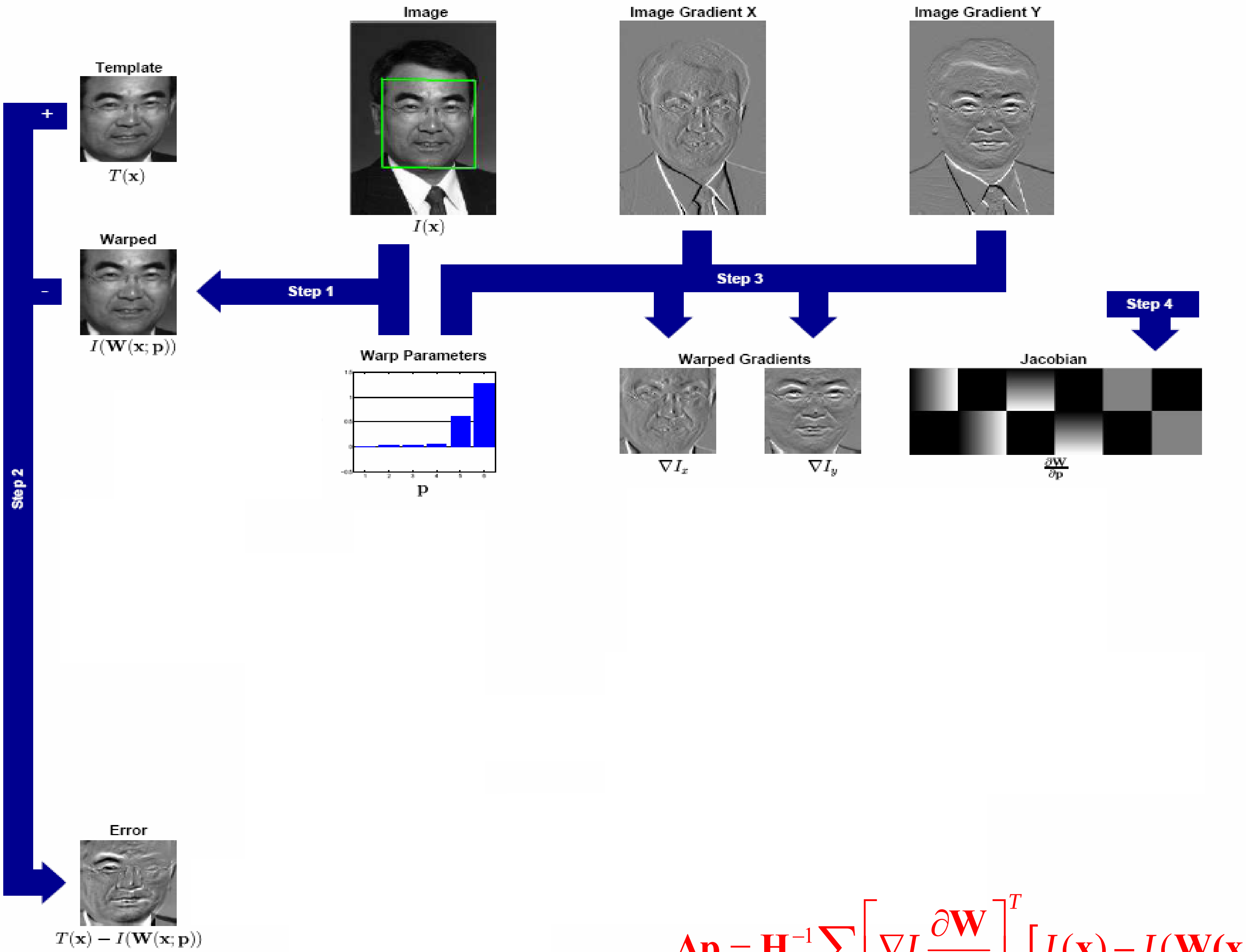
$$\Delta \mathbf{p} = \mathbf{H}^{-1} \sum_{\mathbf{x}} \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^T \left[J(\mathbf{x}) - I(\mathbf{W}(\mathbf{x};\mathbf{p})) \right]$$



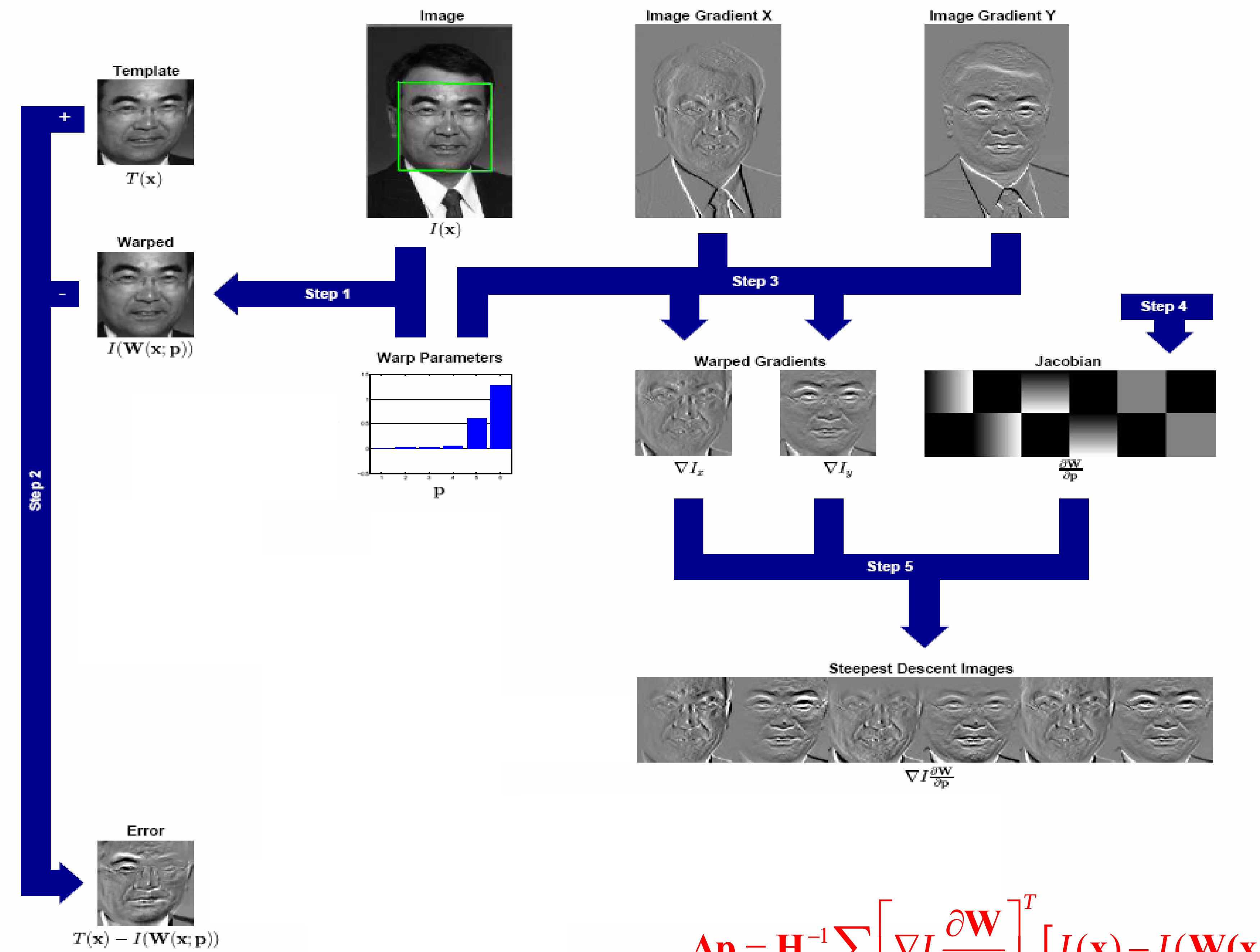
$$\Delta\mathbf{p} = \mathbf{H}^{-1} \sum_{\mathbf{x}} \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^T \left[J(\mathbf{x}) - I(\mathbf{W}(\mathbf{x};\mathbf{p})) \right]$$



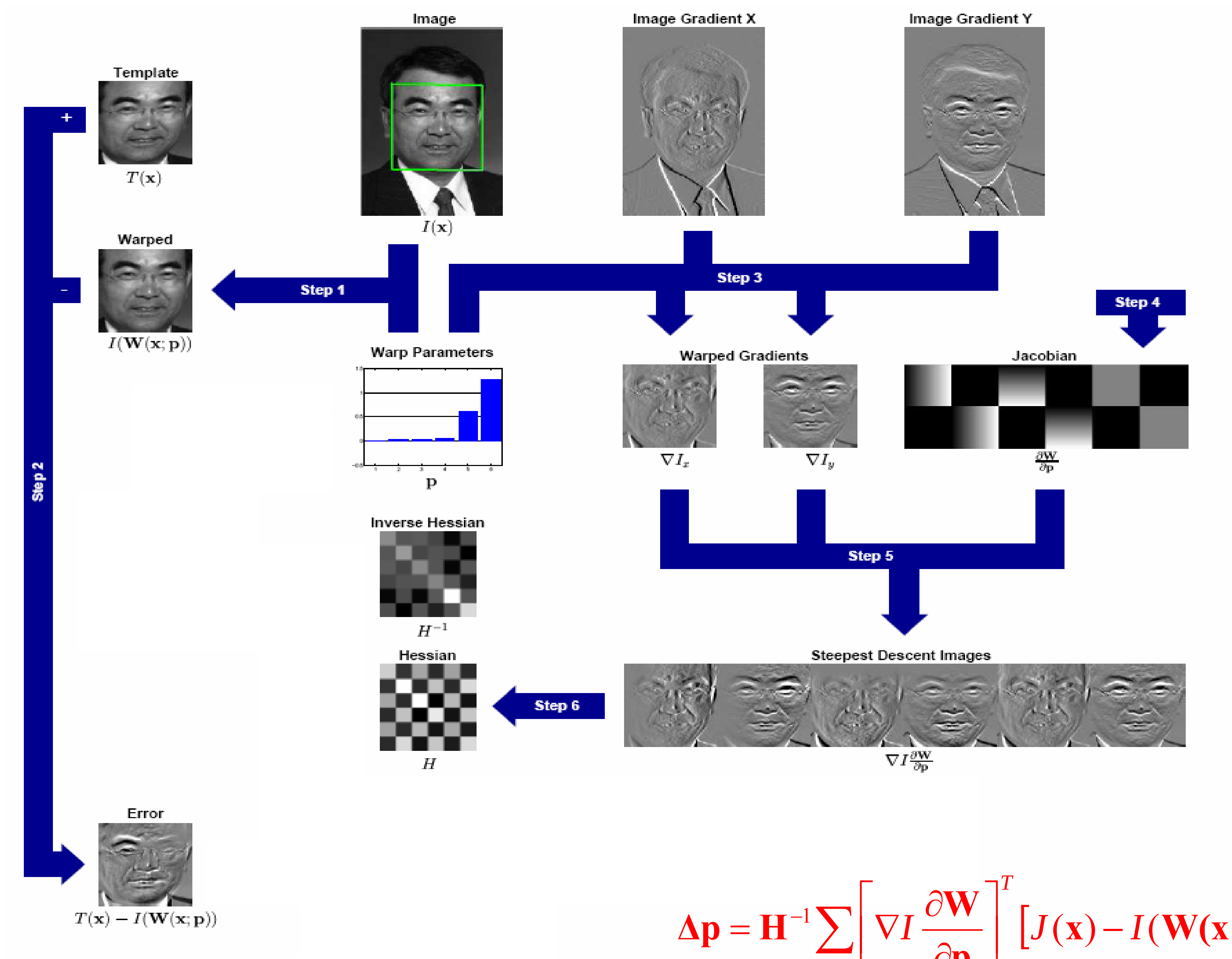
$$\Delta\mathbf{p} = \mathbf{H}^{-1}\sum_{\mathbf{x}}\left[\nabla I\frac{\partial\mathbf{W}}{\partial\mathbf{p}}\right]^T\left[J(\mathbf{x}) - I(\mathbf{W}(\mathbf{x};\mathbf{p}))\right]$$



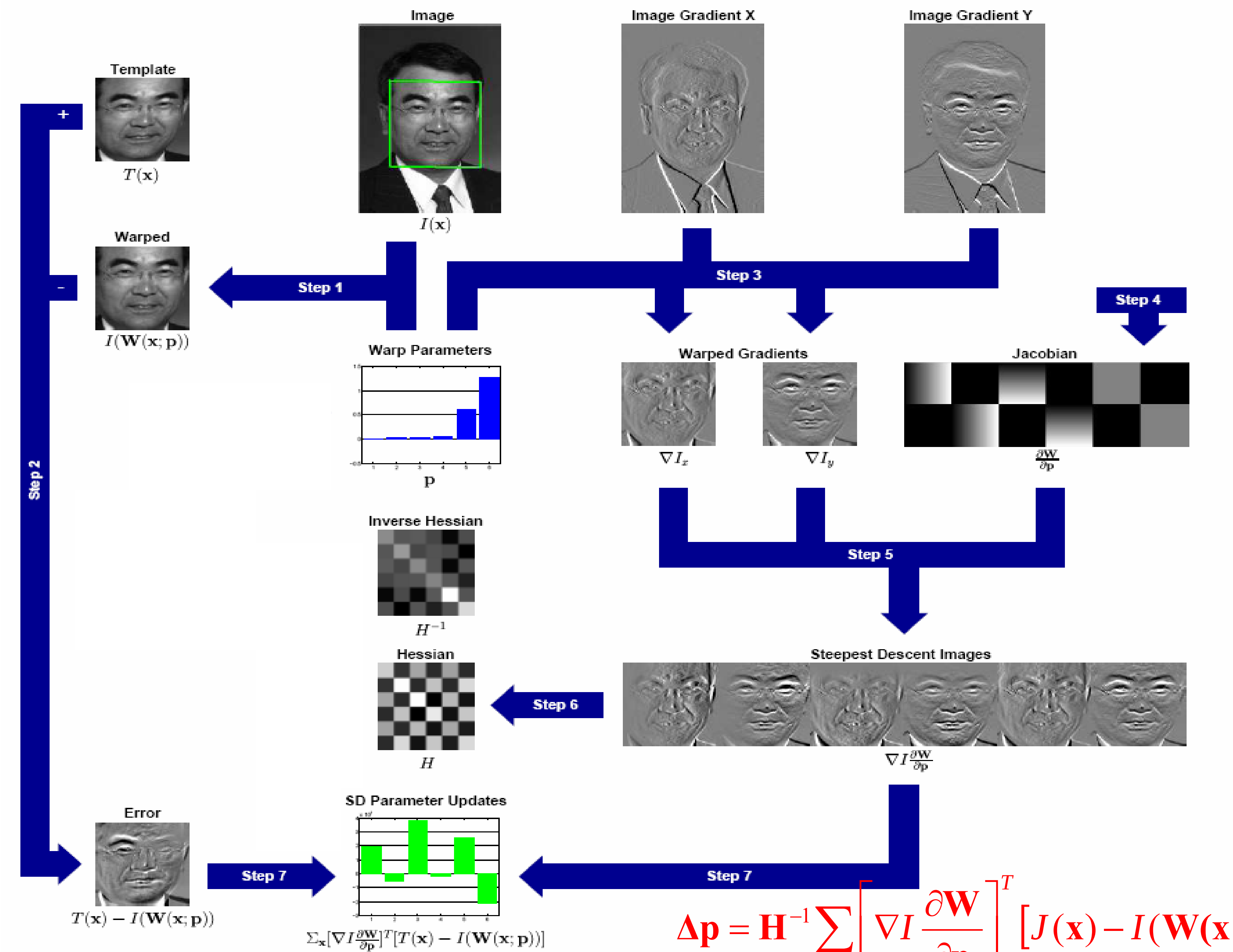
$$\Delta \mathbf{p} = \mathbf{H}^{-1} \sum_{\mathbf{x}} \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^T \left[J(\mathbf{x}) - I(\mathbf{W}(\mathbf{x};\mathbf{p})) \right]$$



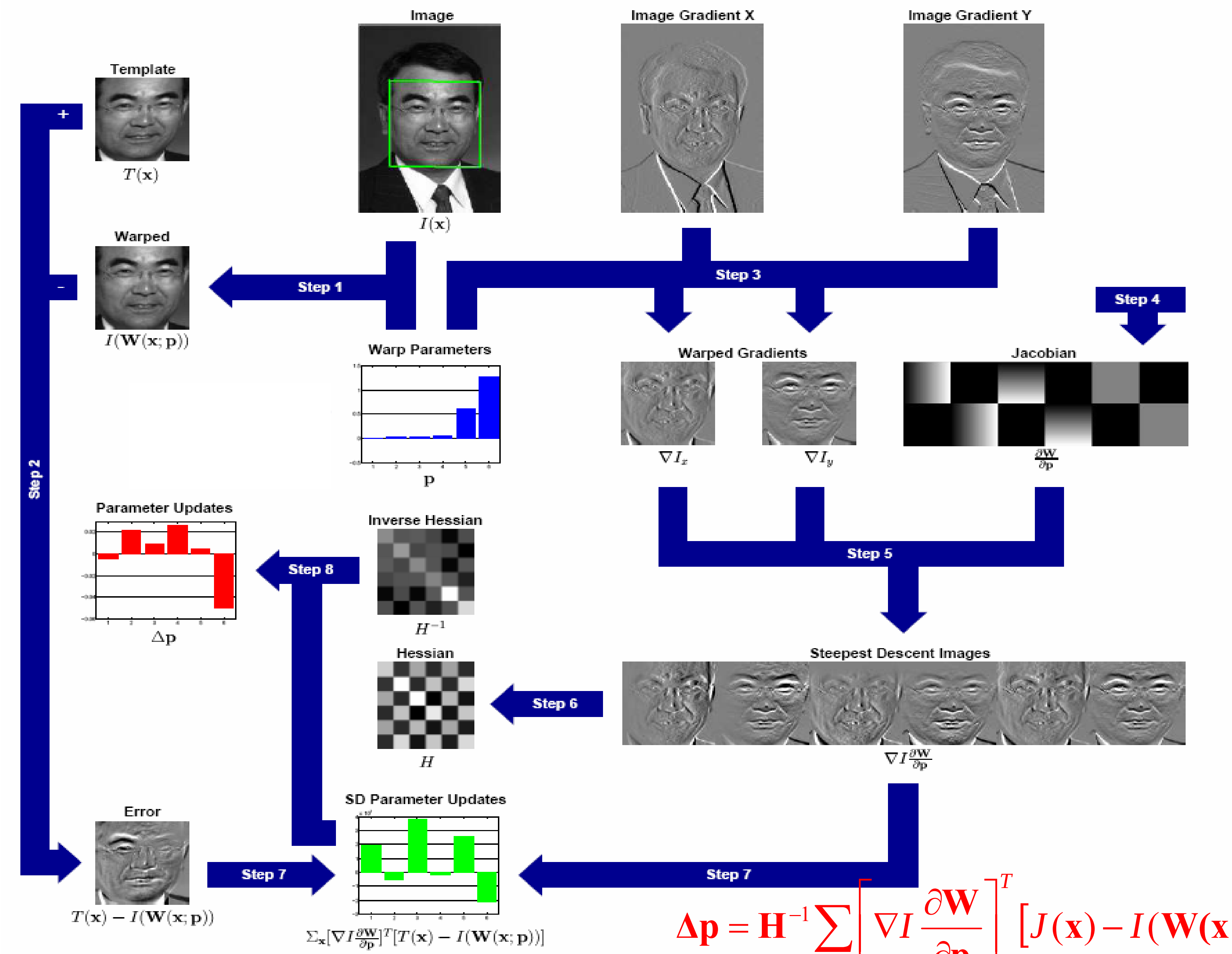
$$\Delta \mathbf{p} = \mathbf{H}^{-1} \sum_{\mathbf{x}} \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^T \left[J(\mathbf{x}) - I(\mathbf{W}(\mathbf{x};\mathbf{p})) \right]$$



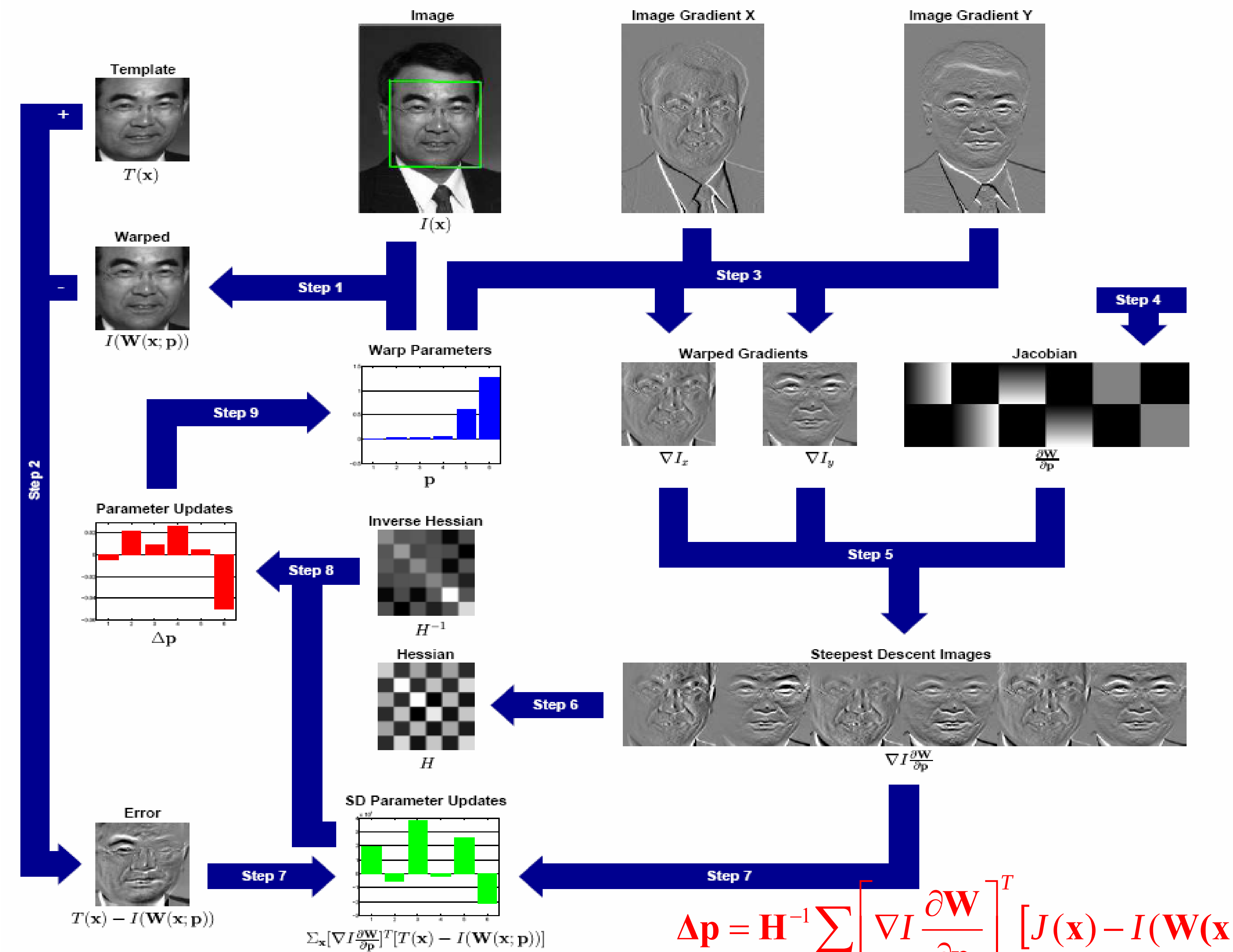
$$\Delta \mathbf{p} = \mathbf{H}^{-1} \sum_{\mathbf{x}} \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^T \left[J(\mathbf{x}) - I(\mathbf{W}(\mathbf{x};\mathbf{p})) \right]$$



$$\Delta \mathbf{p} = \mathbf{H}^{-1} \sum_{\mathbf{x}} \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^T \left[J(\mathbf{x}) - I(\mathbf{W}(\mathbf{x};\mathbf{p})) \right]$$

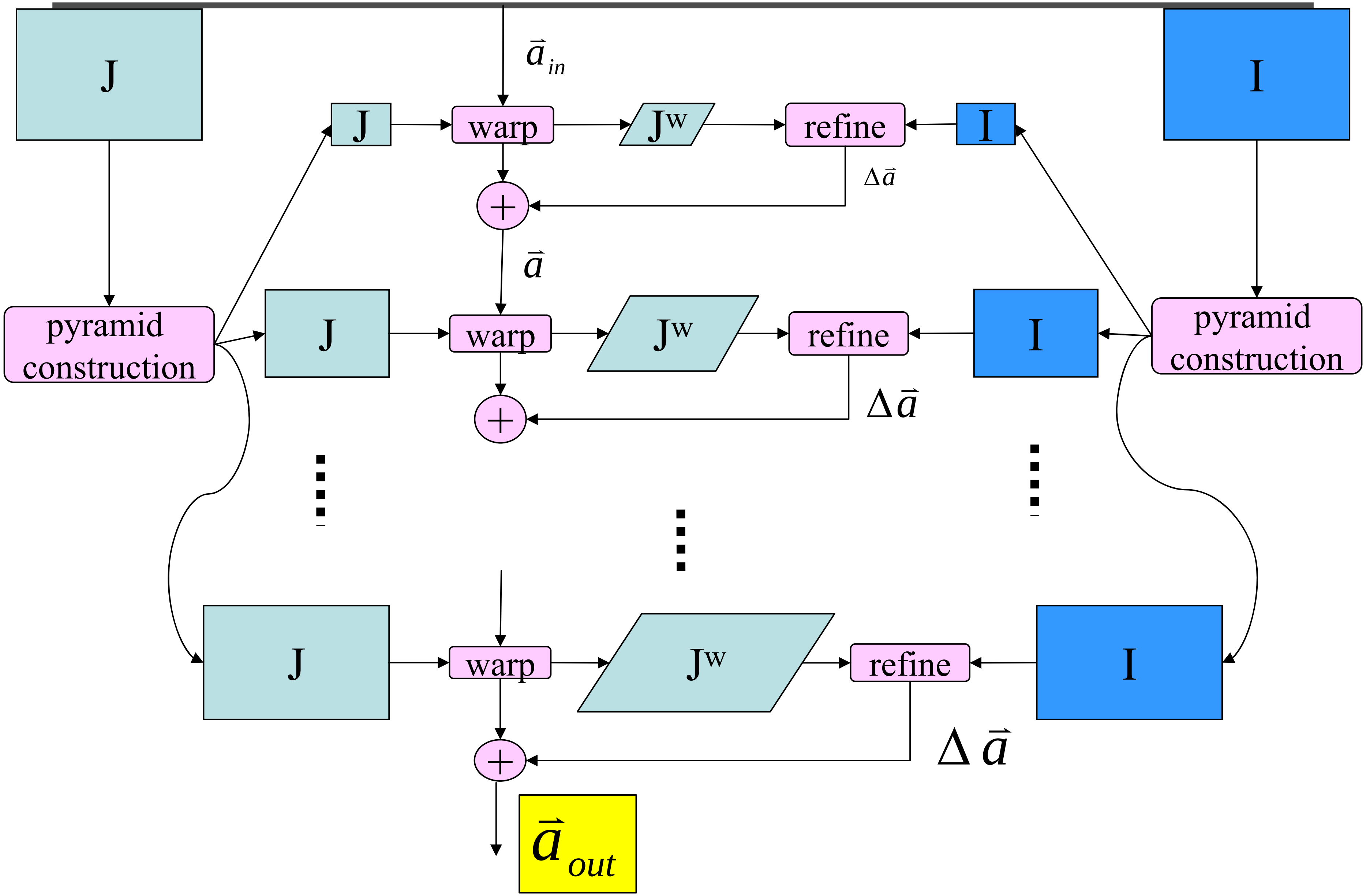


$$\Delta\mathbf{p} = \mathbf{H}^{-1} \sum_{\mathbf{x}} \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^T \left[J(\mathbf{x}) - I(\mathbf{W}(\mathbf{x};\mathbf{p})) \right]$$



$$\Delta \mathbf{p} = \mathbf{H}^{-1} \sum_{\mathbf{x}} \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^T \left[J(\mathbf{x}) - I(\mathbf{W}(\mathbf{x};\mathbf{p})) \right]$$

Coarse-to-fine strategy



Application of image alignment

Direct vs feature-based

- Direct methods use all information and can be very accurate, but they depend on the fragile "brightness constancy" assumption
- Iterative approaches require initialization
- Not robust to illumination change and noise images
- In early days, direct method is better.

- Feature based methods are now more robust and potentially faster
- Even better, it can recognize panorama without initialization

Tracking

Tracking

$I(x,y,t)$ $I(x,y,t+1)$

Tracking

brightness constancy $I(x+u, y+v, t+1) - I(x, y, t) = 0$

$$I(x,y,t) + uI_x(x,y,t) + vI_y(x,y,t) + I_t(x,y,t) - I(x,y,t) \approx 0$$

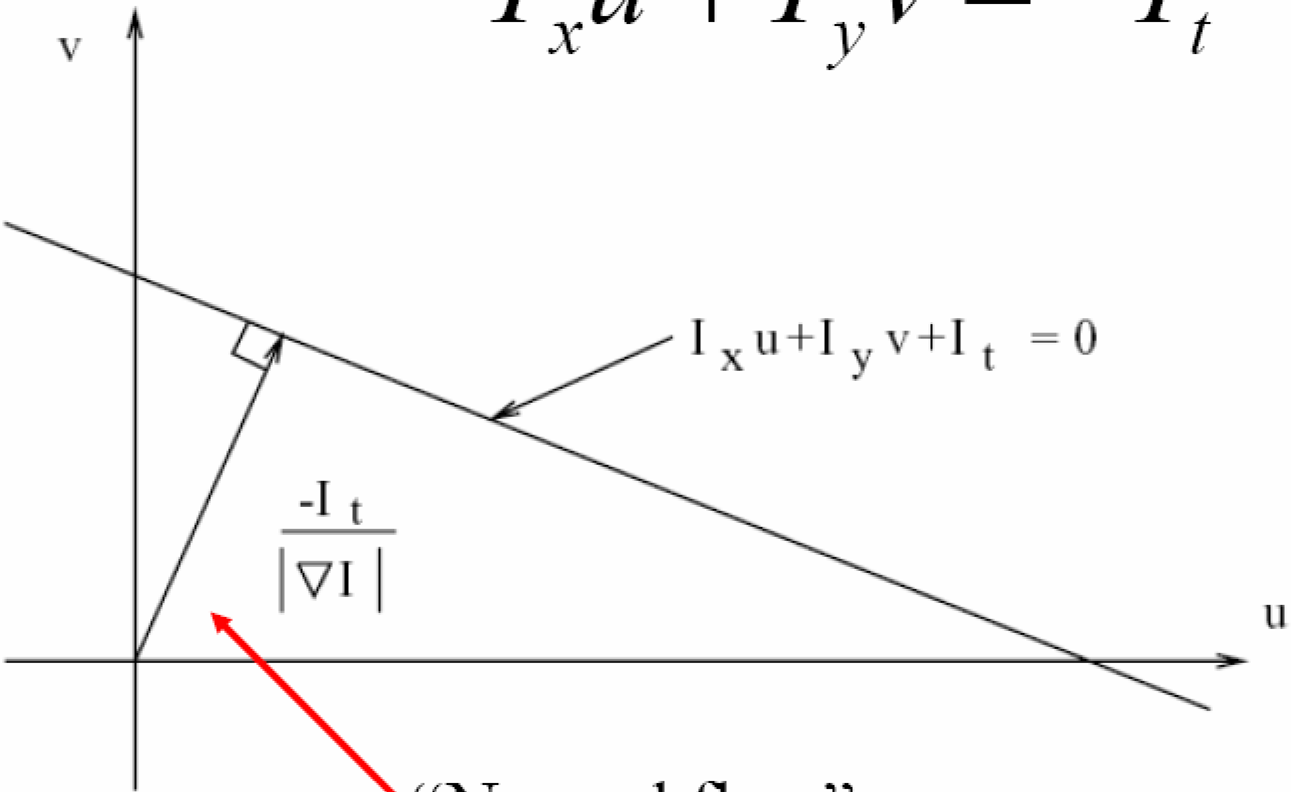
$$uI_x(x,y,t) + vI_y(x,y,t) + I_t(x,y,t) = 0$$

$I_x u + I_y v + I_t = 0$ optical flow constraint equation

Optical flow constraint equation

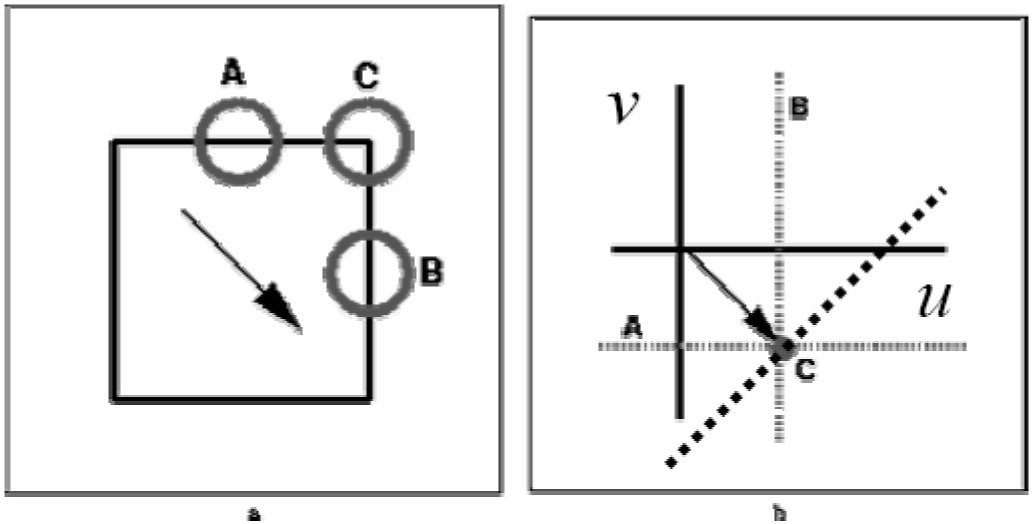
At a single image pixel, we get a line:

$$I_x u + I_y v = -I_t$$



"Normal flow"

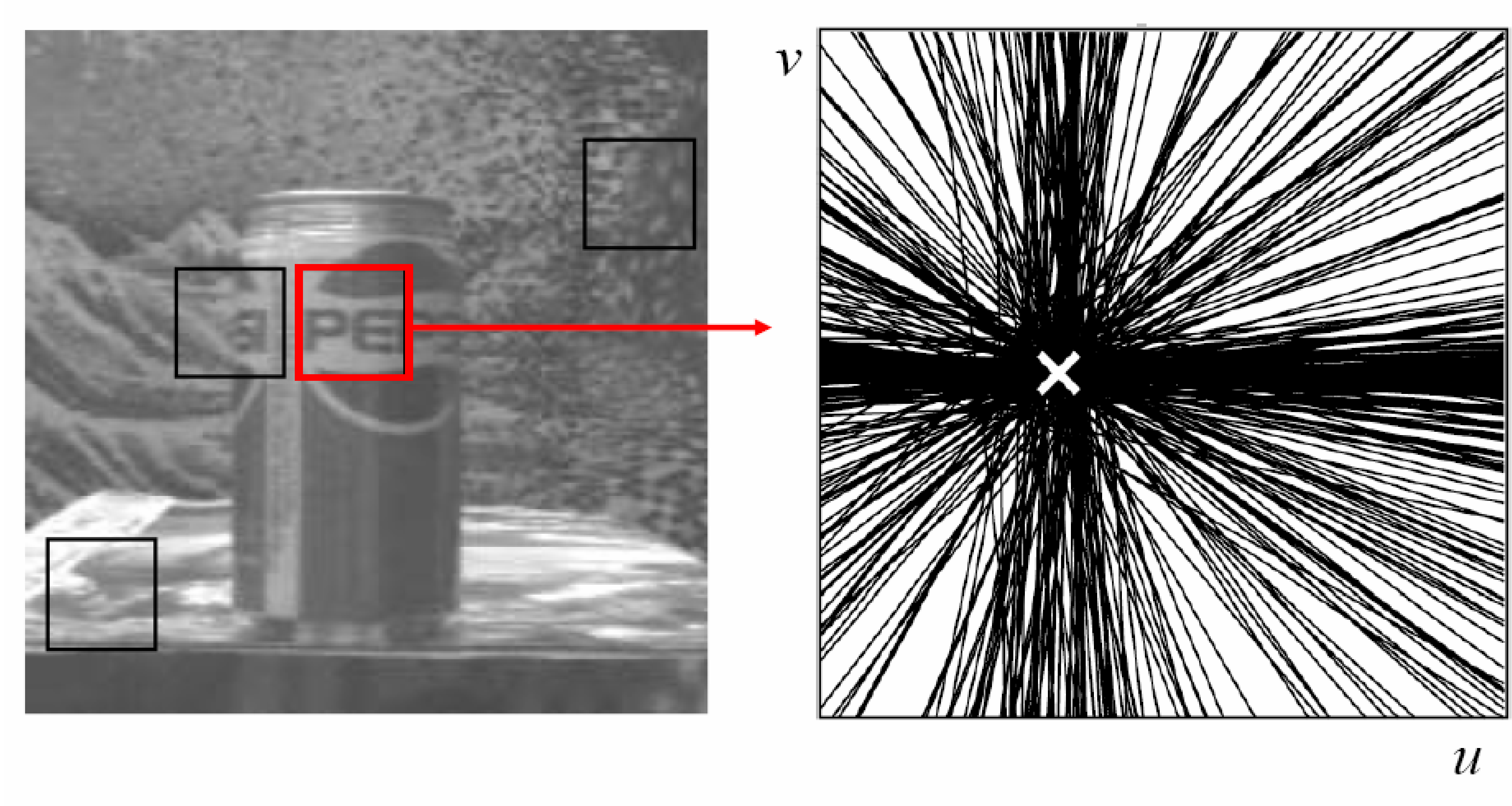
Multiple constraints



Combine constraints to get an estimate of velocity.

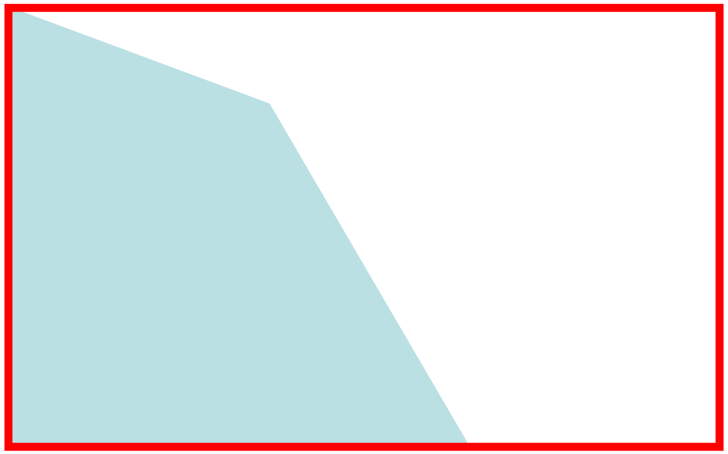
Area-based method

- Assume spatial smoothness

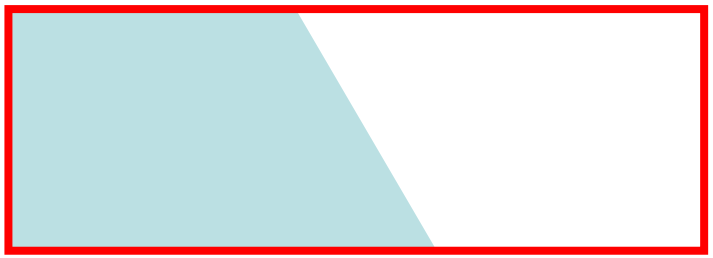


Aperture problem

Aperture problem



Aperture problem



Demo for aperture problem

- http://www.sandlotscience.com/Distortions/Breathing_Square.htm
- http://www.sandlotscience.com/Ambiguous/Barberpole_Illusion.htm

Aperture problem

- Larger window reduces ambiguity, but easily violates spatial smoothness assumption

Area-based method

- Assume spatial smoothness

$$E(u,v) = \sum_{x,y} \left(I_x u + I_y v + I_t \right)^2$$

$$\frac{\partial E}{\partial u} = \sum_R (I_x u + I_y v + I_t) I_x = 0$$

$$\frac{\partial E}{\partial v} = \sum_R (I_x u + I_y v + I_t) I_y = 0$$

Area-based method

$$\left[\sum_R I_x^2\right]u + \left[\sum_R I_x I_y\right]v = -\sum_R I_x I_t$$

$$\left[\sum_R I_x I_y\right]u + \left[\sum_R I_y^2\right]v = -\sum_R I_y I_t$$

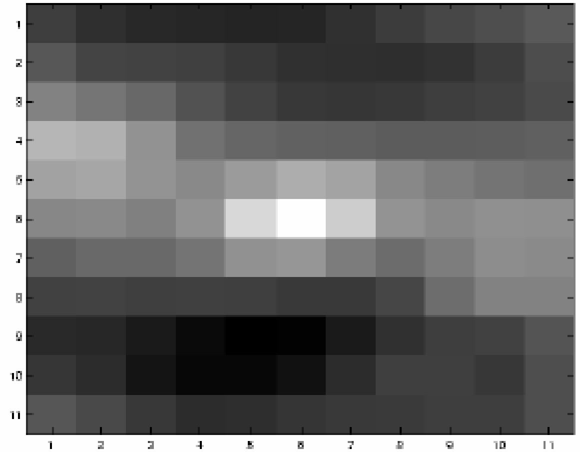
$$\begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_y I_x & \sum I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} -\sum I_x I_t \\ -\sum I_y I_t \end{bmatrix}$$

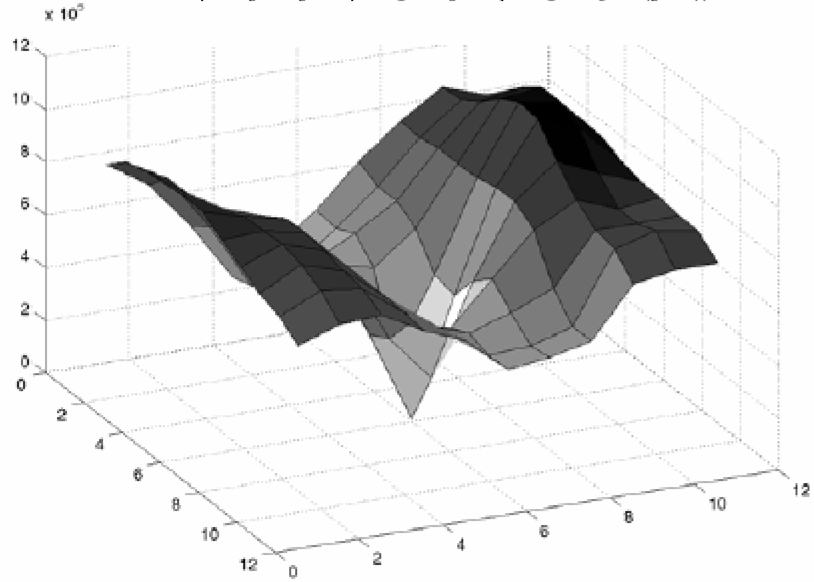
must be invertible

Area-based method

- The eigenvalues tell us about the local image structure.
- They also tell us how well we can estimate the flow in both directions
- Link to Harris corner detector

Textured area

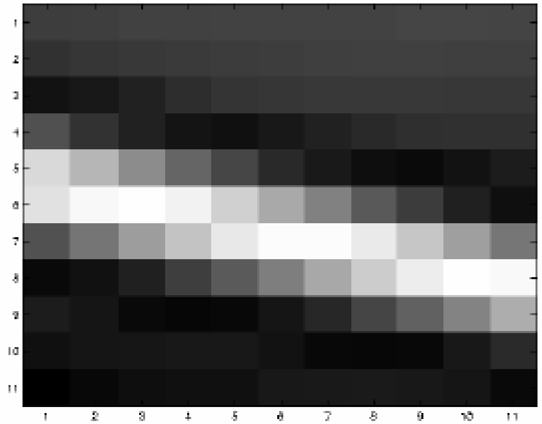


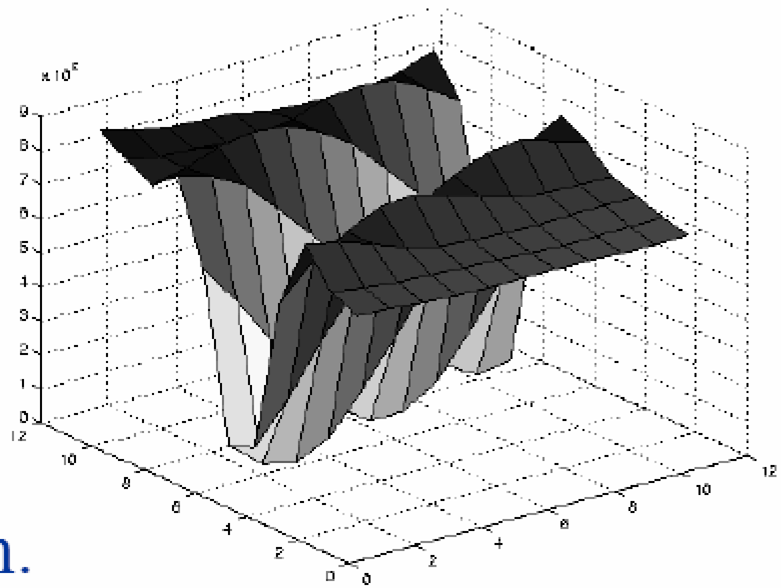


$$\begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_y I_x & \sum I_y^2 \end{bmatrix}$$

Gradients in *x* and *y*.

Edge

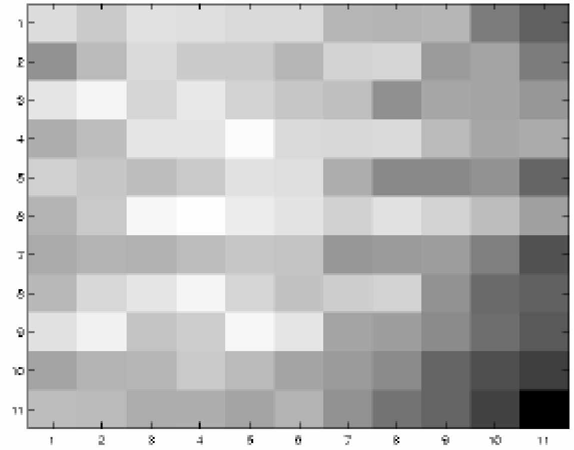


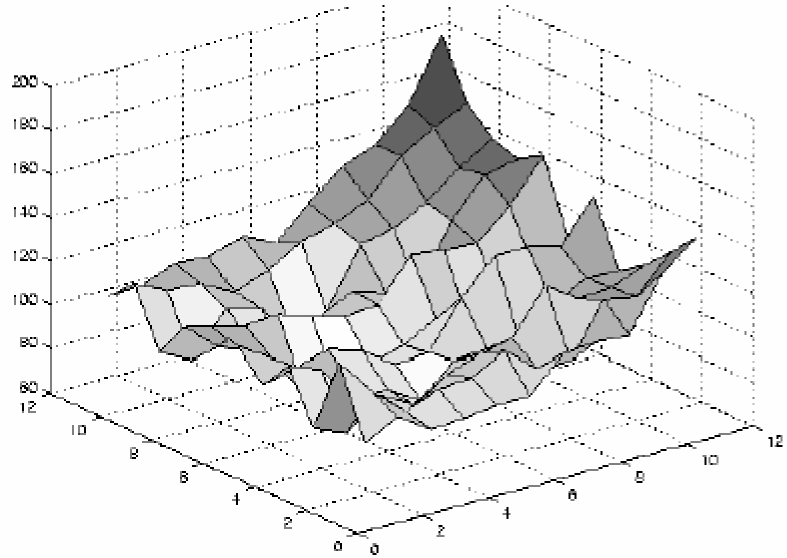


$$\begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_y I_x & \sum I_y^2 \end{bmatrix}$$

Gradients oriented in one direction.

Homogenous area



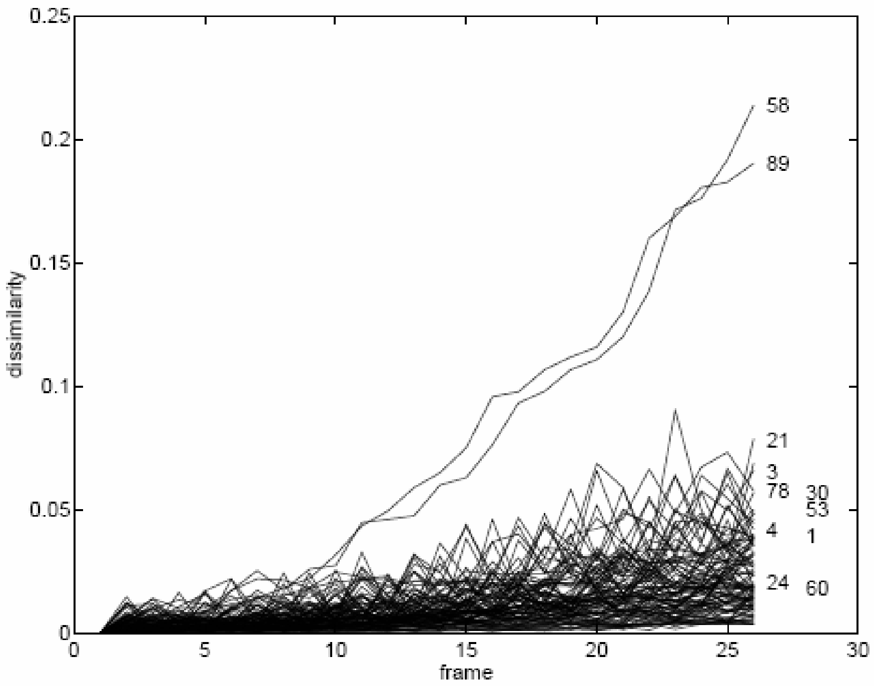


$$\begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_y I_x & \sum I_y^2 \end{bmatrix}$$

Weak gradients everywhere.

KLT tracking

- Select feature by $min(\lambda_1, \lambda_2) > \lambda$
- Monitor features by measuring dissimilarity



Translational Model

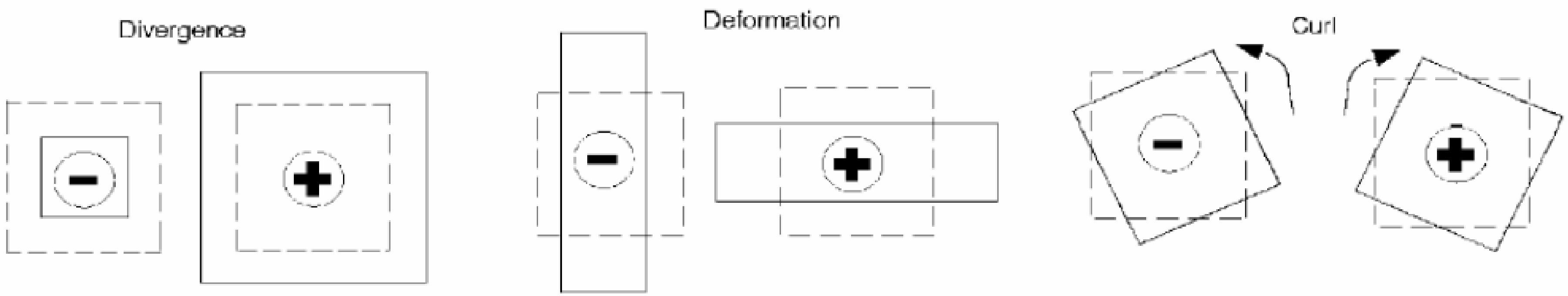
What's wrong with the translational assumption (ie constant motion within a region R)?

How can we generalize it?

Affine Flow

$$E(\mathbf{a}) = \sum_{x,y \in R} (\nabla I^T \mathbf{u}(\mathbf{x};\mathbf{a}) + I_t)^2$$

$$\mathbf{u}(\mathbf{x};\ \mathbf{a}) = \begin{bmatrix} u(\mathbf{x};\mathbf{a}) \\ v(\mathbf{x};\mathbf{a}) \end{bmatrix} = \begin{bmatrix} a_1 + a_2 x + a_3 y \\ a_4 + a_5 x + a_6 y \end{bmatrix}$$



Optimization

$$E(\mathbf{a}) = \sum_{x,y\in R} (I_x a_1 + I_x a_2 x + I_x a_3 y + I_y a_4 + I_y a_5 x + I_y a_6 y + I_t)^2$$

Differentiate wrt the a_i and set equal to zero.

$$\begin{bmatrix} \Sigma I_x^2 & \Sigma I_x^2 x & \Sigma I_x^2 y & \Sigma I_x I_y & \Sigma I_x I_y x & \Sigma I_x I_y y \\ \Sigma I_x^2 x & \Sigma I_x^2 x^2 & \Sigma I_x^2 xy & \Sigma I_x I_y x & \Sigma I_x I_y x^2 & \Sigma I_x I_y xy \\ & & & \vdots & & \\ & & & & & \\ & & & & & \\ & & & & & \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} = \begin{bmatrix} -\Sigma I_x I_t \\ -\Sigma I_x I_t x \\ -\Sigma I_x I_t y \\ -\Sigma I_y I_t \\ -\Sigma I_y I_t x \\ -\Sigma I_y I_t y \end{bmatrix}$$

KLT tracking

http://www.ces.clemson.edu/~stb/klt/

KLT tracking

http://www.ces.clemson.edu/~stb/klt/

SIFT tracking (matching actually)

Frame 0 → Frame 10

SIFT tracking

Frame 0 → Frame 100

SIFT tracking

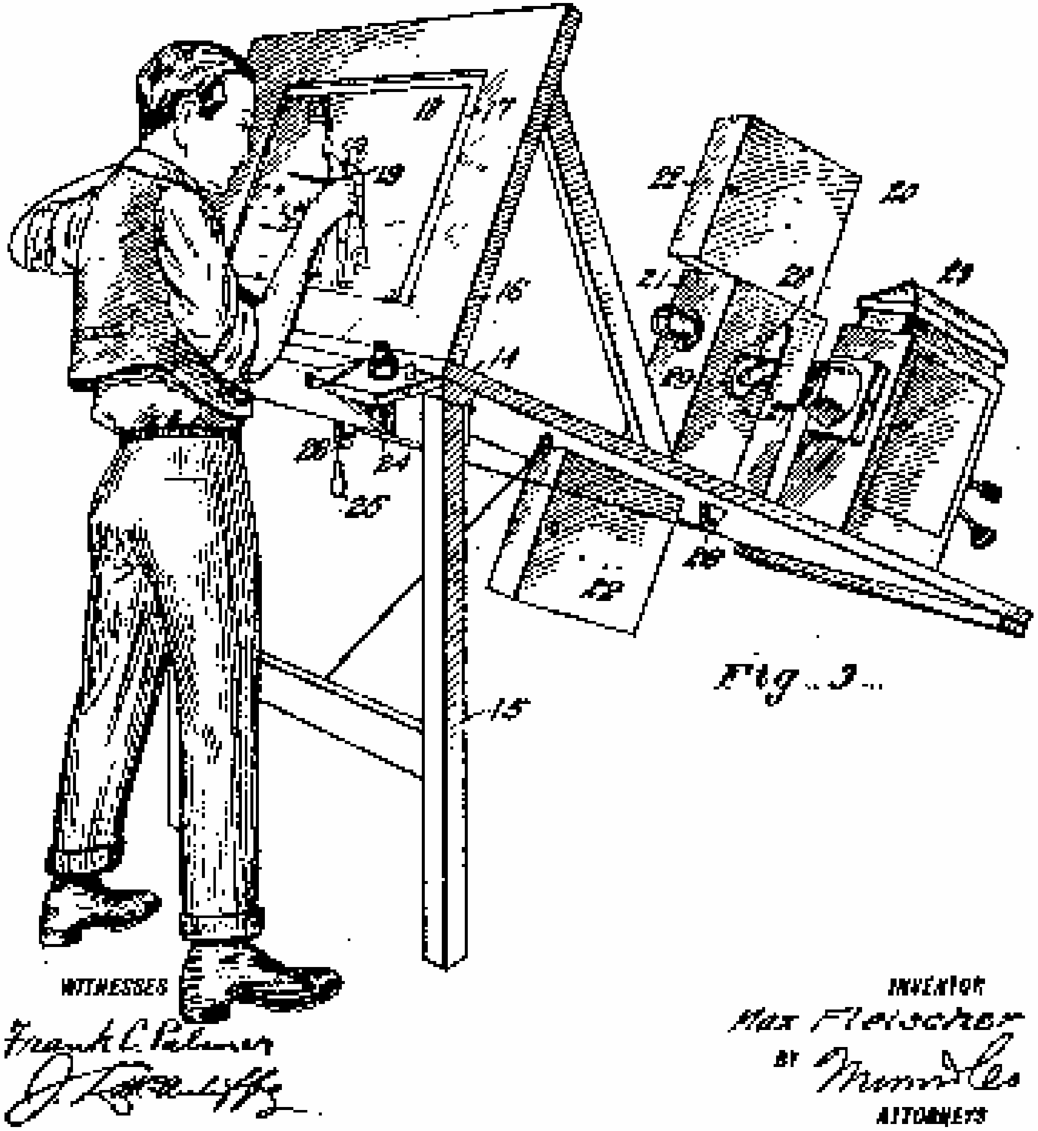
Frame 0 → Frame 200

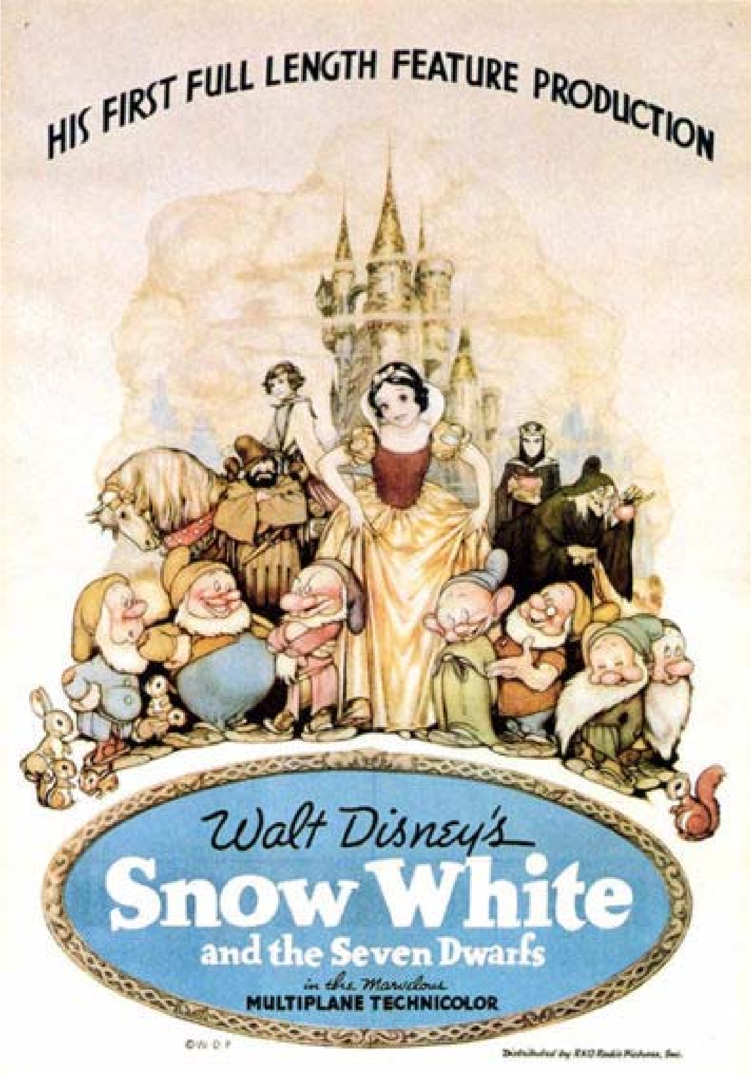
KLT vs SIFT tracking

- KLT has larger accumulating error; partly because our KLT implementation doesn't have affine transformation?
- SIFT is surprisingly robust
- Combination of SIFT and KLT (example)

http://www.frc.ri.cmu.edu/projects/buzzard/smalls/

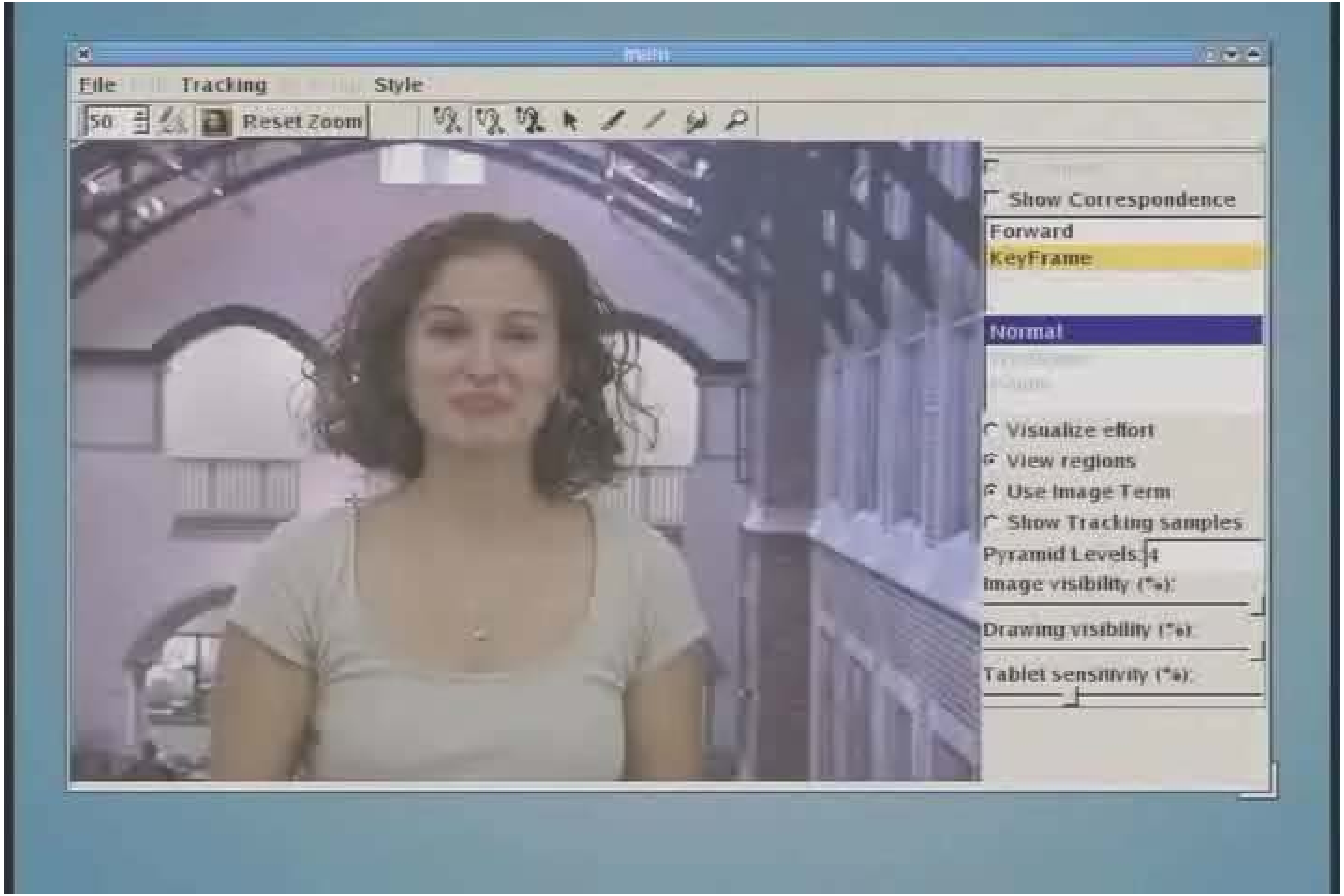
Rotoscoping (Max Fleischer 1914)



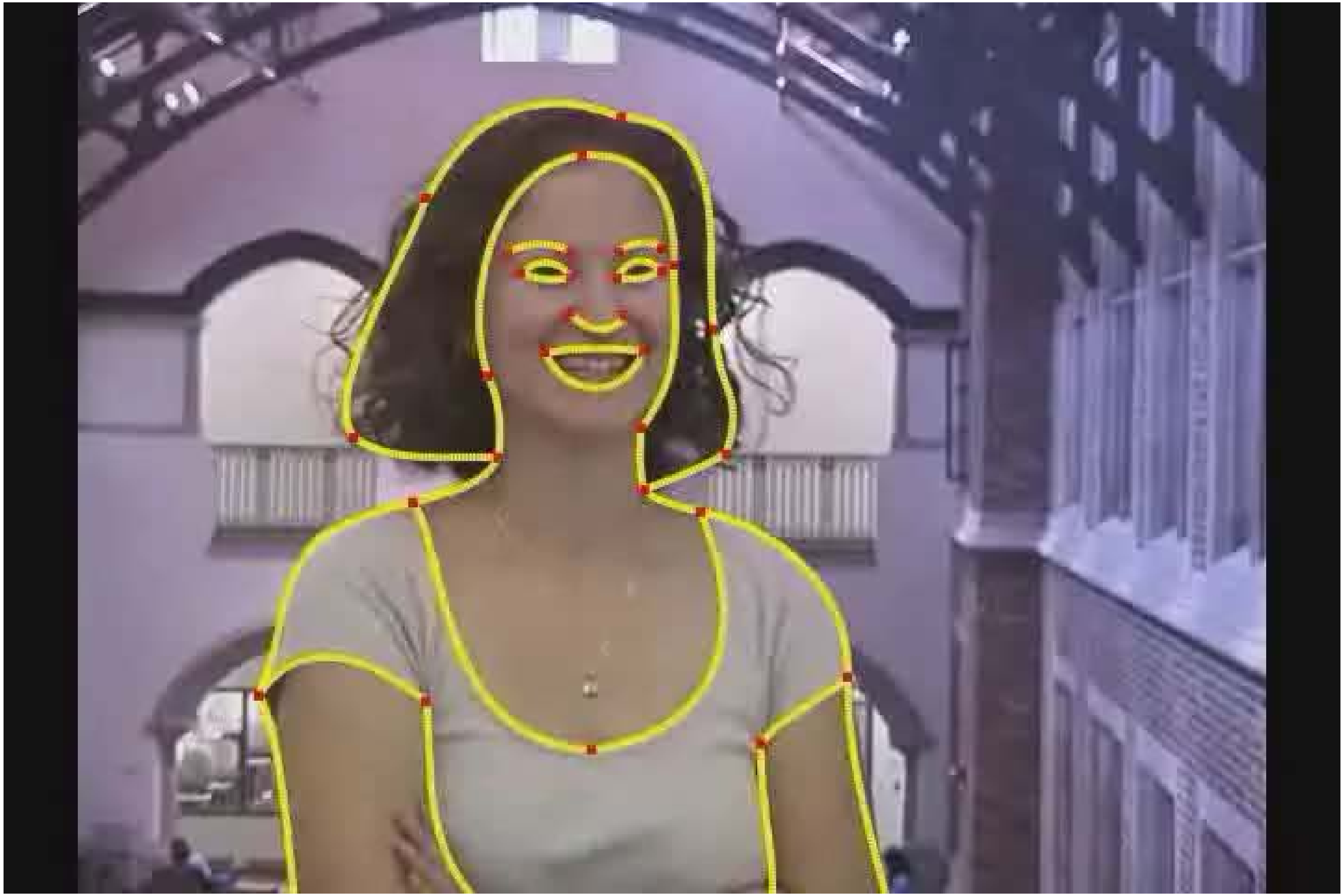


1937

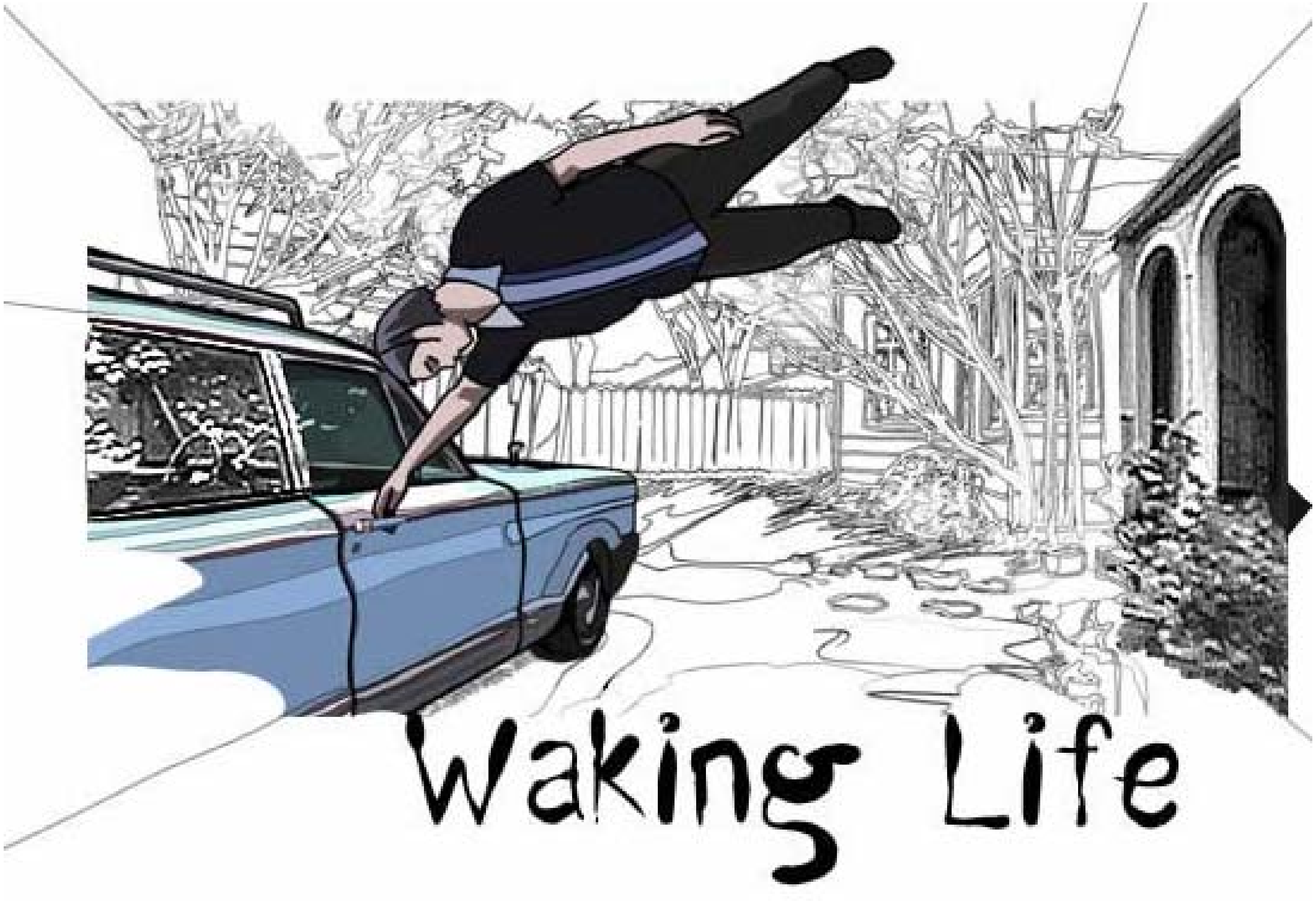
Tracking for rotoscoping



Tracking for rotoscoping



Waking life (2001)



A Scanner Darkly (2006)

- Rotoshop, a proprietary software. Each minute of animation required 500 hours of work.

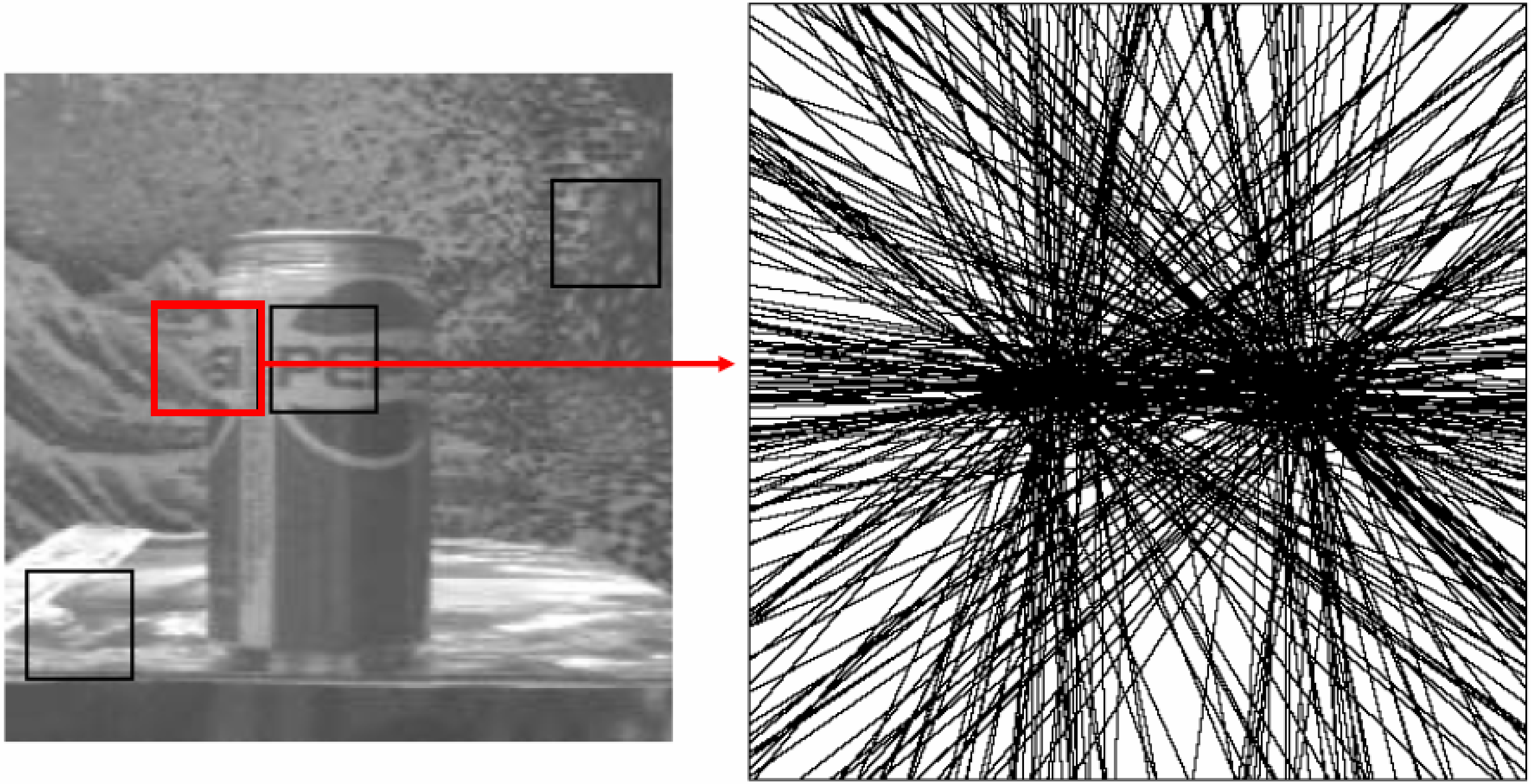
Optical flow

Single-motion assumption

Violated by

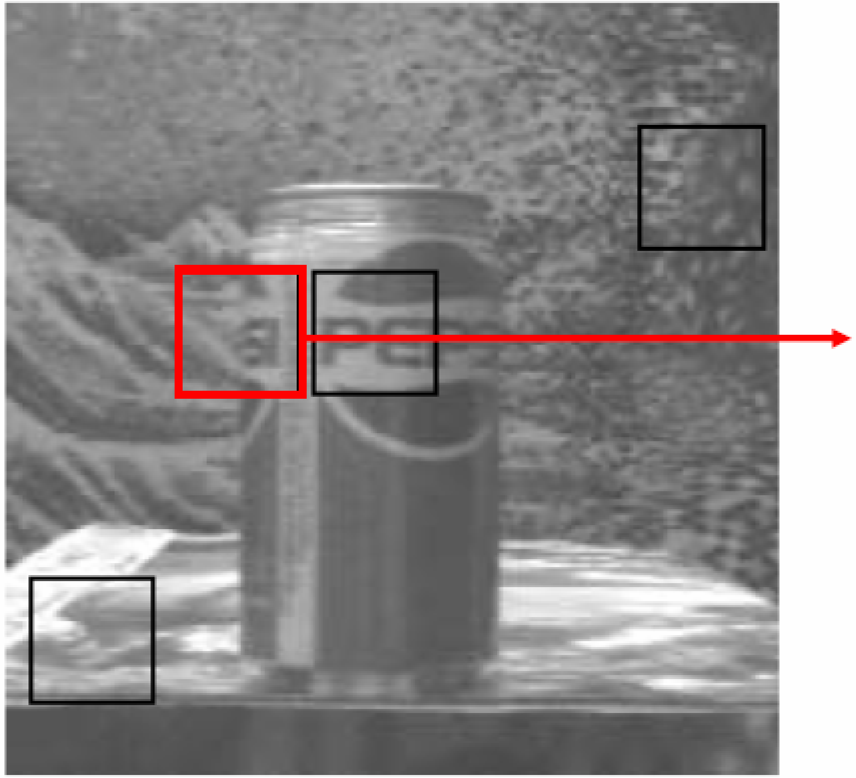
- Motion discontinuity
- Shadows
- Transparency
- Specular reflection
- …

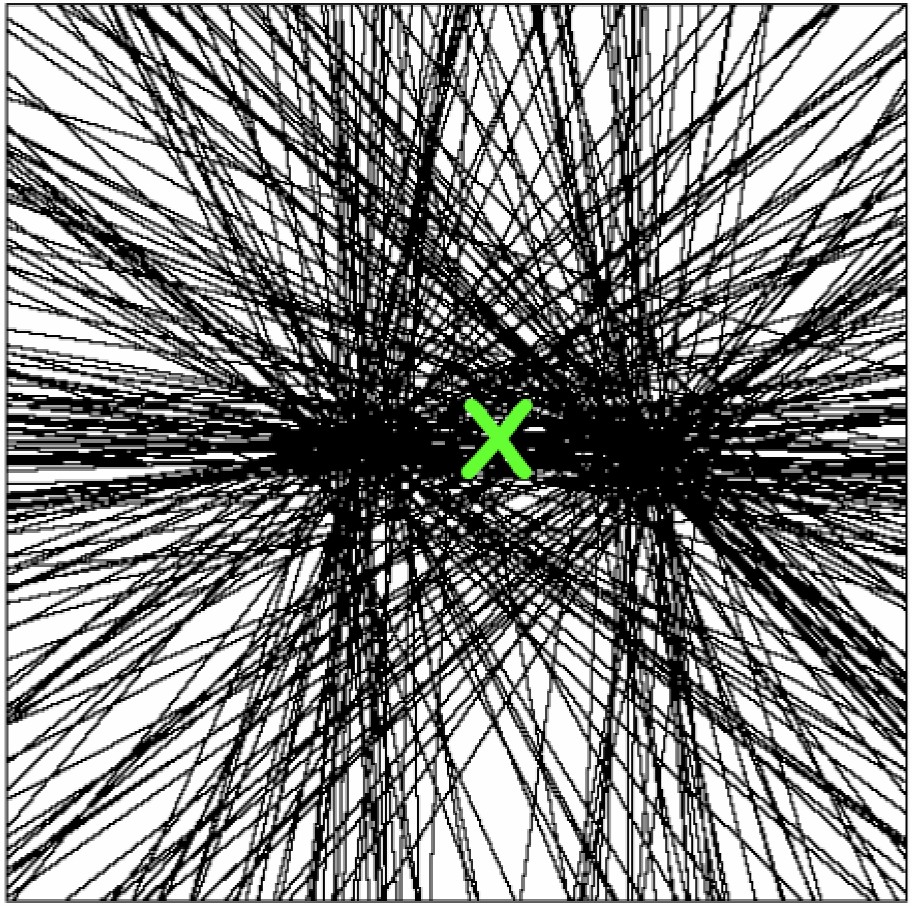
Multiple motion



What is the "best" fitting translational motion?

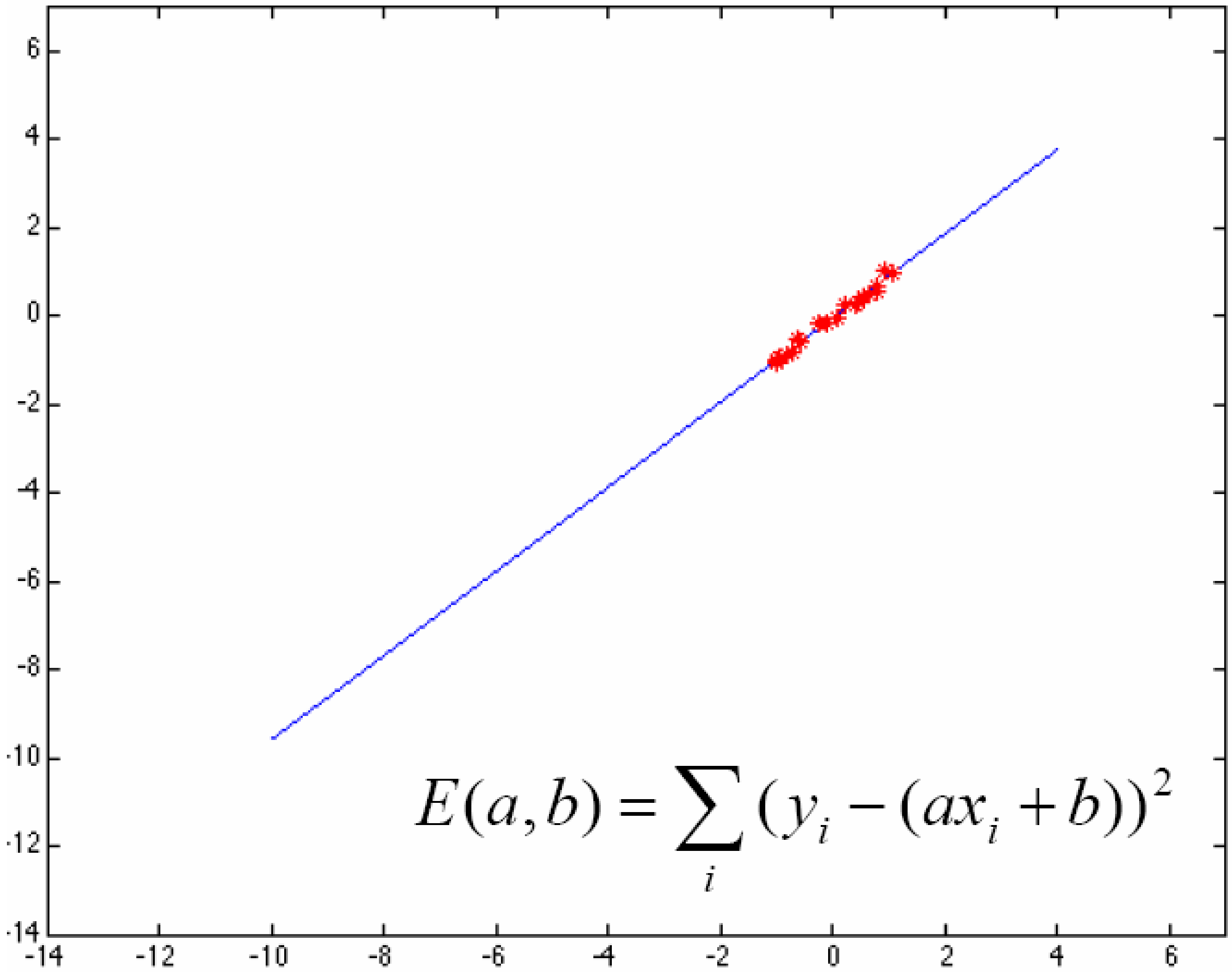
Multiple motion



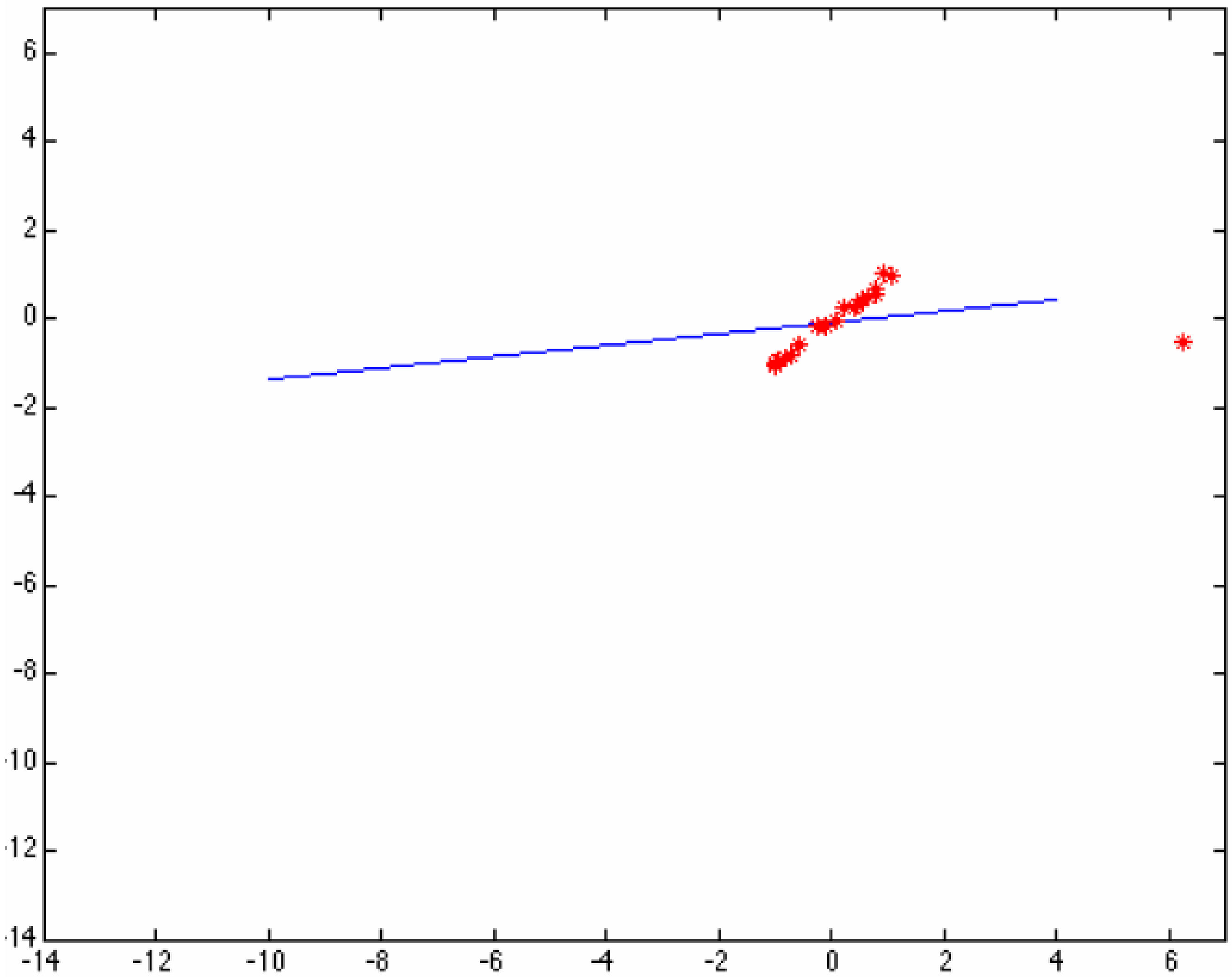


Least squares fit.

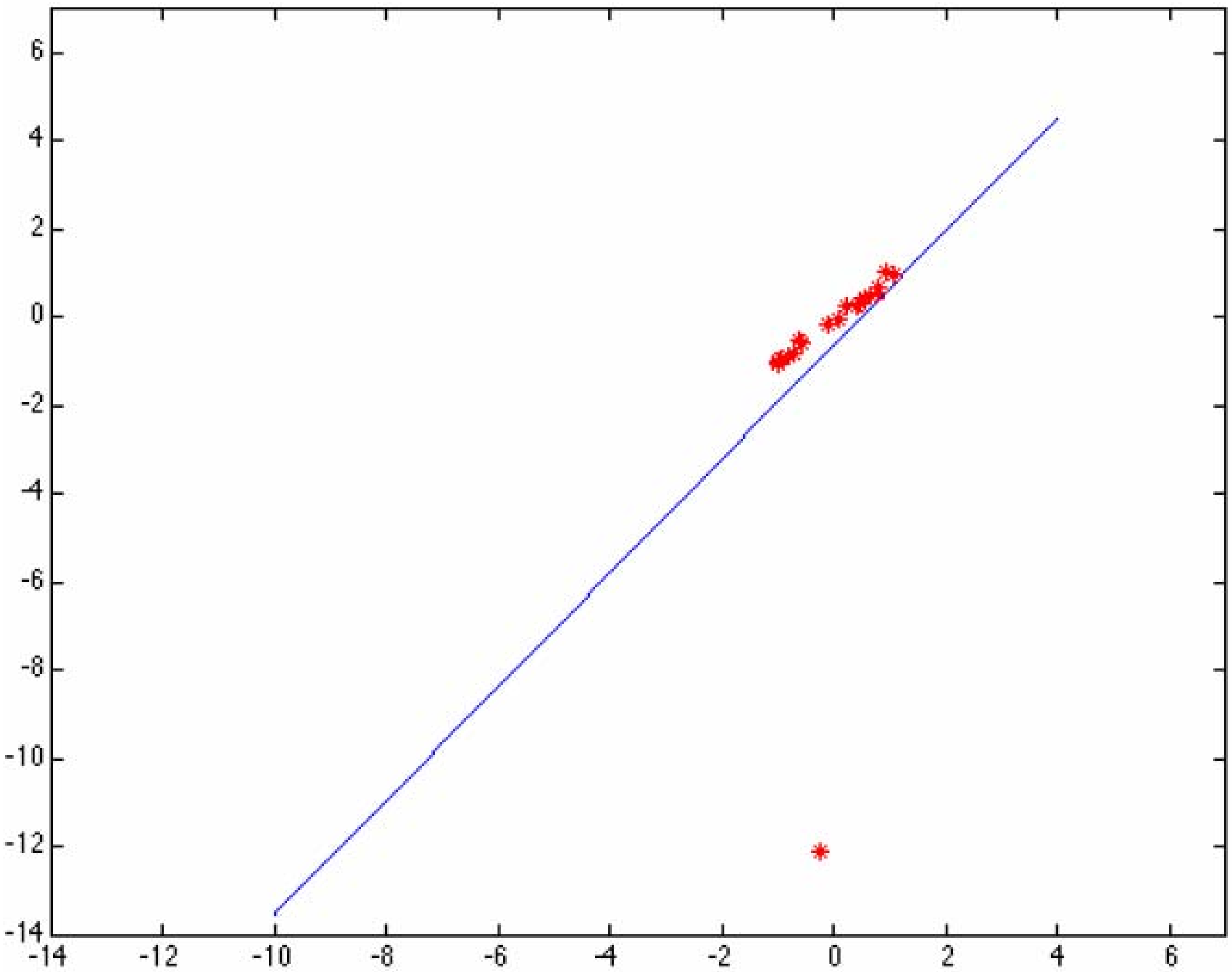
Simple problem: fit a line



Least-square fit



Least-square fit

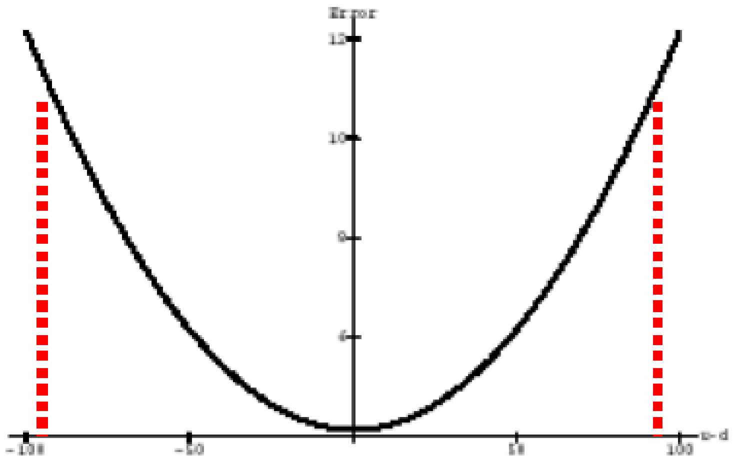


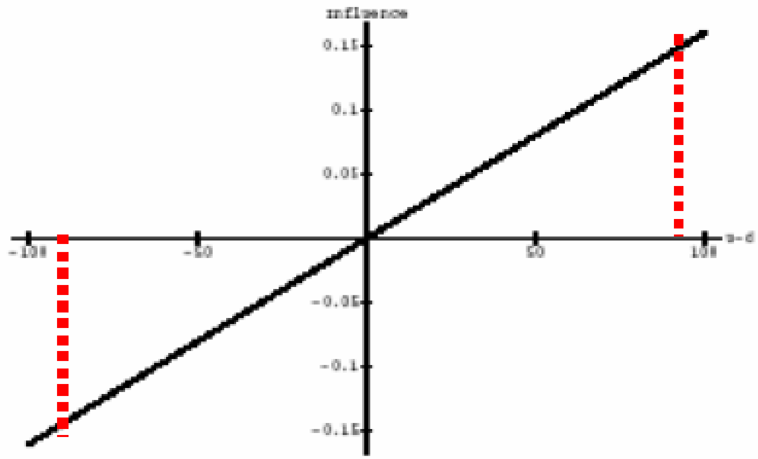
Robust statistics

- Recover the best fit for the majority of the data
- Detect and reject outliers

Approach

Influence is proportional to the derivative of the ρ function.

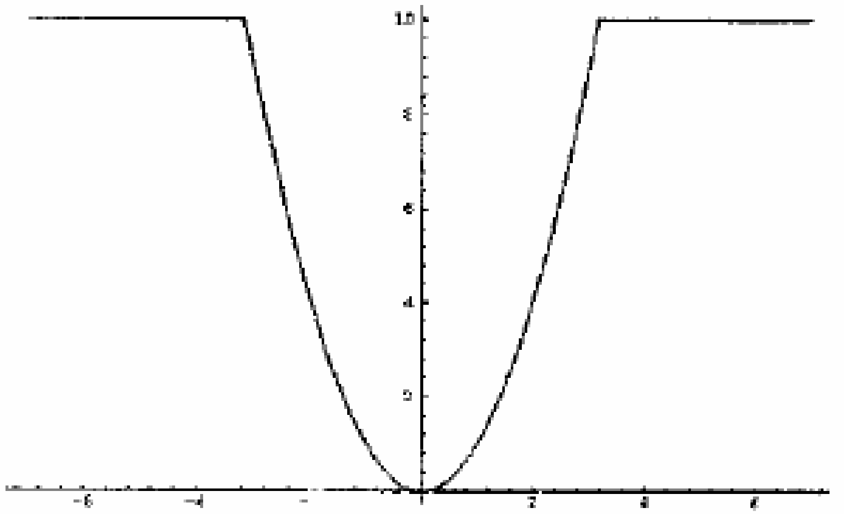


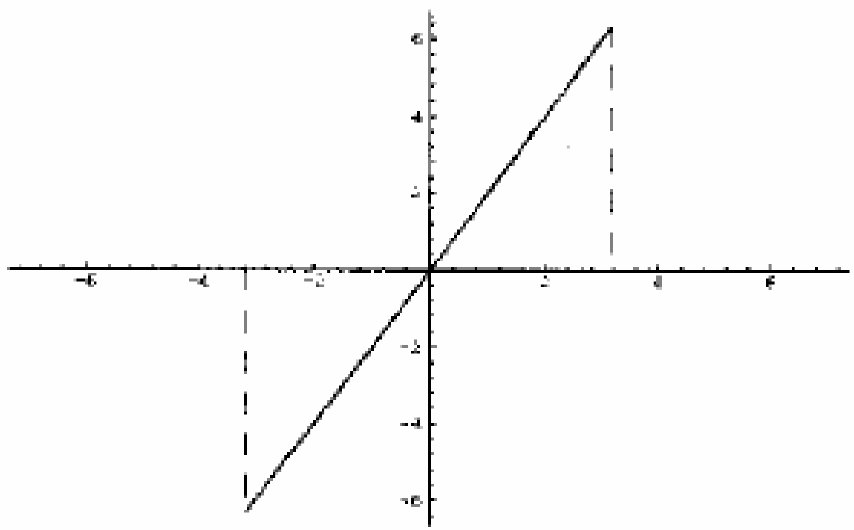


Want to give less influence to points beyond some value.

Robust weighting

$$\rho(x,\alpha,\lambda) = \begin{cases} \lambda x^2 & \text{if } |x| < \frac{\sqrt{\alpha}}{\sqrt{\lambda}}, \\ \alpha & \text{otherwise.} \end{cases} \qquad \psi(x,\alpha,\lambda) = \begin{cases} 2\lambda x & \text{if } |x| < \frac{\sqrt{\alpha}}{\sqrt{\lambda}}, \\ 0 & \text{otherwise.} \end{cases}$$



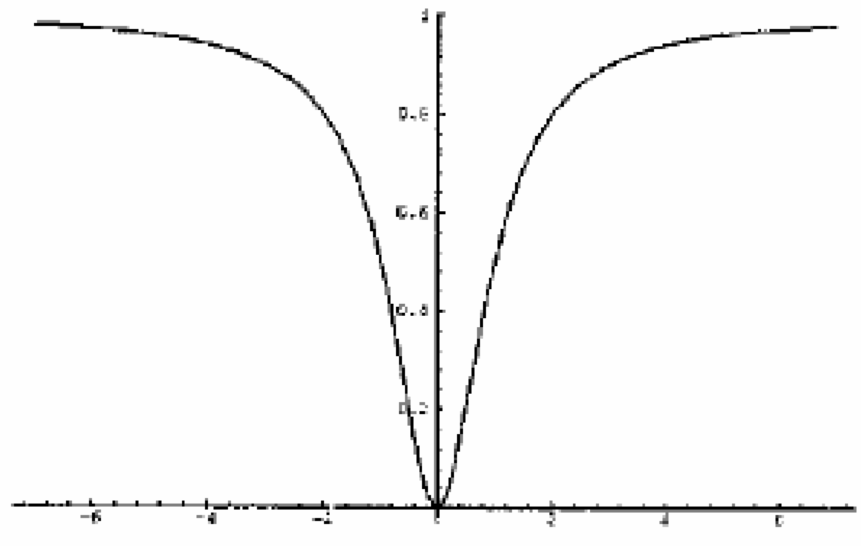


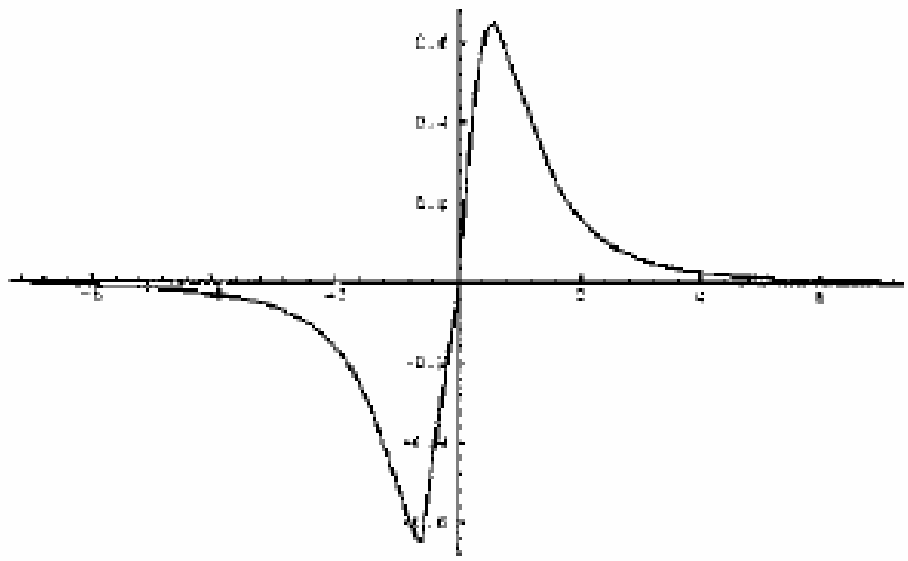
Truncated quadratic

Robust weighting

$$\rho(x,\sigma) = \frac{x^2}{\sigma + x^2}$$

$$\psi(x,\sigma) = \frac{2x\sigma}{(\sigma + x^2)^2}$$





Geman & McClure

Robust estimation

$$E(\mathbf{a}) = \sum_{x,y \in R} \rho(I_x u + I_y v + I_t, \sigma)$$

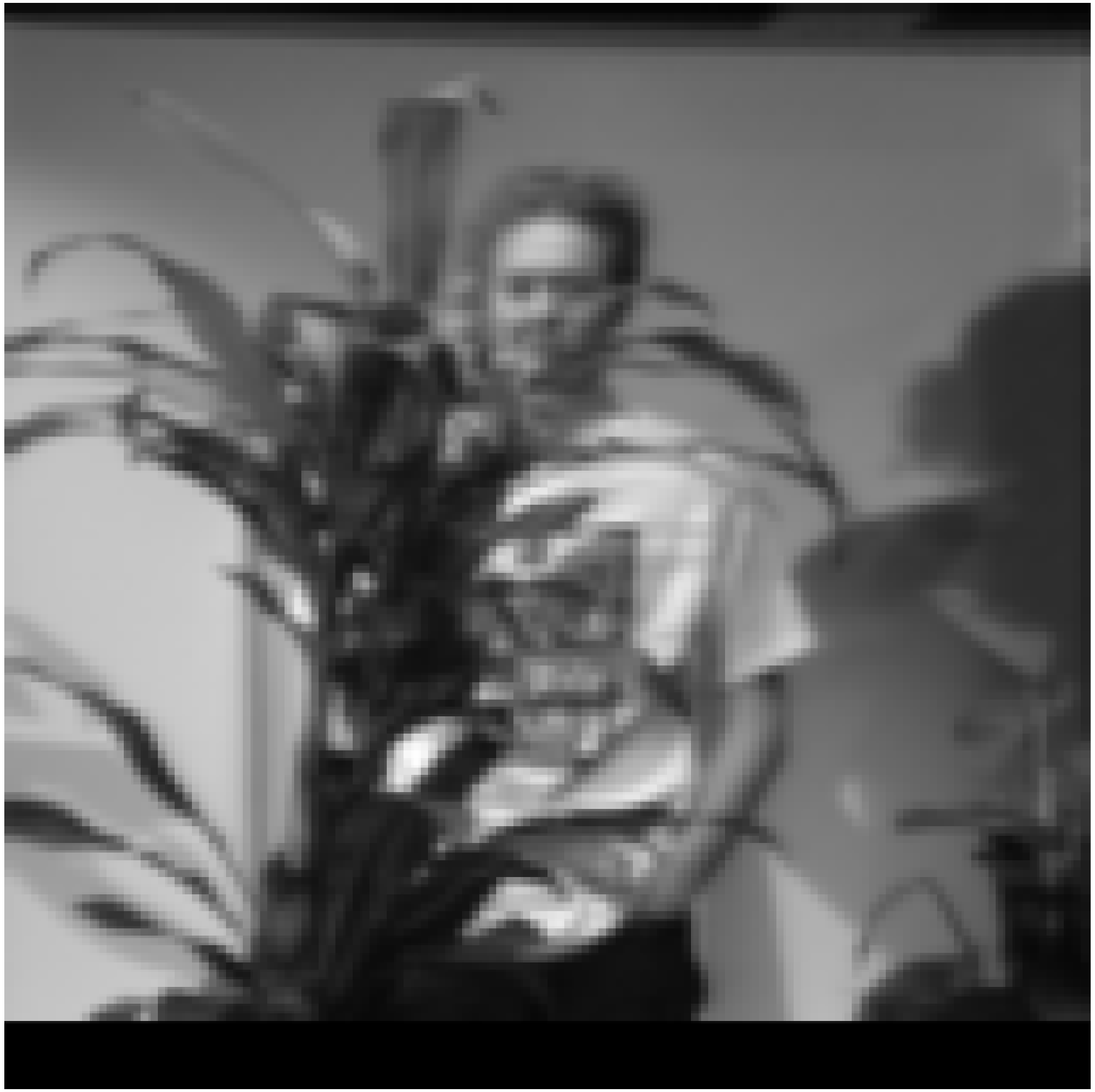
Minimize: differentiate and set equal to zero:

$$\frac{\partial E}{\partial u} = \sum_{x,y \in R} \psi(I_x u + I_y v + I_t, \sigma) I_x = 0$$

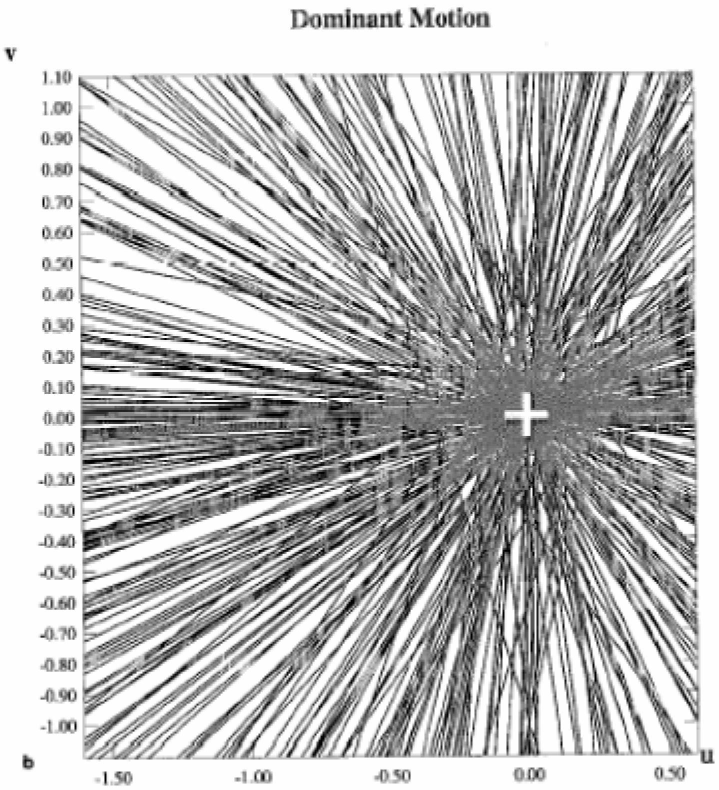
$$\frac{\partial E}{\partial v} = \sum_{x,y \in R} \psi(I_x u + I_y v + I_t, \sigma) I_y = 0$$

No closed form solution!

Fragmented occlusion



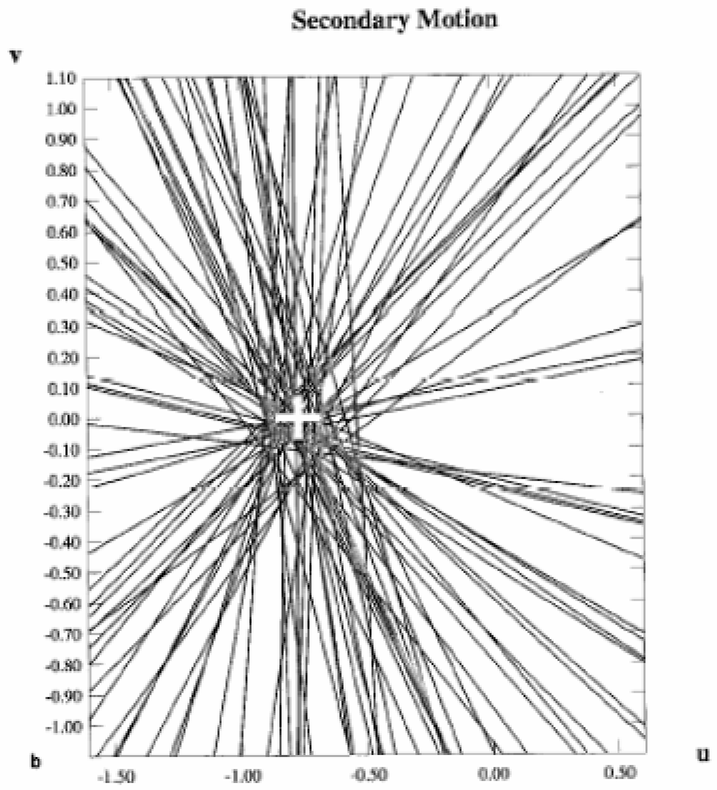
Results

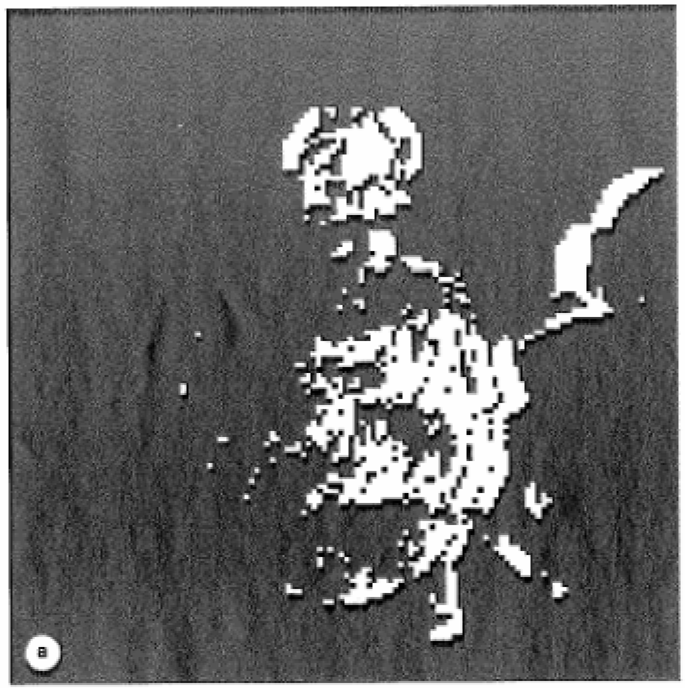


Dominant Motion
v
1.10
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
-0.40
-0.50
-0.60
-0.70
-0.80
-0.90
-1.00
b
-1.50
-1.00
-0.50
0.00
0.50
u

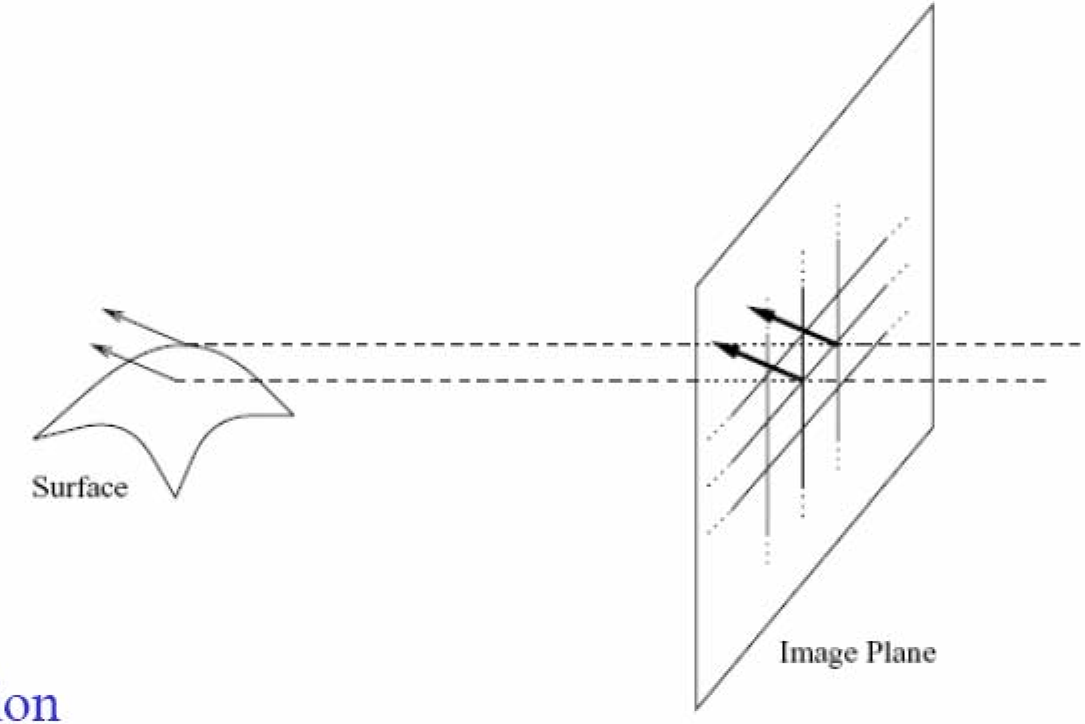
a

Results


Secondary Motion
v
1.10
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
-0.40
-0.50
-0.60
-0.70
-0.80
-0.90
-1.00
b
-1.50
-1.00
-0.50
0.00
0.50
u


a

Regularization and dense optical flow



Assumption

* Neighboring points in the scene typically belong to the same surface and hence typically have similar motions.
* Since they also project to nearby points in the image, we expect spatial coherence in image flow.

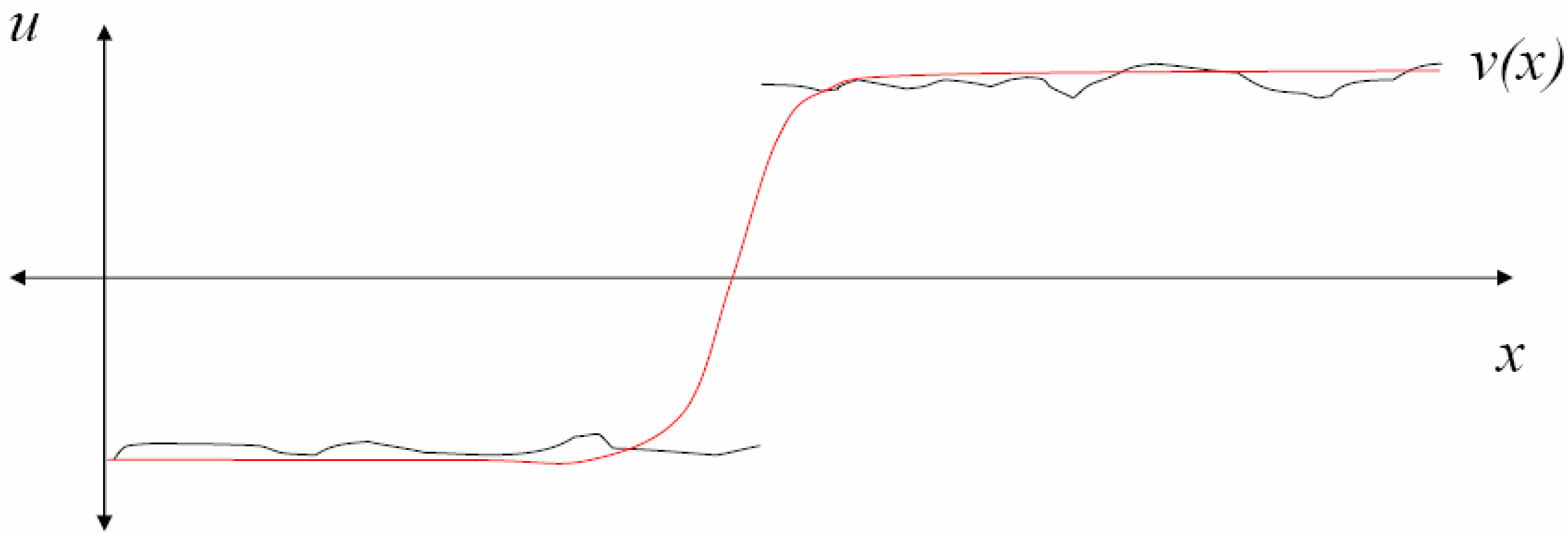
Formalize this Idea

Noisy 1D signal:

Noisy measurements *u(x)*

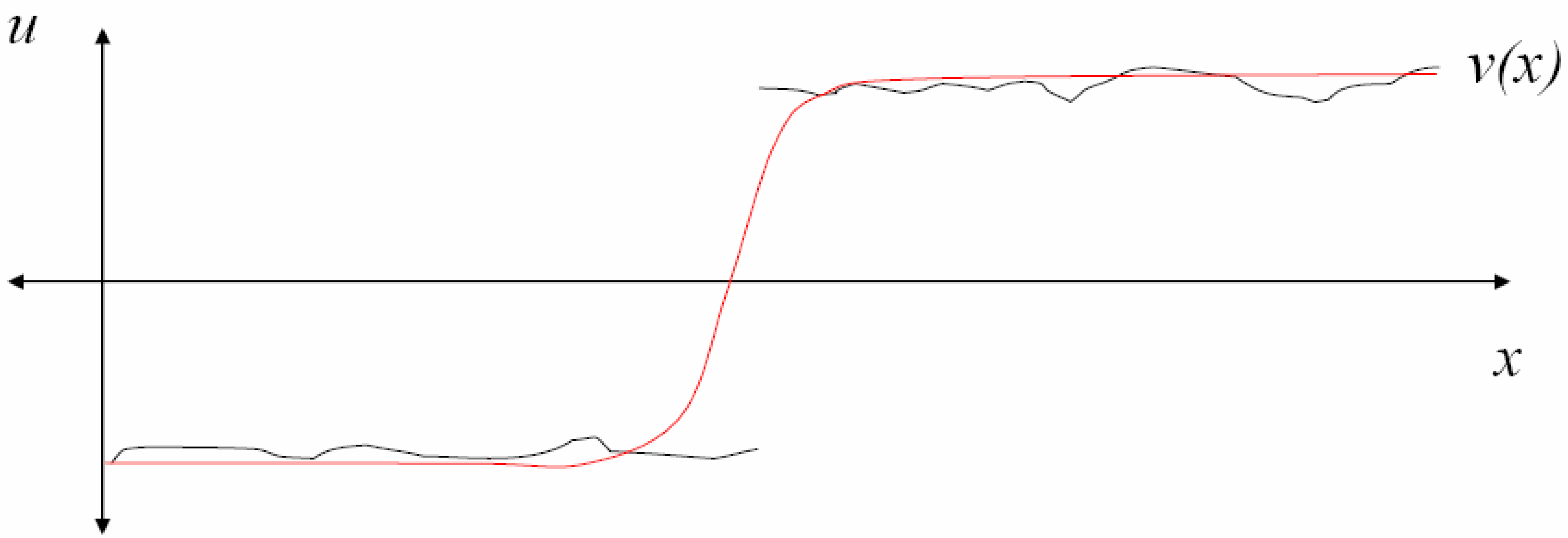
Regularization

Find the "best fitting" smoothed function *v(x)*



Noisy measurements *u(x)*

Regularization



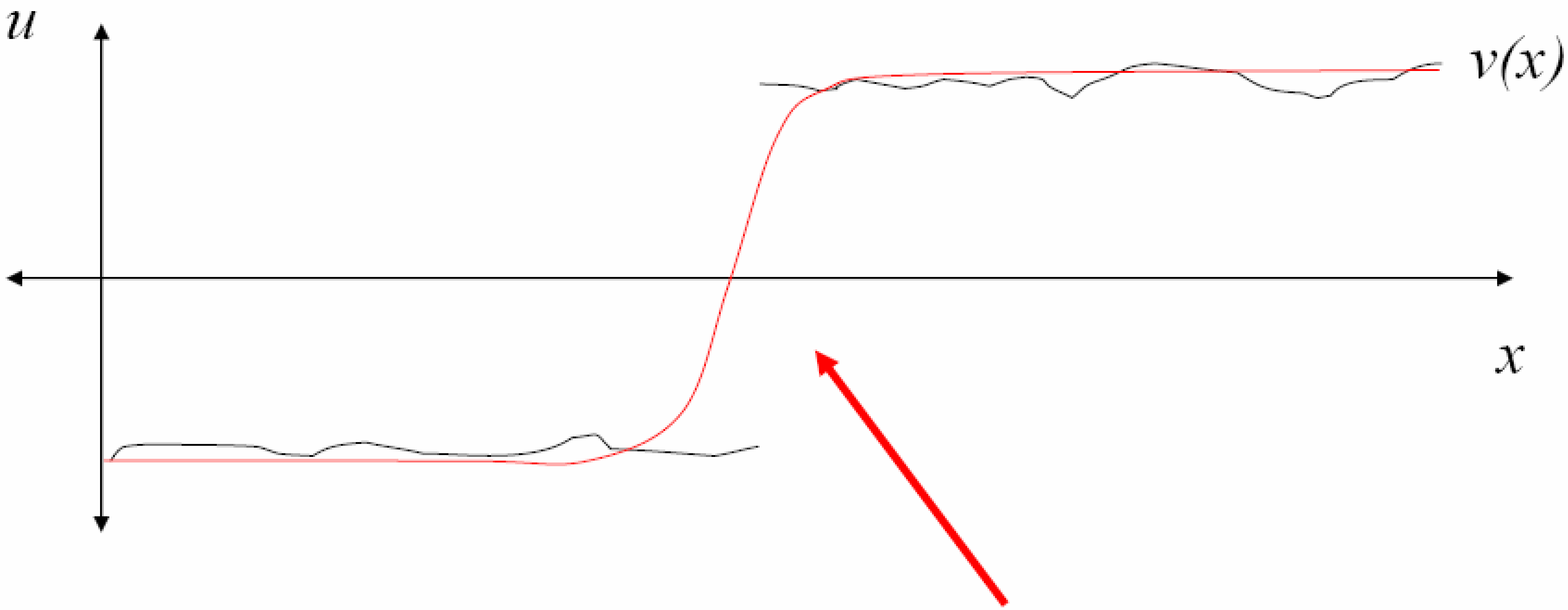
Minimize:

Faithful to the data

Spatial smoothness assumption

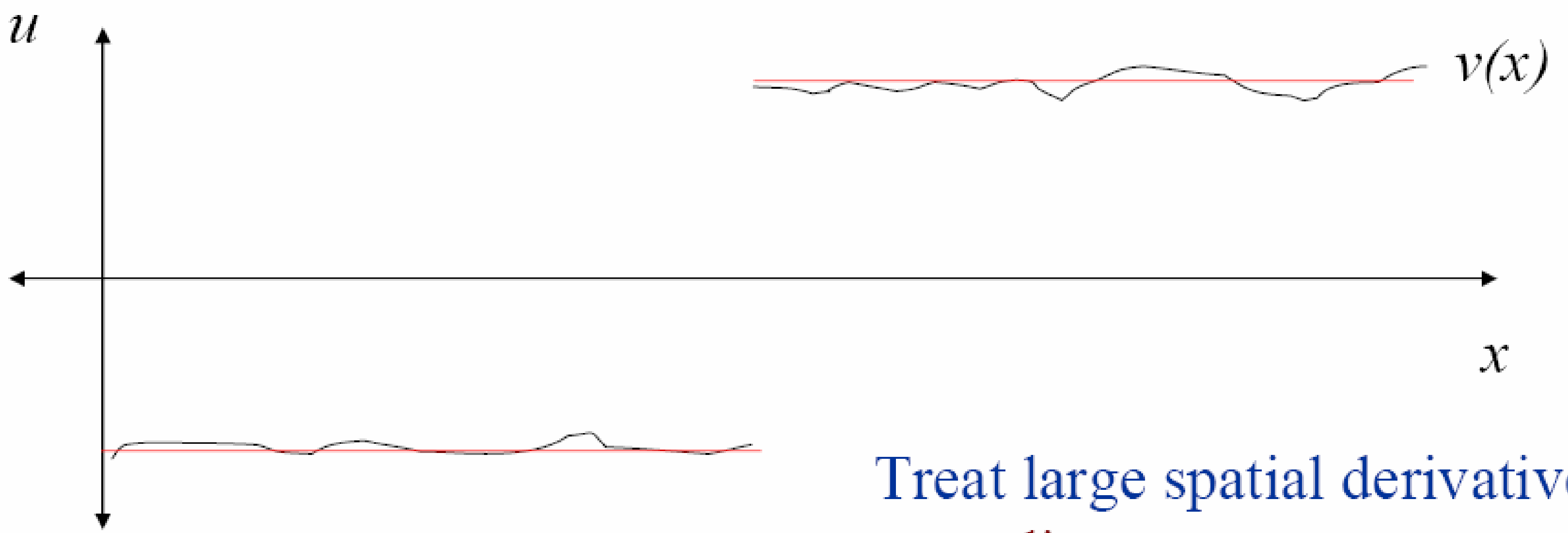
$$E(v)=\sum_{x=1}^{N}(v(x)-u(x))^2+\lambda\sum_{x=1}^{N-1}(v(x+1)-v(x))^2$$

Discontinuities



What about this discontinuity?
What is happening here?
What can we do?

Robust Regularization



Treat large spatial derivatives as outliers.

Minimize:

$$E(v)=\sum_{x=1}^{N}\rho(v(x)-u(x),\sigma_1)+\lambda\sum_{x=1}^{N-1}\rho(v(x+1)-v(x),\sigma_2)$$

"Dense" Optical Flow

$$E_D(\mathbf{u}(\mathbf{x})) = \rho(I_x(\mathbf{x})u(\mathbf{x}) + I_y(\mathbf{x})v(\mathbf{x}) + I_t(\mathbf{x}), \sigma_D)$$

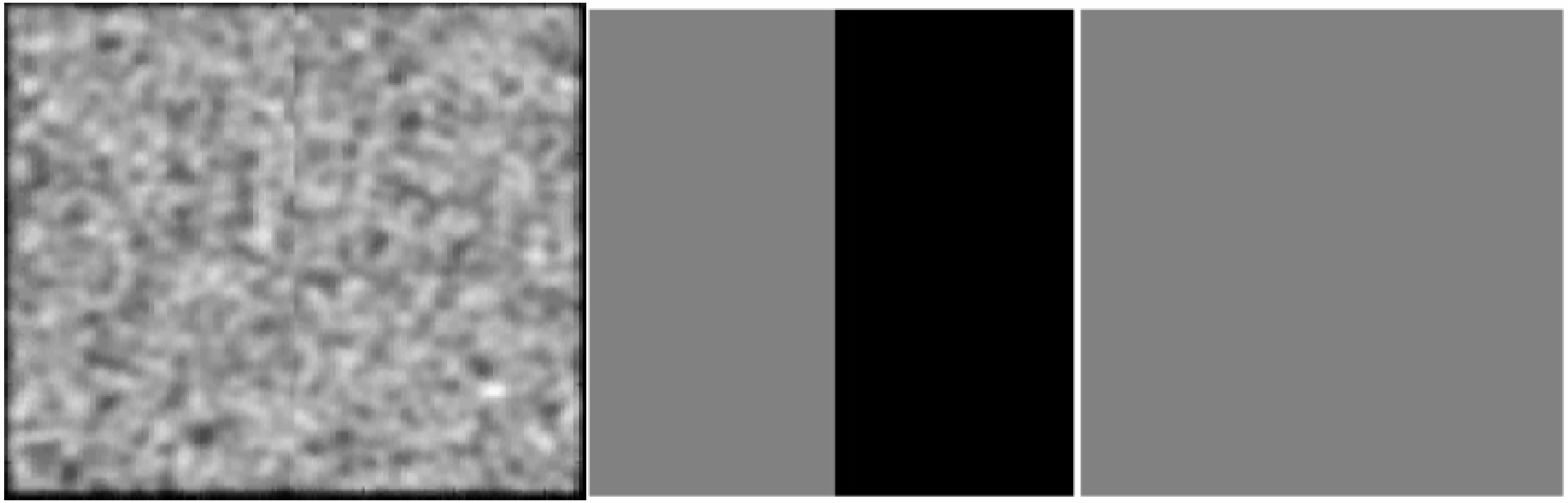
$$E_S(u,v) = \sum_{\mathbf{y}\in G(\mathbf{x})} [\rho(u(\mathbf{x}) - u(\mathbf{y}), \sigma_S) + \rho(v(\mathbf{x}) - v(\mathbf{y}), \sigma_S)]$$

Objective function:

$$E(\mathbf{u}) = \sum_{\mathbf{x}} E_D(\mathbf{u}(\mathbf{x})) + \lambda E_S(\mathbf{u}(\mathbf{x}))$$

When ρ is quadratic = "Horn and Schunck"

Example

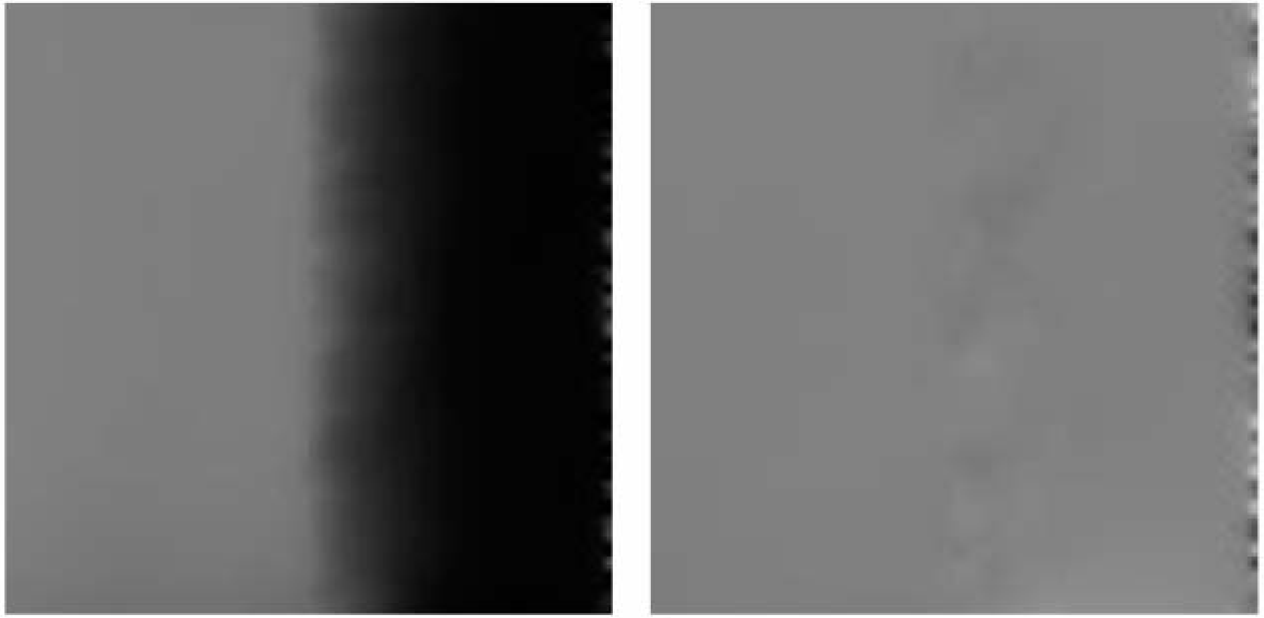


Input image

Horizontal motion

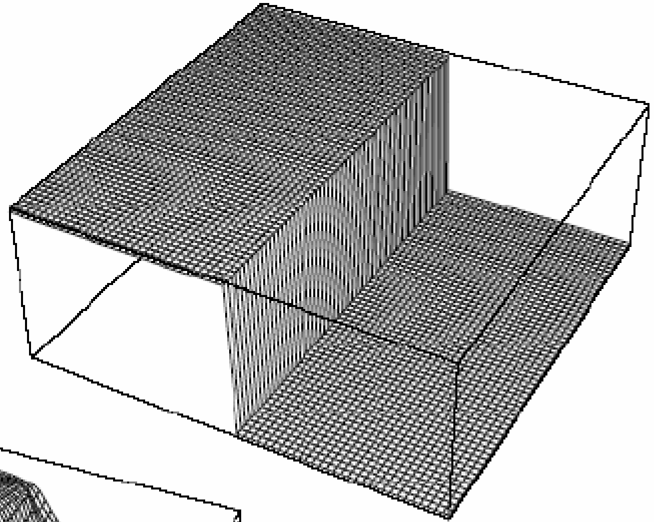
Vertical motion

Quadratic:

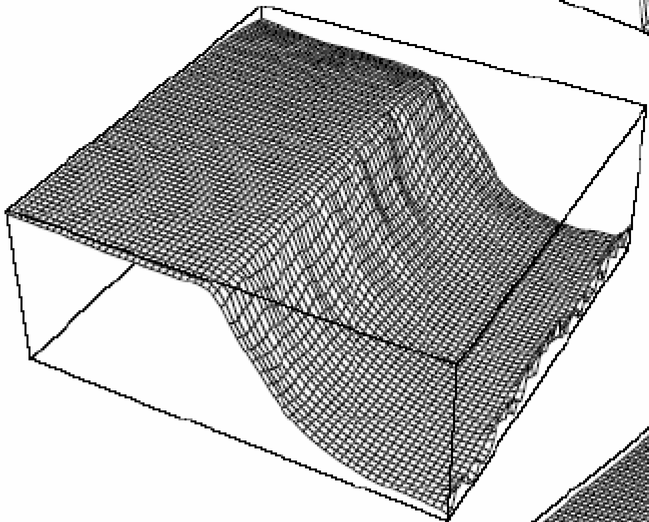


Robust:

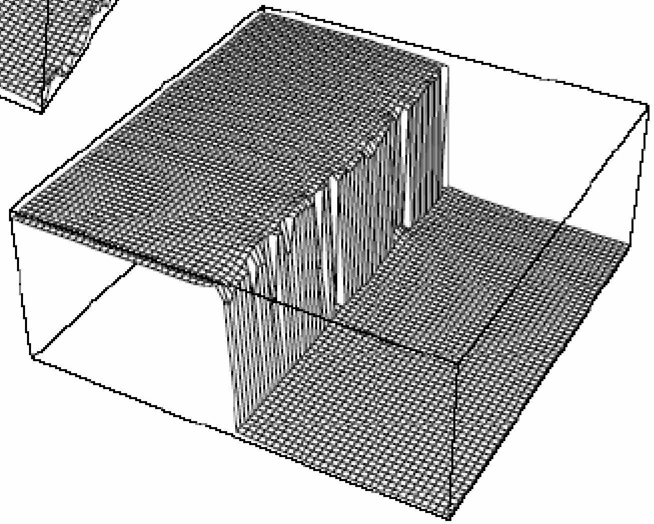
Magnitude of horizontal flow



a

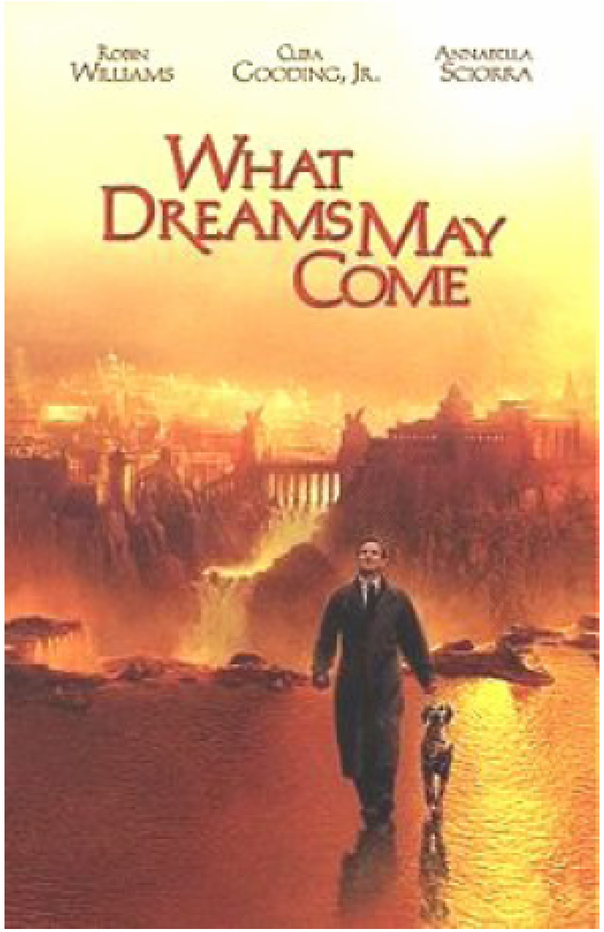


b



c

Applications of Optical Flow



Impressionist effect.
Transfer motion of real world to a painting

Input for the NPR algorithm

Brushes

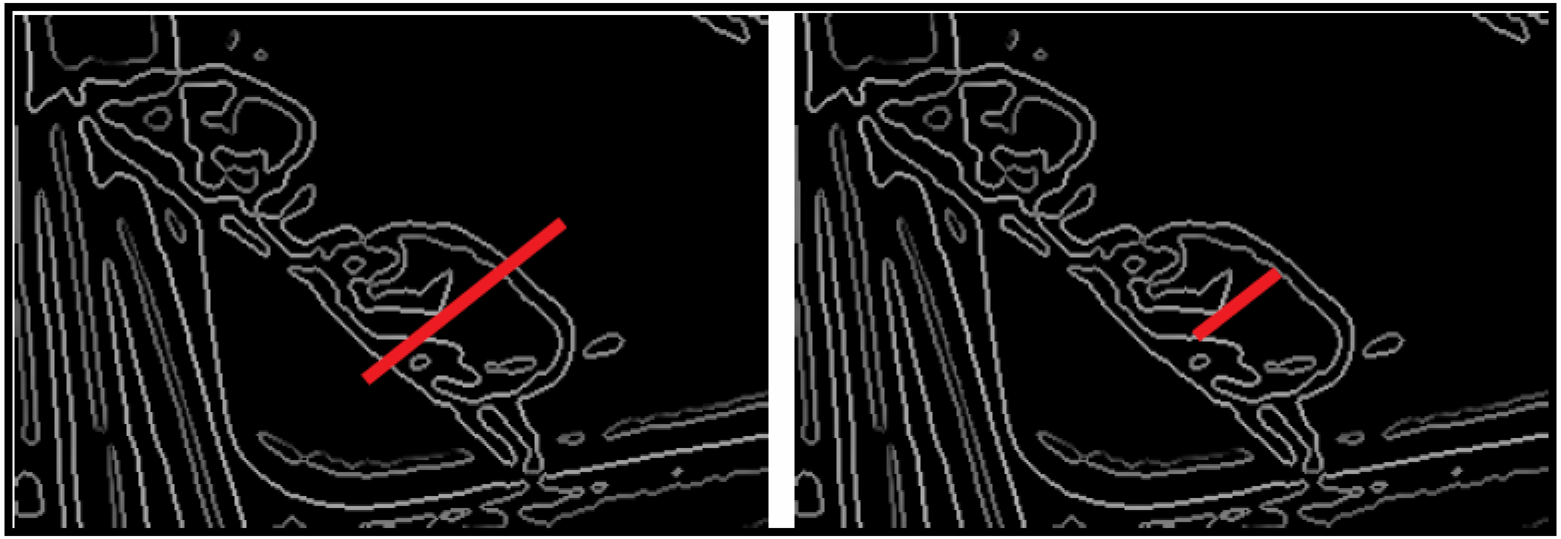
Edge clipping

Gradient

Smooth gradient

Textured brush

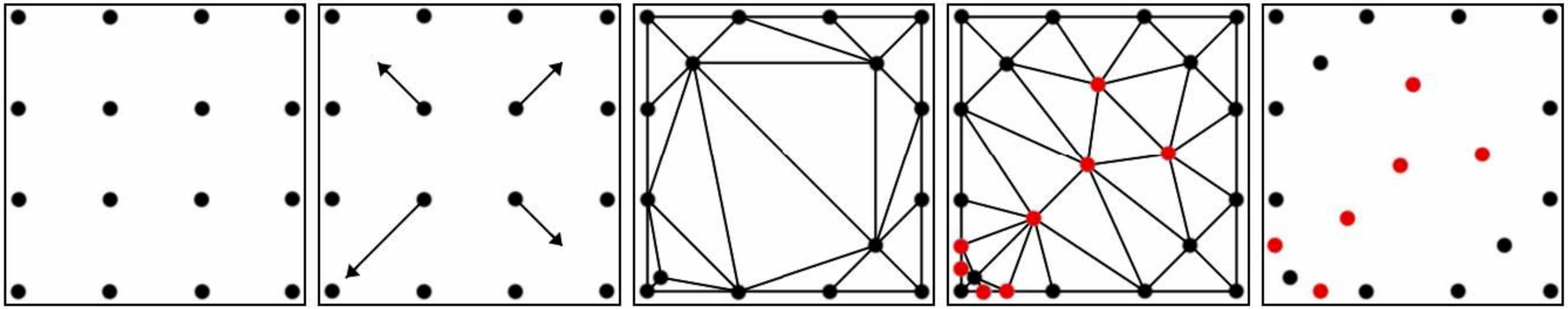
Edge clipping



Temporal artifacts

Frame-by-frame application of the NPR algorithm

Temporal coherence



RE:Vision

Video Gogh™ by RE:Vision Effects
Footage supplied by Videometry

What dreams may come

Reference

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- Bergen, J. R. and Anandan, P. and Hanna, K. J. and Hingorani, R., Hierarchical Model-Based Motion Estimation, ECCV 1992, pp237-252.
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