More on Features

Digital Visual Effects, Spring 2007

Yung-Yu Chuang

2007/3/27

with slides by Trevor Darrell Cordelia Schmid, David Lowe, Darya Frolova, Denis Simakov, Robert Collins, Brad Osgood, W W L Chen, and Jiwon Kim

Announcements

- Project #1 was due at noon today. You have a total of 10 delay days without penalty, but you are advised to use them wisely.
- We reserve the rights for not including late homework for artifact voting.
- Project #2 handout will be available on the web today.
- We may not have class next week. I will send out mails if the class is canceled.

Outline

- Harris corner detector
- SIFT
- SIFT extensions
- MSOP

Three components for features

- Feature detection
- Feature description
- Feature matching

Harris corner detector

Harris corner detector

- Consider all small shifts by Taylor's expansion

$$E(u,v)=\sum_{x,y} w(x,y)\left[I(x+u,y+v)-I(x,y)\right]^2$$

$$=\sum_{x,y} w(x,y)\left[I_x u+I_y v+O(u^2,v^2)\right]^2$$

$$E(u,v)=Au^2+2Cuv+Bv^2$$

$$A=\sum_{x,y} w(x,y)I_x^2(x,y)$$

$$B=\sum_{x,y} w(x,y)I_y^2(x,y)$$

$$C=\sum_{x,y} w(x,y)I_x(x,y)I_y(x,y)$$

Harris corner detector

Equivalently, for small shifts [*u*,*v*] we have a *bilinear* approximation:

$$E(u,v) \cong [u,v] \; M \begin{bmatrix} u \\ v \end{bmatrix}$$

, where *M* is a 2×2 matrix computed from image derivatives:

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

Harris corner detector (matrix form)

$$E(\mathbf{u}) = |I(\mathbf{x_0} + \mathbf{u}) - I(\mathbf{x_0})|^2$$

$$= \left| \left(I_0 + \frac{\partial I}{\partial \mathbf{u}}^T \mathbf{u} \right) - I_0 \right|^2$$

$$= \left| \frac{\partial I}{\partial \mathbf{u}}^T \mathbf{u} \right|^2$$

$$= \mathbf{u}^T \frac{\partial I}{\partial \mathbf{u}} \frac{\partial I}{\partial \mathbf{u}}^T \mathbf{u}$$

$$= \mathbf{u}^T \mathbf{H} \mathbf{u}$$

Quadratic forms

- Quadratic form (homogeneous polynomial of degree two) of n variables x_i

$$\sum_{i=1}^{n} \sum_{\substack{j=1 \\ i \le j}}^{n} c_{ij} x_i x_j$$

- E $4x_1^2 + 5x_2^2 + 3x_3^2 + 2x_1x_2 + 4x_1x_3 + 6x_2x_3$

$$= \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} \begin{pmatrix} 4 & 1 & 2 \\ 1 & 5 & 3 \\ 2 & 3 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

Symmetric matrices

- Quadratic forms can be represented by a real symmetric matrix $\mathbf{A}$ where

$$a_{ij} = \begin{cases} c_{ij} & \text{if } i = j, \\ \frac{1}{2}c_{ij} & \text{if } i < j, \\ \frac{1}{2}c_{ji} & \text{if } i > j. \end{cases}$$

$$\sum_{\substack{i=1 \\ i \le j}}^{n} \sum_{j=1}^{n} c_{ij} x_i x_j = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} x_i x_j$$

$$= \begin{pmatrix} x_1 & \dots & x_n \end{pmatrix} \begin{pmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \dots & a_{nn} \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

$$= \mathbf{x}^t A \mathbf{x}$$

Eigenvalues of symmetric matrices

DigiVFX

suppose $A \in \mathbf{R}^{n \times n}$ is symmetric, $i.e.$, $A = A^T$

fact: the eigenvalues of A are real

suppose $Av = \lambda v$, $v \neq 0$, $v \in \mathbf{C}^n$

$$\overline{v}^T A v = \overline{v}^T (Av) = \lambda \overline{v}^T v = \lambda \sum_{i=1}^{n} |v_i|^2$$

$$\overline{v}^T A v = \overline{(Av)}^T v = \overline{(\lambda v)}^T v = \overline{\lambda} \sum_{i=1}^{n} |v_i|^2$$

we have $\lambda = \overline{\lambda}$, $i.e.$, $\lambda \in \mathbf{R}$

(hence, can assume $v \in \mathbf{R}^n$)

Brad Osgood

Eigenvectors of symmetric matrices

suppose $A \in \mathbf{R}^{n\times n}$ is symmetric, *i.e.*, $A = A^T$

fact: there is a set of orthonormal eigenvectors of A

$$A = Q\Lambda Q^T$$

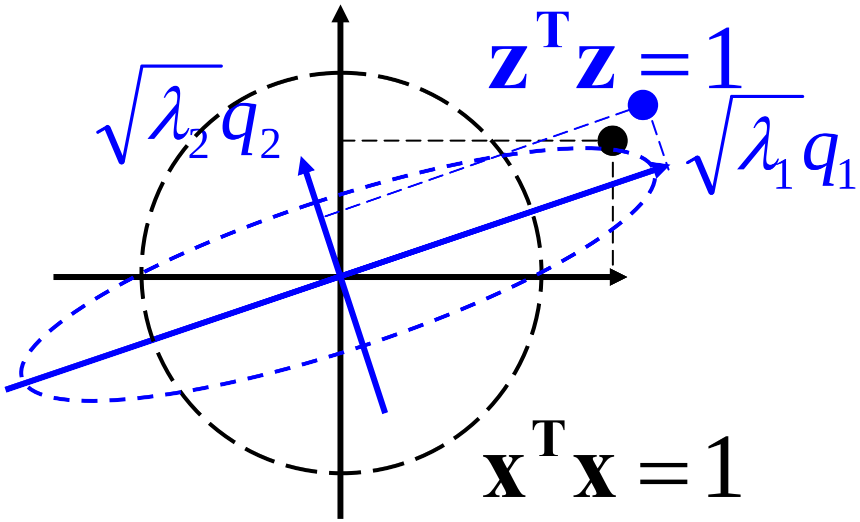
$$\mathbf{x}^\mathbf{T}\mathbf{A}\mathbf{x}$$

$$= \mathbf{x}^\mathbf{T}\mathbf{Q}\,\boldsymbol{\Lambda}\,\mathbf{Q}^\mathbf{T}\mathbf{x}$$

$$= \left(\mathbf{Q}^\mathbf{T}\mathbf{x}\right)^\mathbf{T}\boldsymbol{\Lambda}\left(\mathbf{Q}^\mathbf{T}\mathbf{x}\right)$$

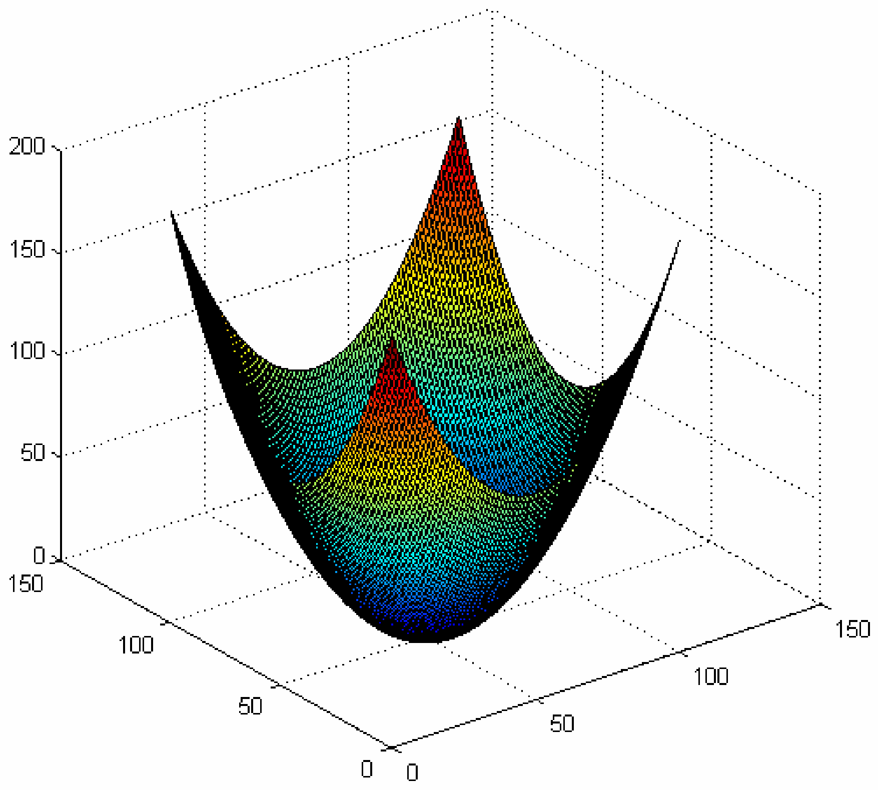
$$= \mathbf{y}^\mathbf{T}\boldsymbol{\Lambda}\mathbf{y}$$

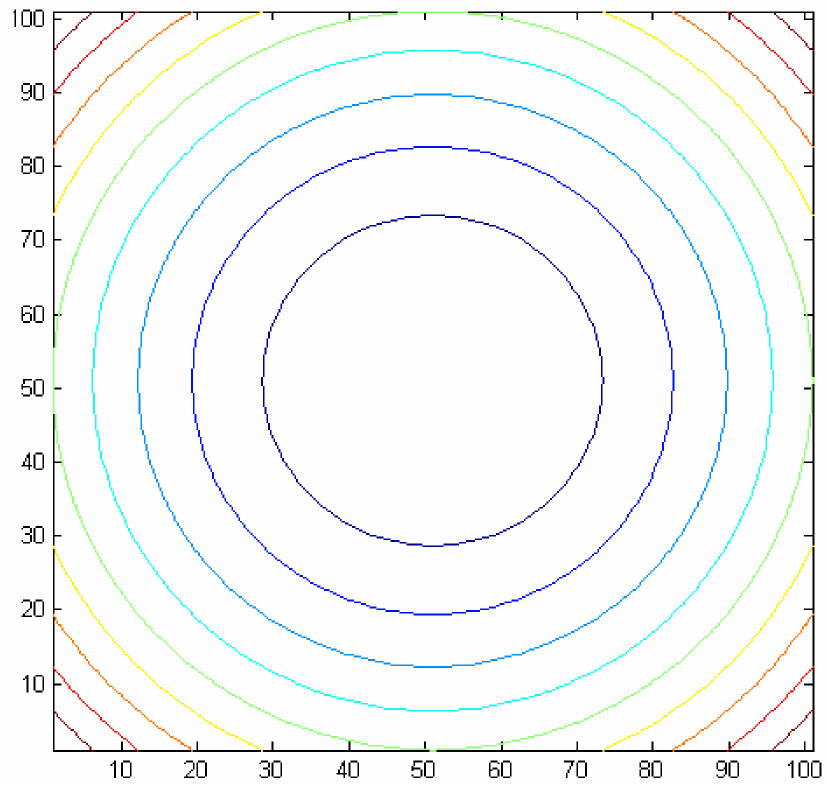
$$= \left(\boldsymbol{\Lambda}^{\frac{1}{2}}\mathbf{y}\right)^\mathbf{T}\left(\boldsymbol{\Lambda}^{\frac{1}{2}}\mathbf{y}\right)$$



Visualize quadratic functions

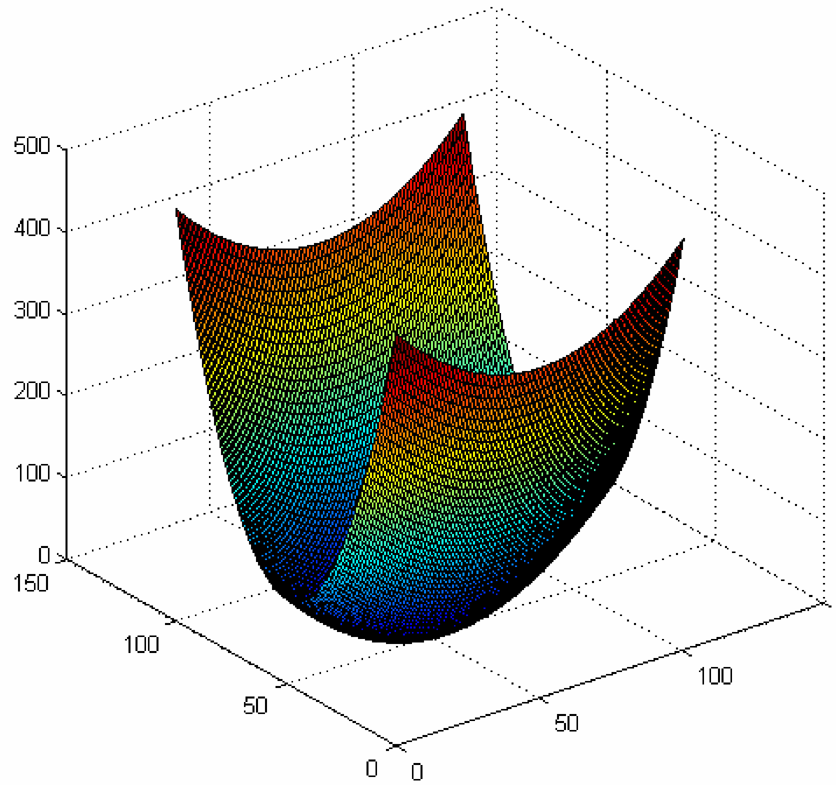
$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^T$$

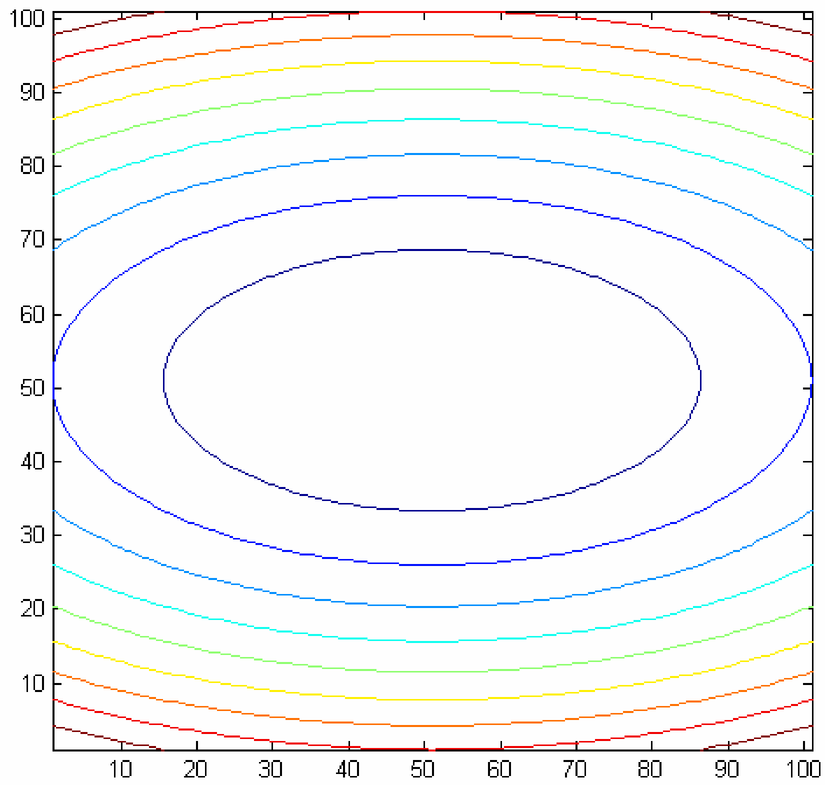




Visualize quadratic functions

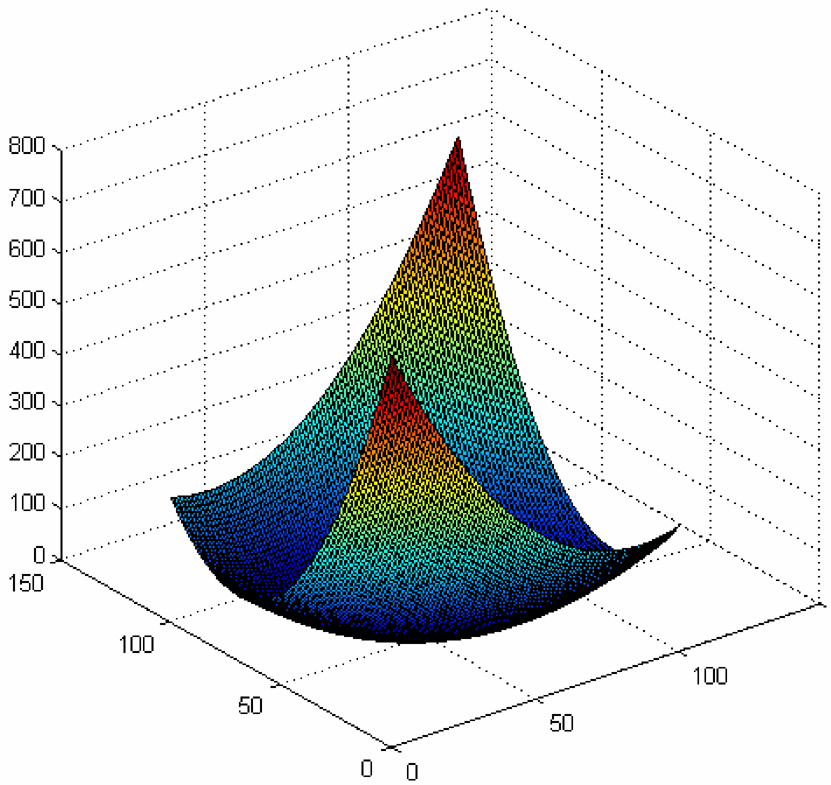
$$A = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^T$$

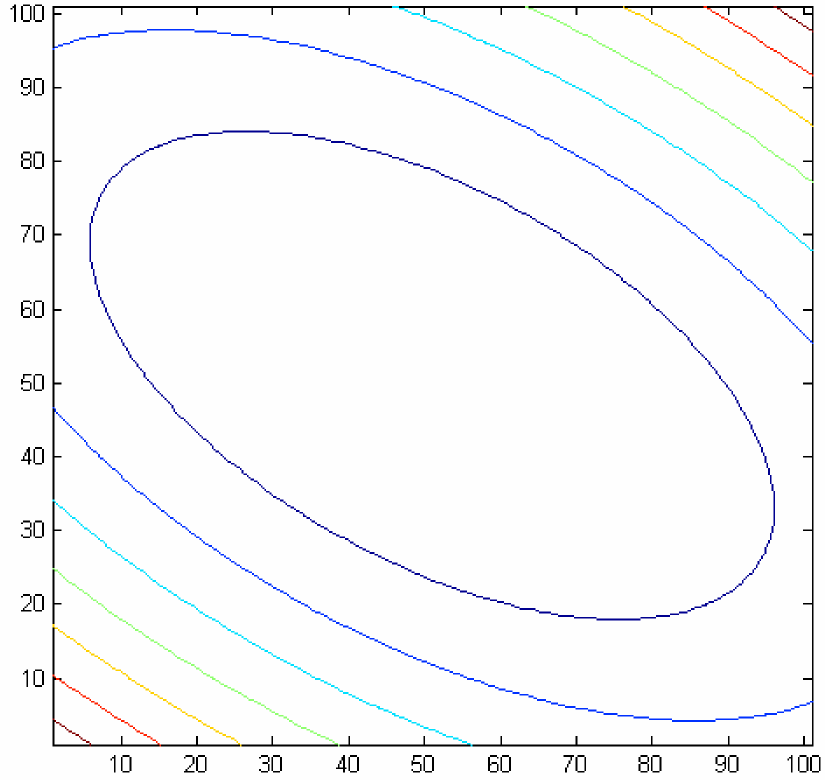




Visualize quadratic functions

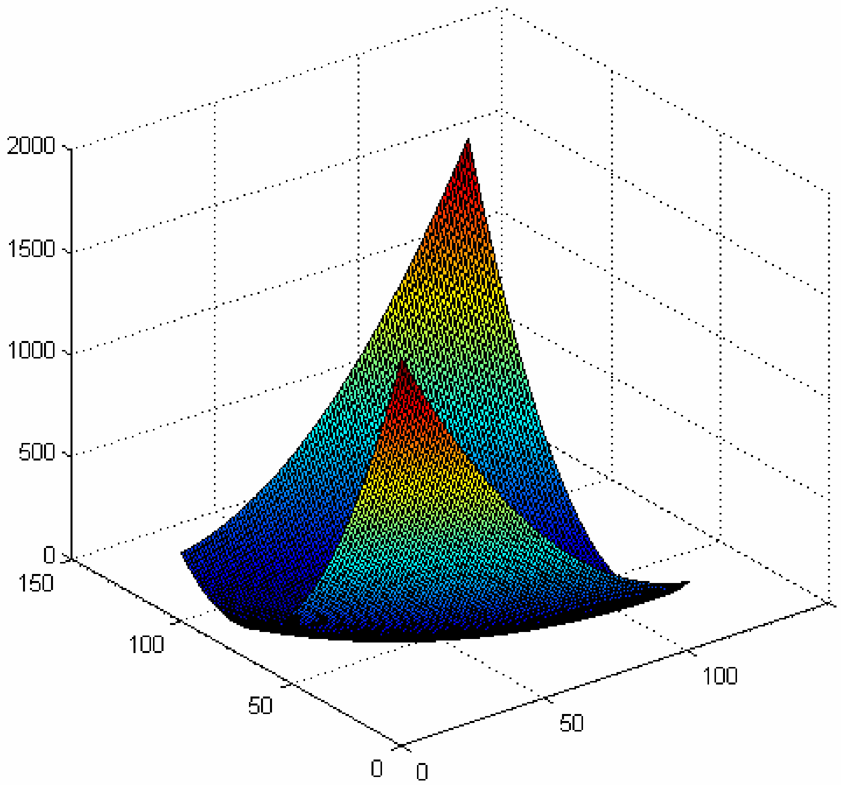
$$A=\begin{bmatrix}3.25 & 1.30\\ 1.30 & 1.75\end{bmatrix}=\begin{bmatrix}0.50 & -0.87\\ -0.87 & -0.50\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 4\end{bmatrix}\begin{bmatrix}0.50 & -0.87\\ -0.87 & -0.50\end{bmatrix}^T$$

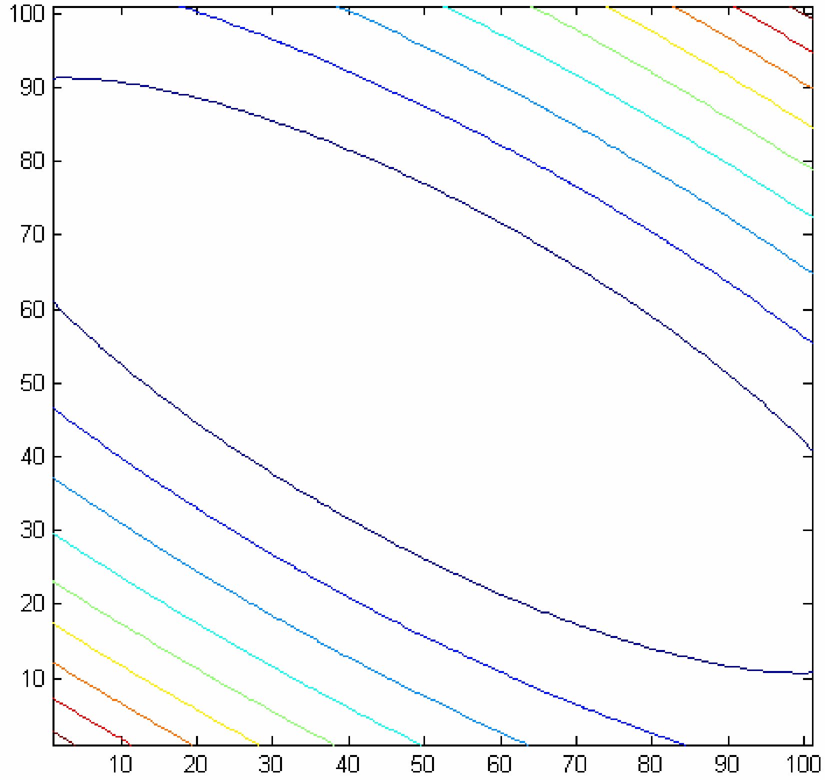




Visualize quadratic functions

$$A=\begin{bmatrix}7.75 & 3.90\\ 3.90 & 3.25\end{bmatrix}=\begin{bmatrix}0.50 & -0.87\\ -0.87 & -0.50\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 10\end{bmatrix}\begin{bmatrix}0.50 & -0.87\\ -0.87 & -0.50\end{bmatrix}^T$$





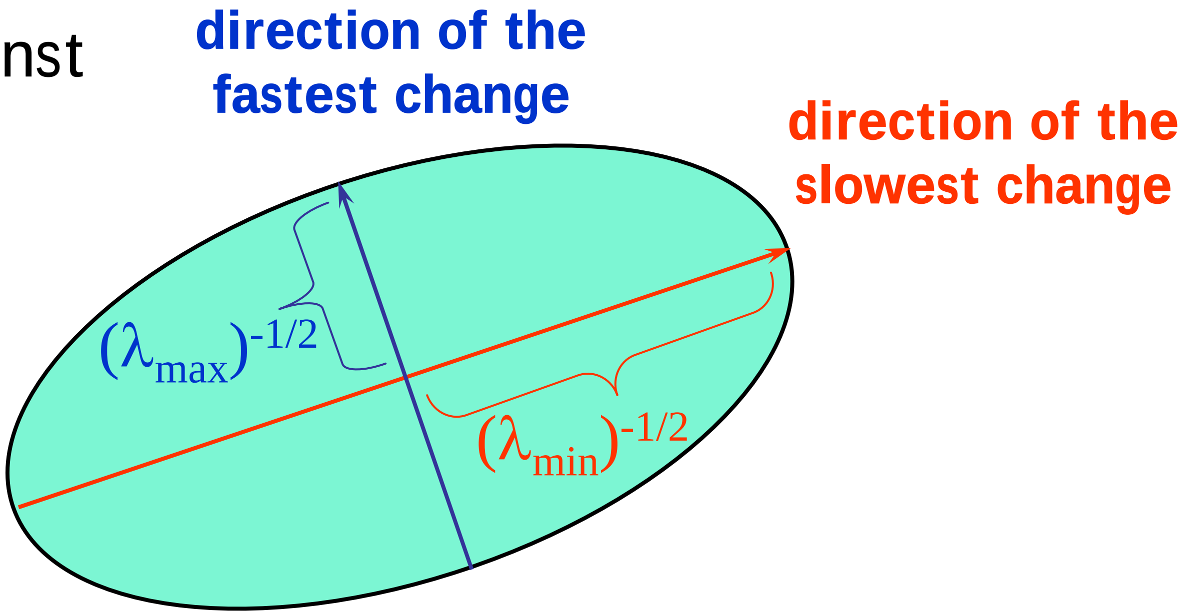
Harris corner detector

Intensity change in shifting window: eigenvalue analysis

$$E(u,v) \cong [u,v] \; M \begin{bmatrix} u \\ v \end{bmatrix}$$

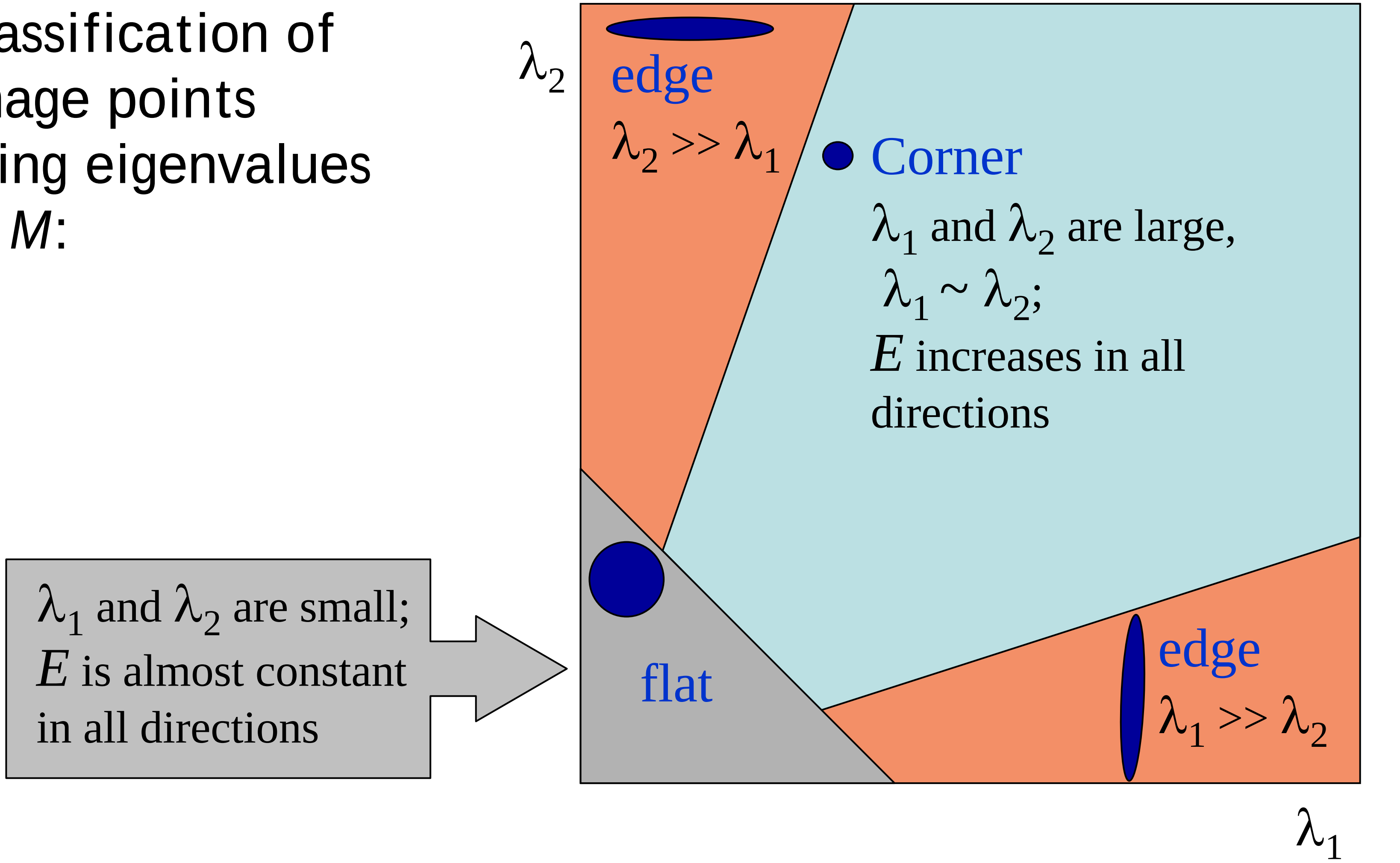
λ_1, λ_2 – eigenvalues of M

Ellipse $E(u,v)$ = const



Harris corner detector

Classification of image points using eigenvalues of *M*:



Harris corner detector

$$\lambda = \frac{a_{00} + a_{11} \pm \sqrt{(a_{00} - a_{11})^2 + 4a_{10}a_{01}}}{2}$$

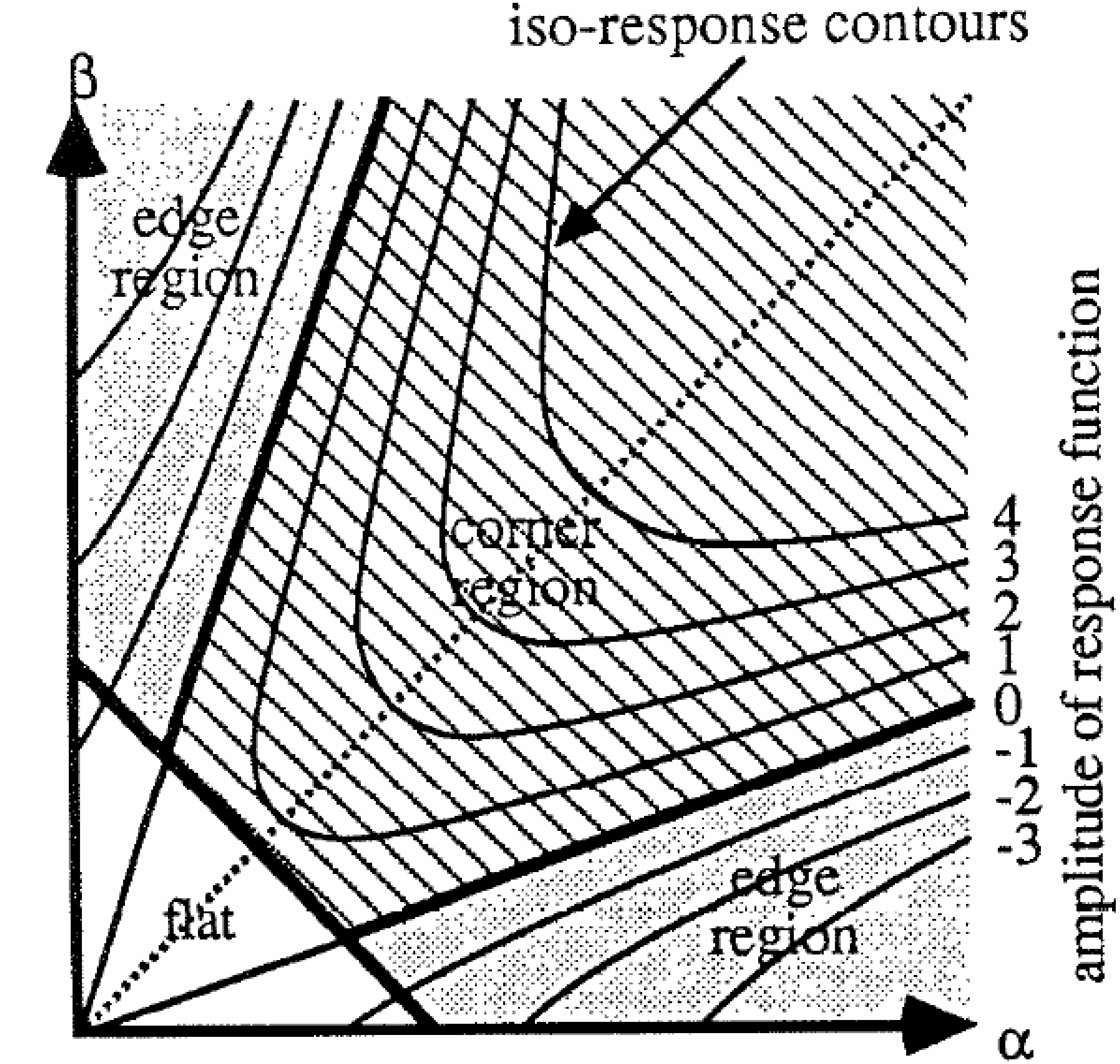
Measure of corner response:

$$R = \det M - k\left(\operatorname{trace} M\right)^2$$

$$\det M = \lambda_1 \lambda_2$$
$$\operatorname{trace} M = \lambda_1 + \lambda_2$$

(*k* – empirical constant, *k* = 0.04-0.06)

Harris corner detector



Summary of Harris detector

DigiVFX

1. Compute x and y derivatives of image

$$I_x = G_\sigma^x * I \quad I_y = G_\sigma^y * I$$

2. Compute products of derivatives at every pixel

$$I_{x2} = I_x . I_x \quad I_{y2} = I_y . I_y \quad I_{xy} = I_x . I_y$$

3. Compute the sums of the products of derivatives at each pixel

$$S_{x2} = G_{\sigma\prime} * I_{x2} \quad S_{y2} = G_{\sigma\prime} * I_{y2} \quad S_{xy} = G_{\sigma\prime} * I_{xy}$$

4. Define at each pixel (x, y) the matrix

$$H(x,y) = \begin{bmatrix} S_{x2}(x,y) & S_{xy}(x,y) \\ S_{xy}(x,y) & S_{y2}(x,y) \end{bmatrix}$$

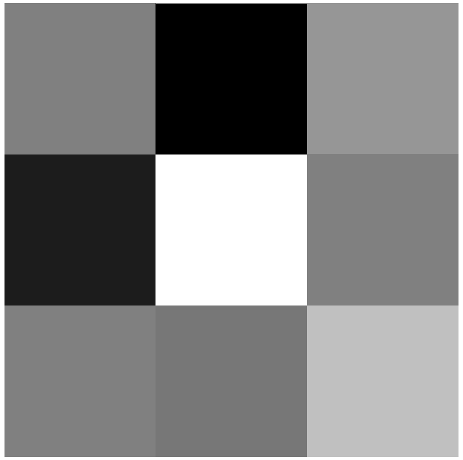
5. Compute the response of the detector at each pixel

$$R = Det(H) - k(Trace(H))^2$$

6. Threshold on value of R. Compute nonmax suppression.

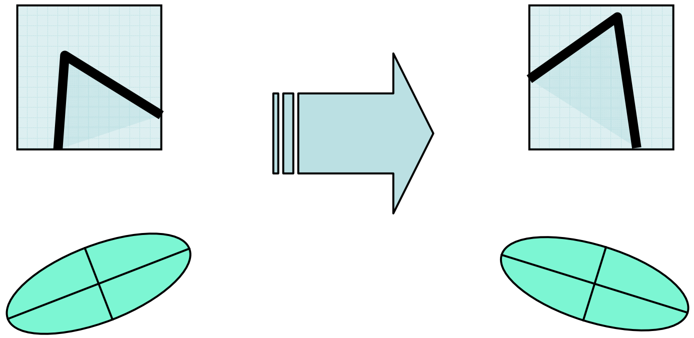
Now we know where features are

- But, how to match them?
- What is the descriptor for a feature? The simplest solution is the intensities of its spatial neighbors. This might not be robust to brightness change or small shift/rotation.



Harris Detector: Some Properties

- Rotation invariance

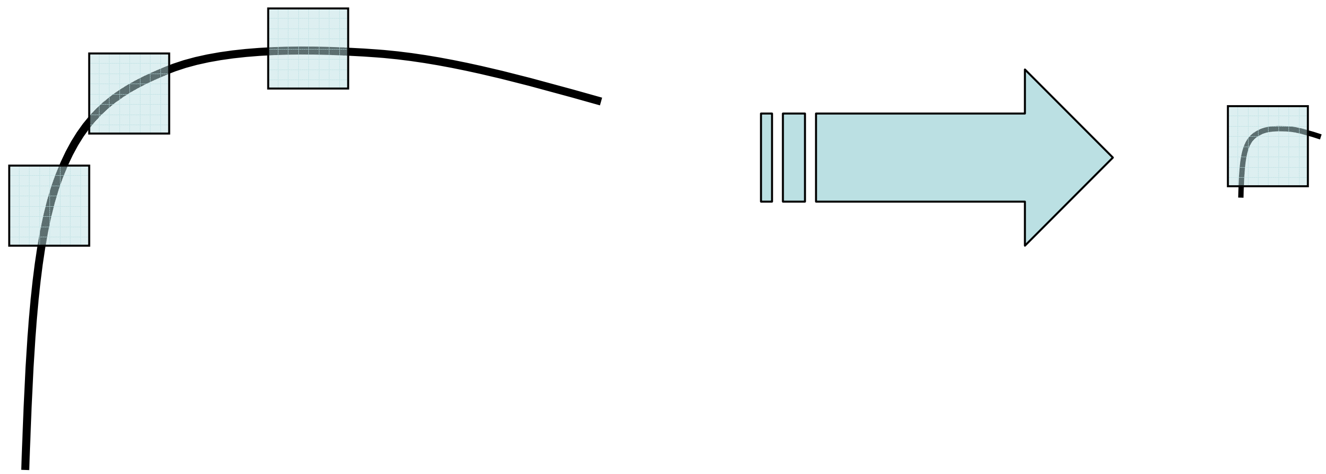


Ellipse rotates but its shape (i.e. eigenvalues) remains the same

Corner response R is invariant to image rotation

Harris Detector: Some Properties

- But: non-invariant to *image scale*!

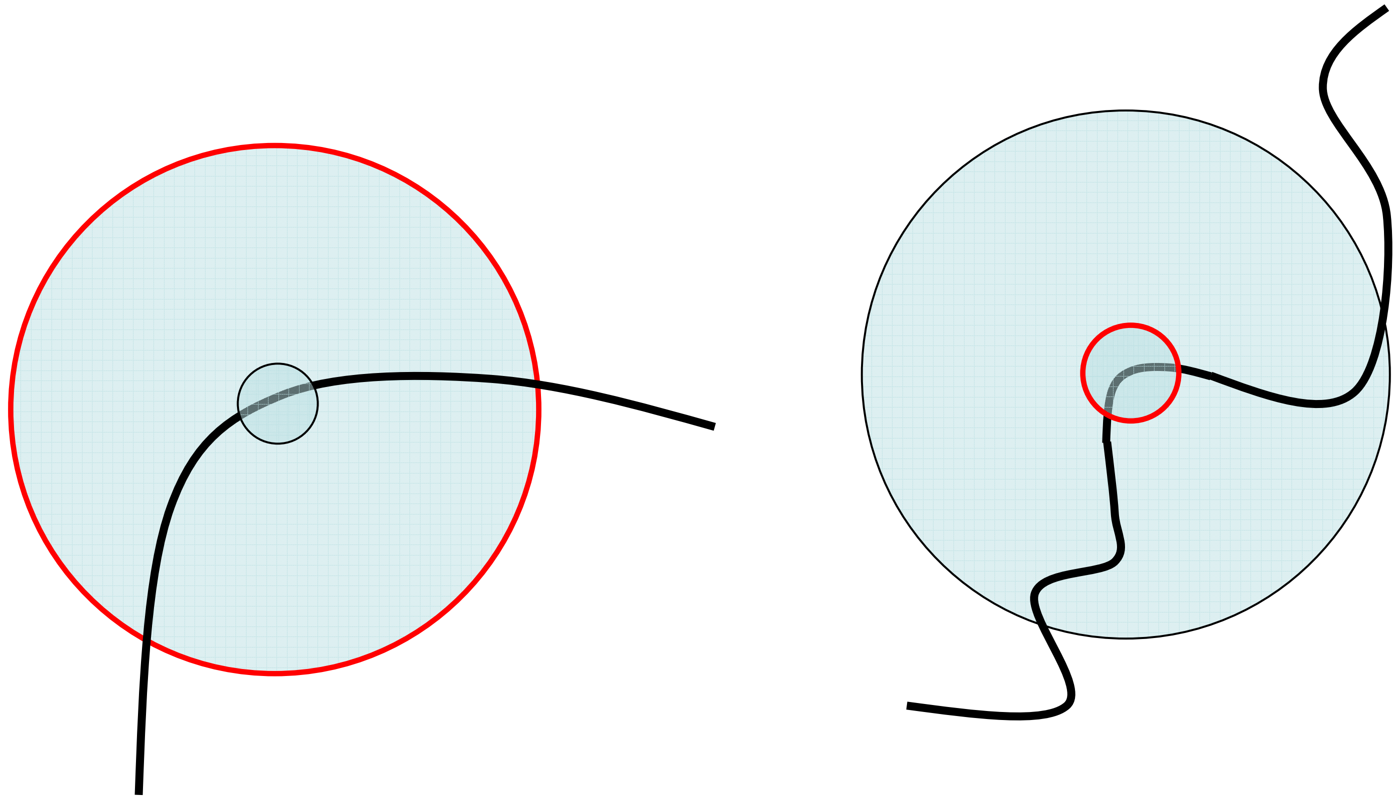


All points will be classified as edges

Corner !

Scale invariant detection

- The problem: how do we choose corresponding circles *independently* in each image?
- Aperture problem



SIFT
(Scale Invariant Feature Transform)

SIFT

- SIFT is an carefully designed procedure with empirically determined parameters for the invariant and distinctive features.

SIFT stages:

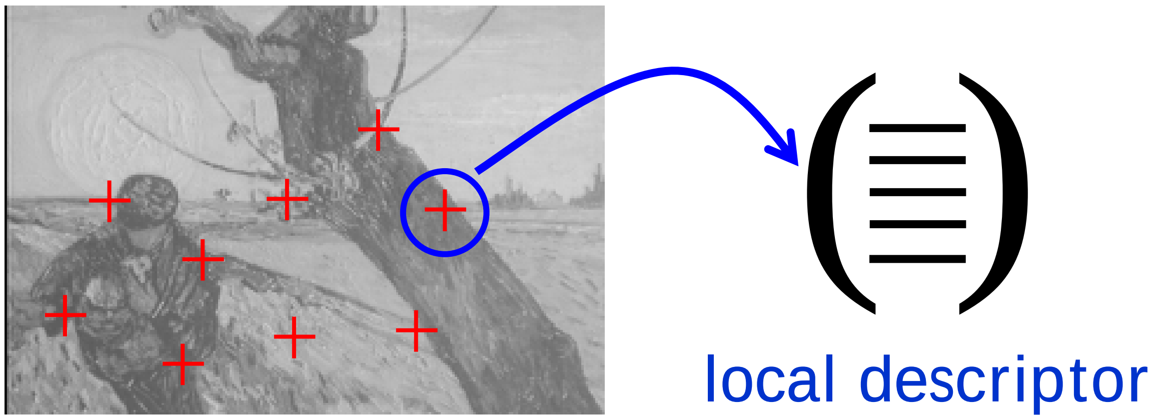
- Scale-space extrema detection
- Keypoint localization

detector

- Orientation assignment
- Keypoint descriptor

descriptor

matching



A 500x500 image gives about 2000 features

1. Detection of scale-space extrema

- For scale invariance, search for stable features across all possible scales using a continuous function of scale, scale space.
- SIFT uses DoG filter for scale space because it is efficient and as stable as scale-normalized Laplacian of Gaussian.

DoG filtering

Convolution with a variable-scale Gaussian

$$L(x, y, \sigma) = G(x, y, \sigma) * I(x, y),$$

$$G(x, y, \sigma) = 1/(2\pi\sigma^2) \exp^{-(x^2+y^2)/\sigma^2}$$

Difference-of-Gaussian (DoG) filter

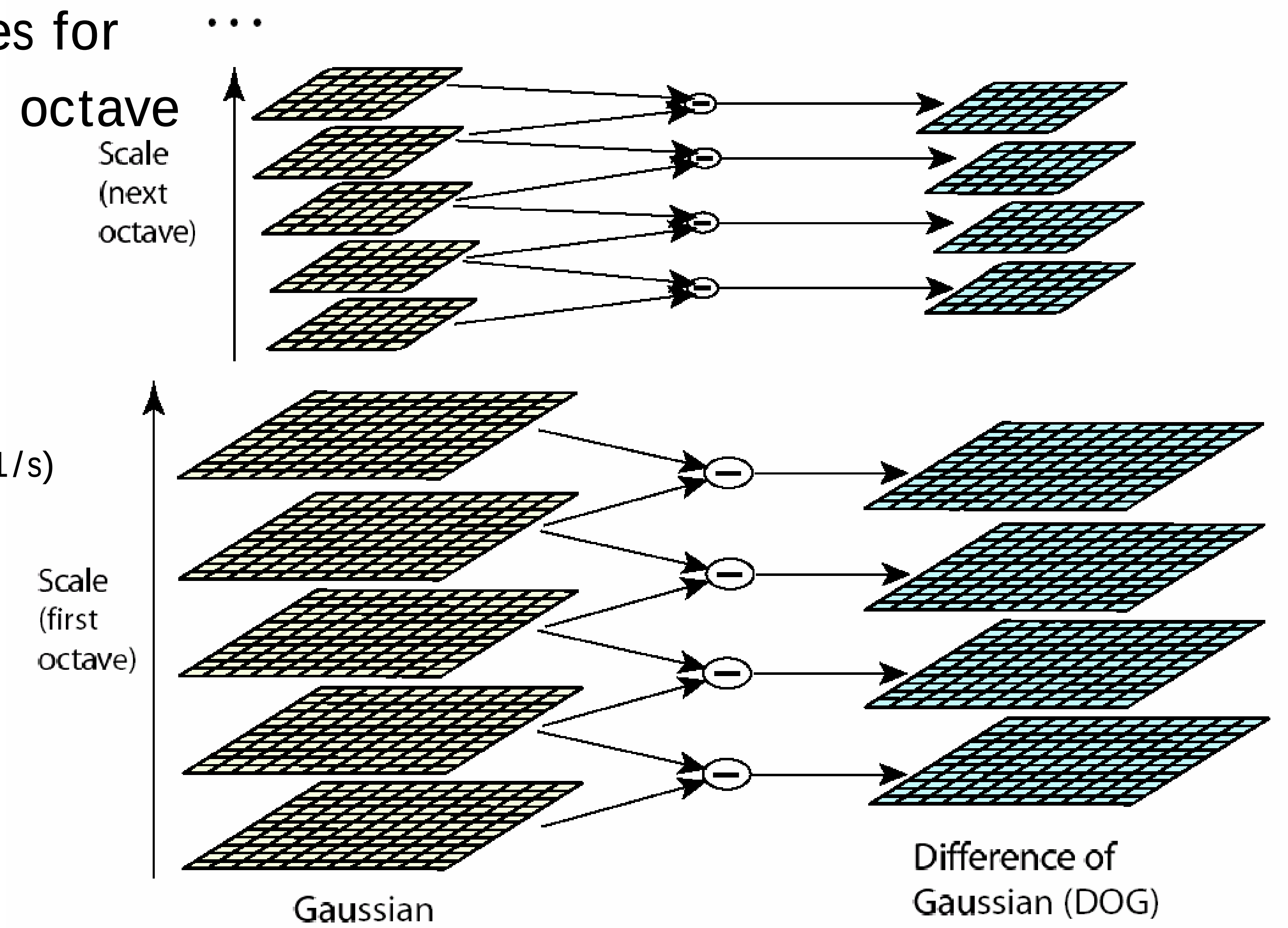
$$G(x, y, k\sigma) - G(x, y, \sigma)$$

Convolution with the DoG filter

$$\begin{aligned} D(x, y, \sigma) &= (G(x, y, k\sigma) - G(x, y, \sigma)) * I(x, y) \\ &= L(x, y, k\sigma) - L(x, y, \sigma). \end{aligned}$$

Scale space

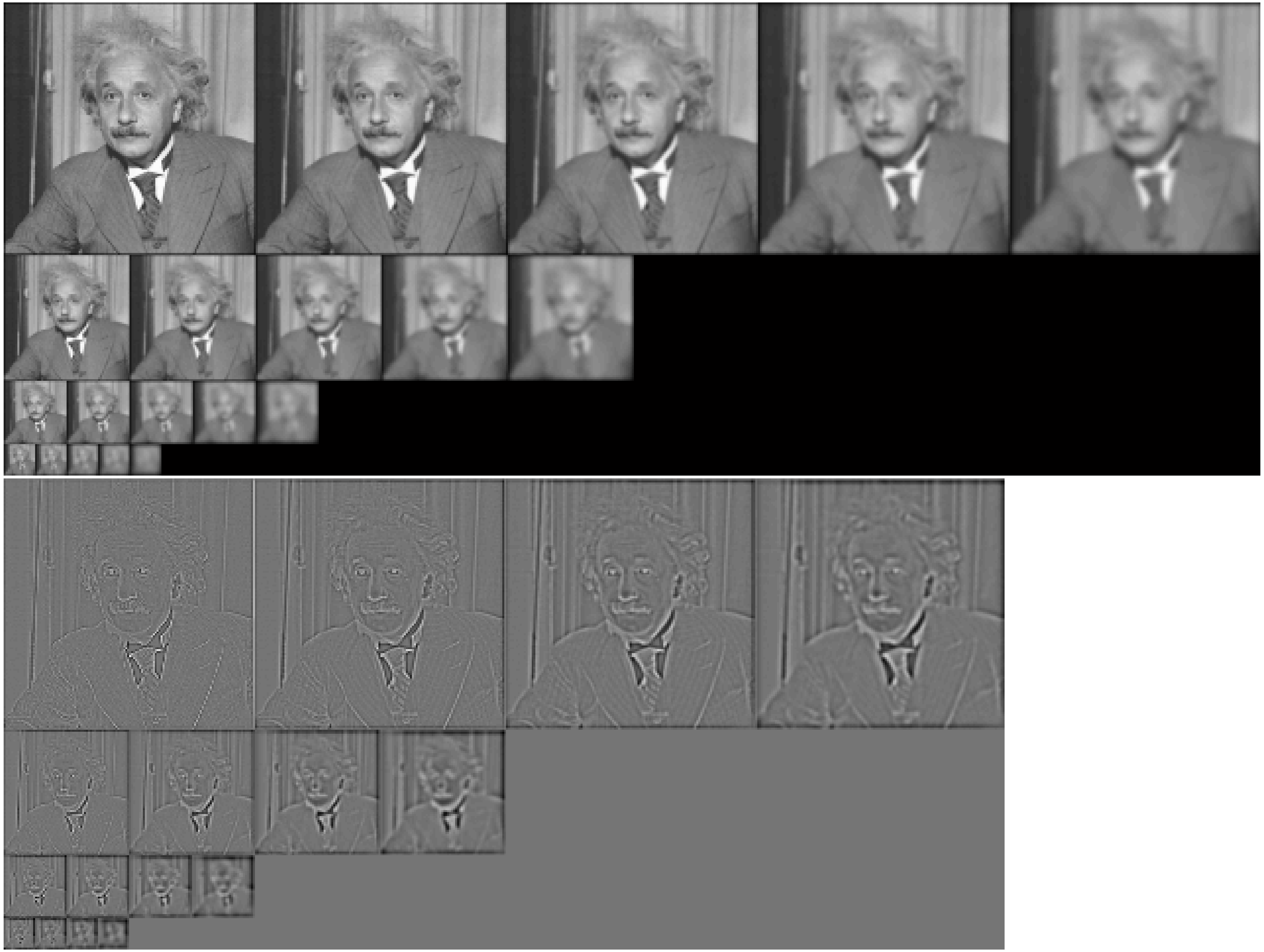
σ doubles for the next octave



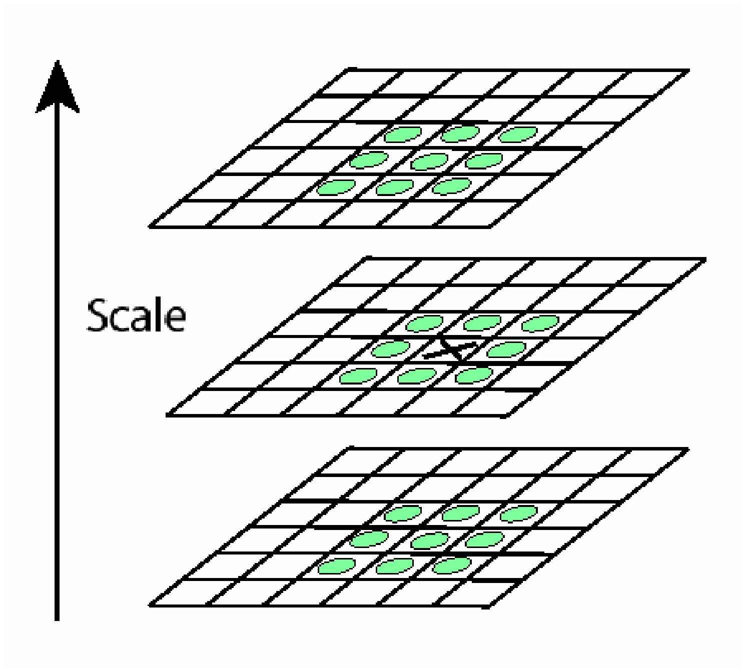
$K=2^{(1/s)}$

Dividing into octave is for efficiency only.

Detection of scale-space extrema



Keypoint localization

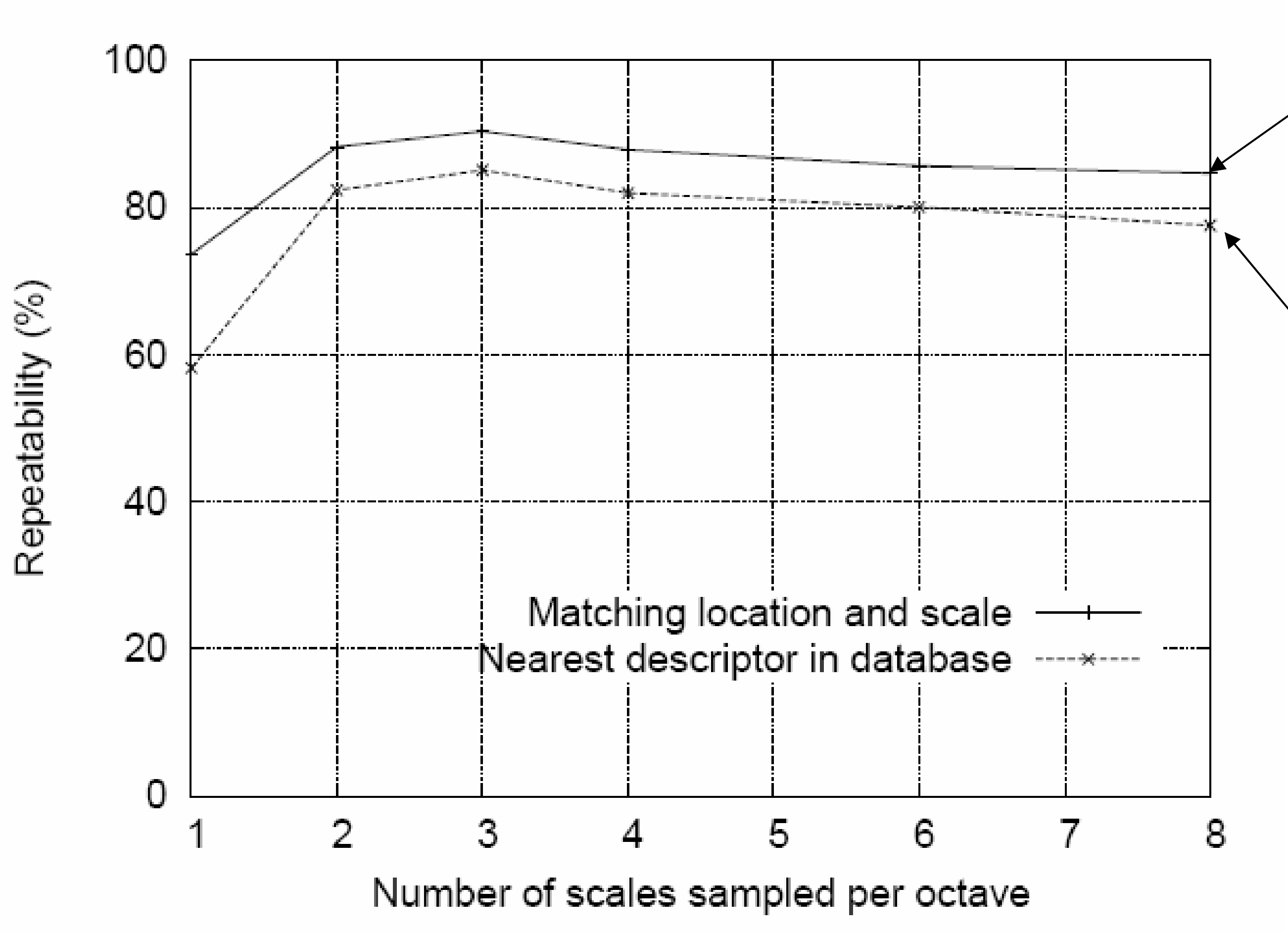


X is selected if it is larger or smaller than all 26 neighbors

Decide scale sampling frequency

- It is impossible to sample the whole space, tradeoff efficiency with completeness.
- Decide the best sampling frequency by experimenting on 32 real image subject to synthetic transformations. (rotation, scaling, affine stretch, brightness and contrast change, adding noise…)

Decide scale sampling frequency

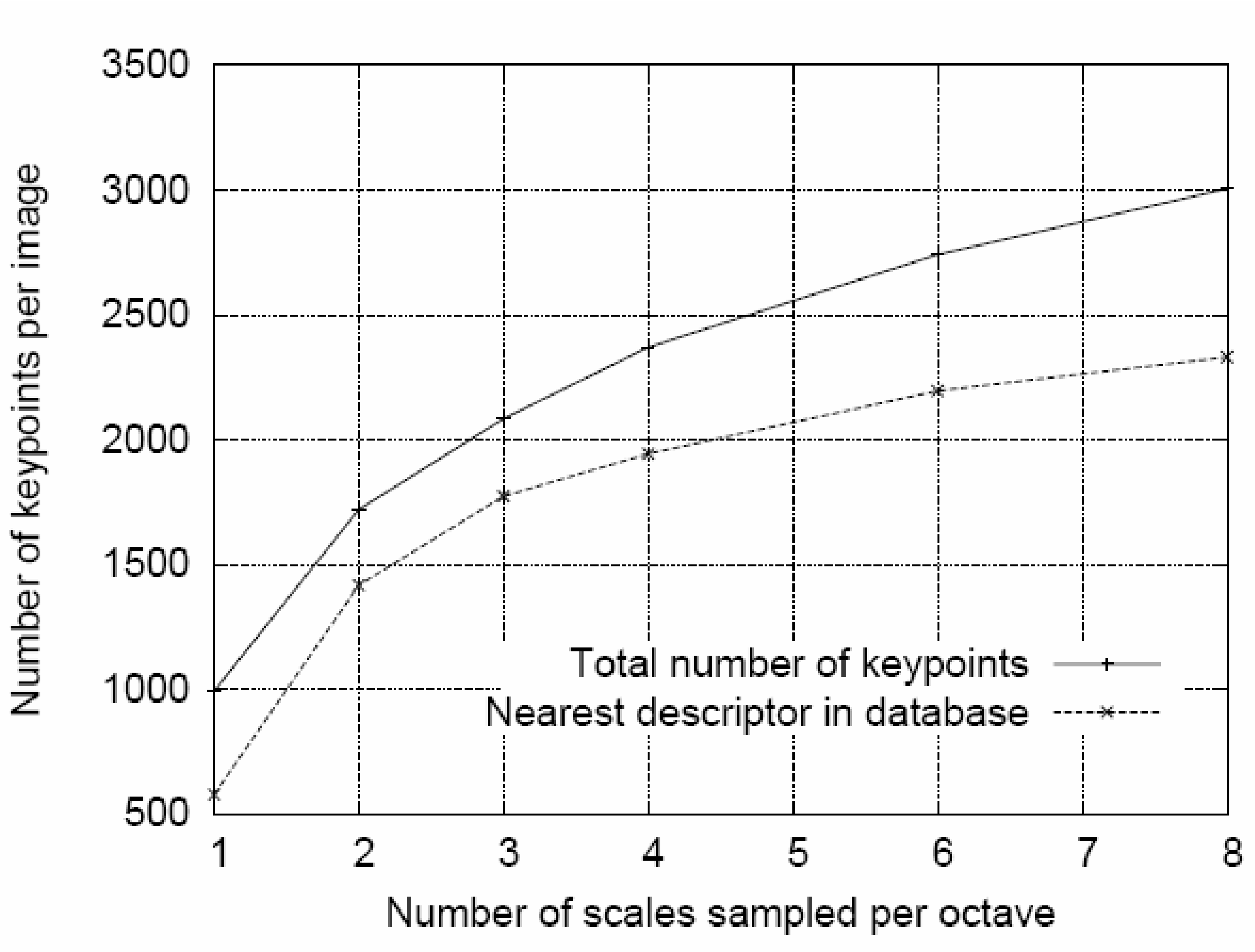


for detector, repeatability

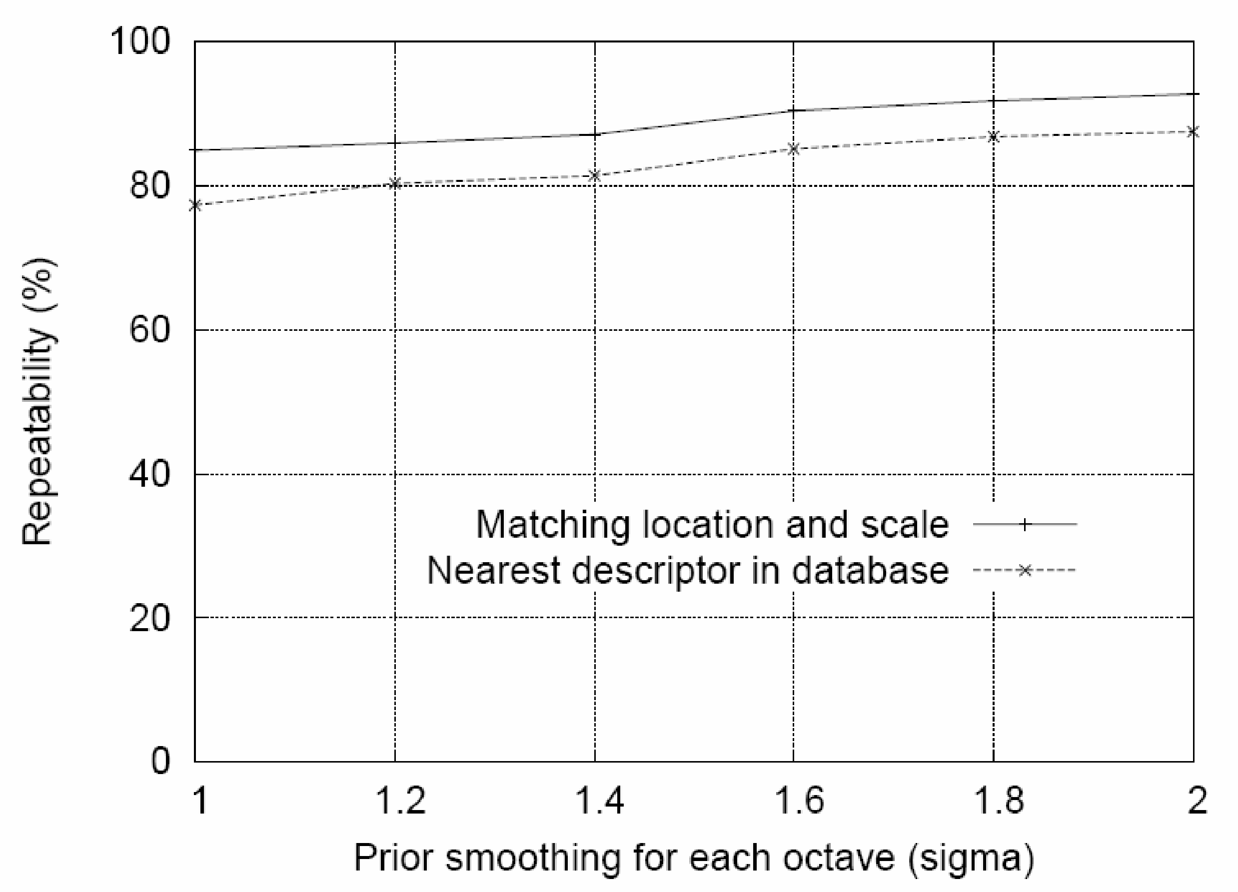
for descriptor, distinctiveness

S=3, for larger s, too many unstable features

Decide scale sampling frequency

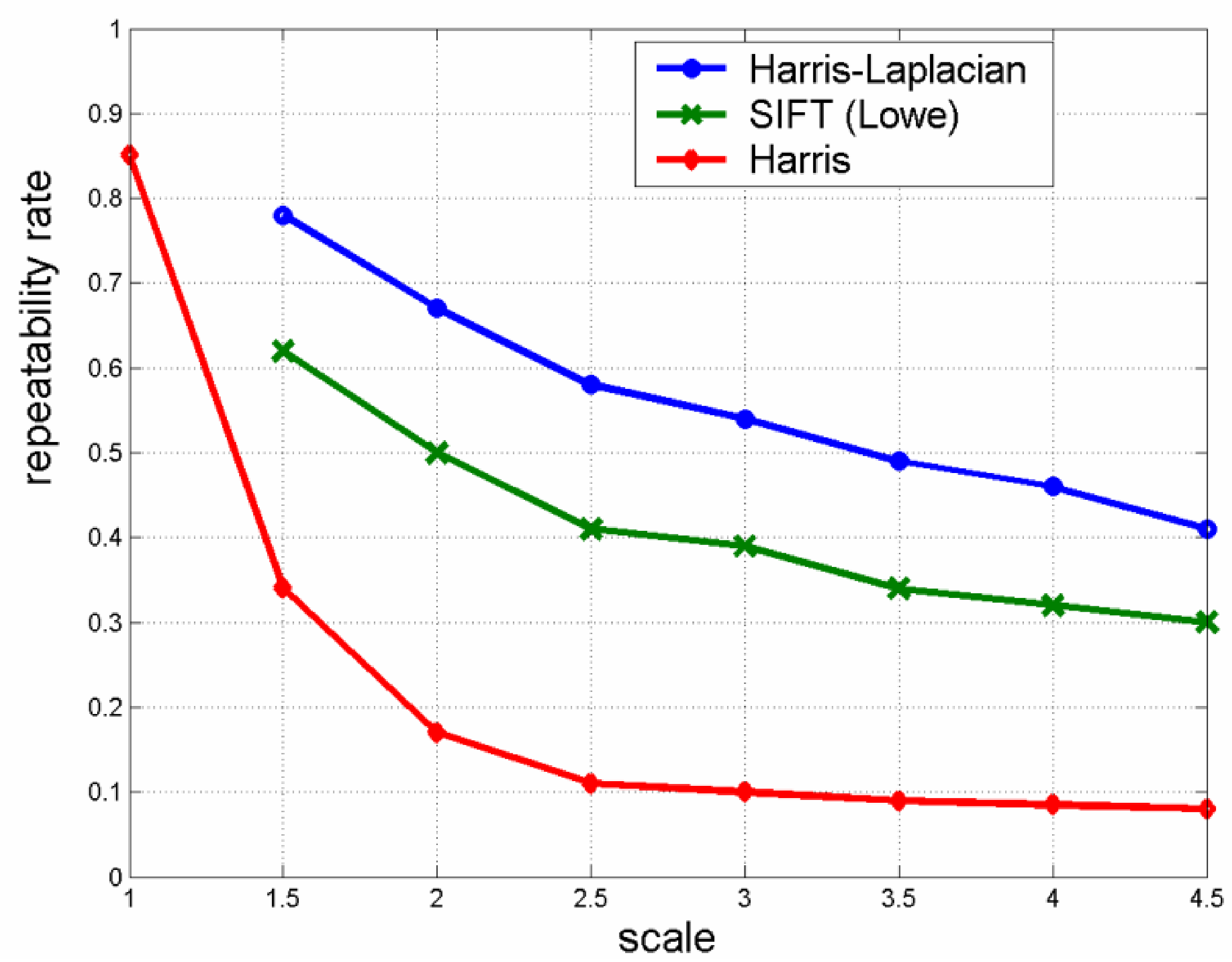


Pre-smoothing



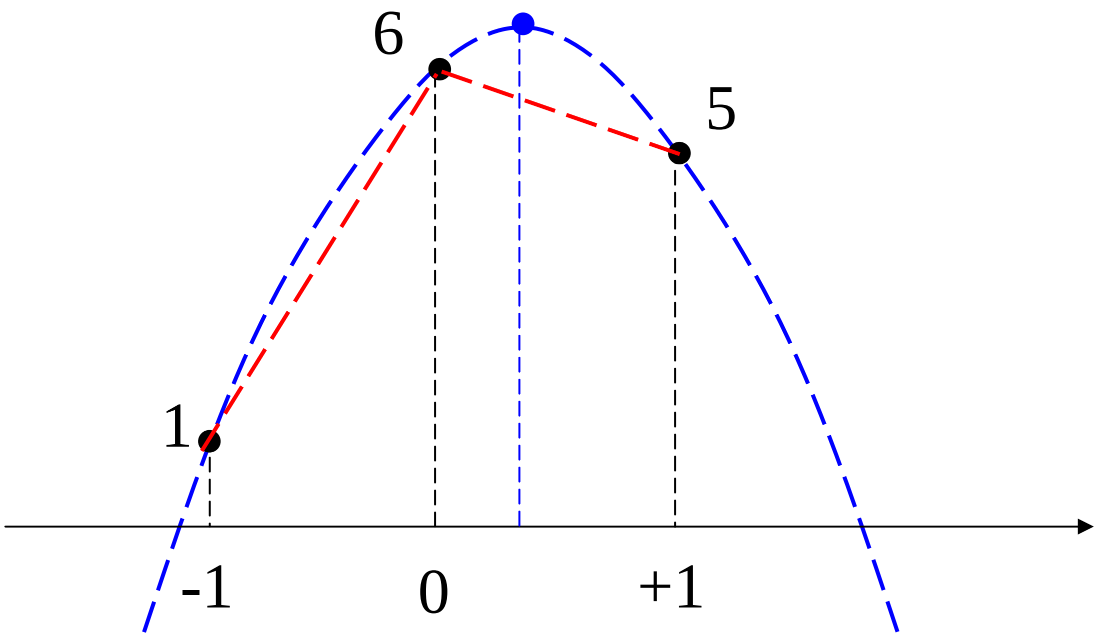
σ =1.6, plus a double expansion

Scale invariance



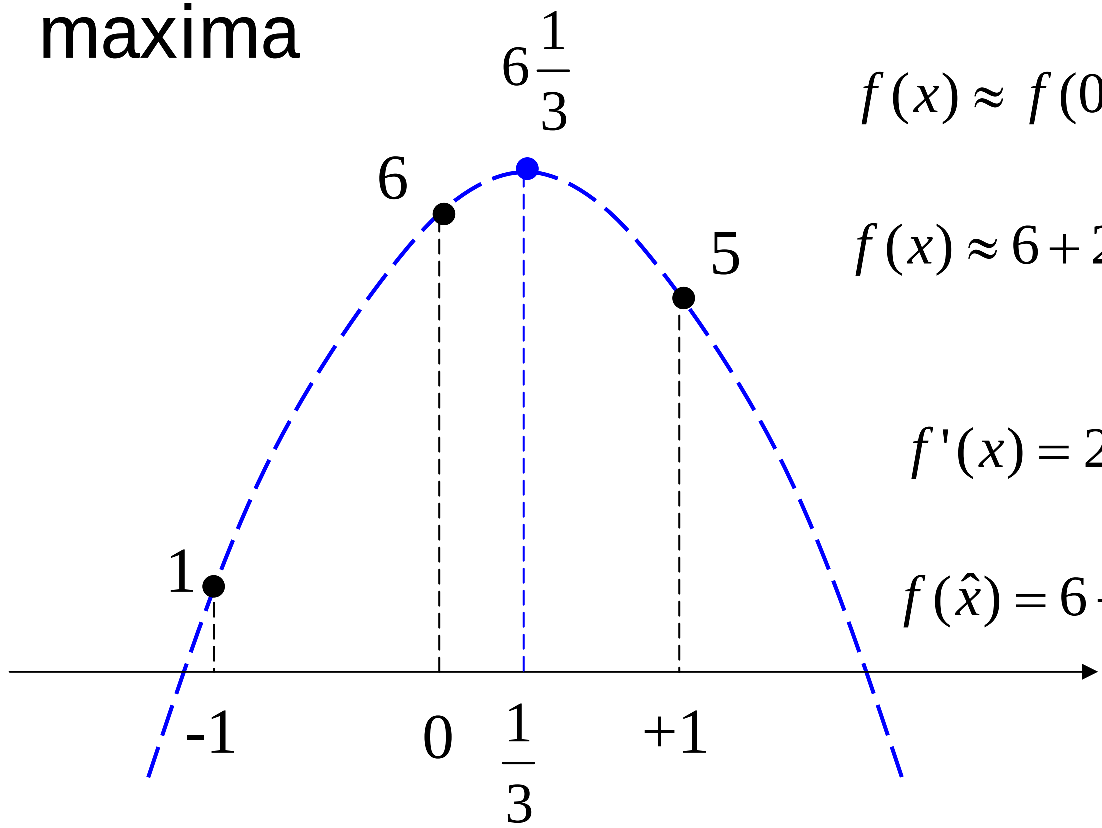
2. Accurate keypoint localization

- Reject points with low contrast (flat) and poorly localized along an edge (edge)
- Fit a 3D quadratic function for sub-pixel maxima



2. Accurate keypoint localization

- Reject points with low contrast and poorly localized along an edge
- Fit a 3D quadratic function for sub-pixel maxima



$$f(x) \approx f(0) + f'(0)x + \frac{f''(0)}{2}x^2$$

$$f(x) \approx 6 + 2x + \frac{-6}{2}x^2 = 6 + 2x - 3x^2$$

$$f'(x) = 2 - 6x = 0 \longrightarrow \hat{x} = \frac{1}{3}$$

$$f(\hat{x}) = 6 + 2 \cdot \frac{1}{3} - 3 \cdot \left(\frac{1}{3}\right)^2 = 6\frac{1}{3}$$

2. Accurate keypoint localization

- Taylor series of several variables

$$T(x_1, \cdots, x_d) = \sum_{n_1=0}^{\infty} \cdots \sum_{n_d=0}^{\infty} \frac{\partial^{n_1}}{\partial x_1^{n_1}} \cdots \frac{\partial^{n_d}}{\partial x_d^{n_d}} \frac{f(a_1, \cdots, a_d)}{n_1! \cdots n_d!} (x_1 - a_1)^{n_1} \cdots (x_d - a_d)^{n_d}$$

- Two variables

$$f(x,y) \approx f(0,0) + \left(\frac{\partial f}{\partial x} x + \frac{\partial f}{\partial y} y \right) + \frac{1}{2} \left(\frac{\partial^2 f}{\partial x \partial x} x^2 + 2 \frac{\partial^2 f}{\partial x \partial y} xy + \frac{\partial^2 f}{\partial y \partial y} y^2 \right)$$

$$f\left(\begin{bmatrix} x \\ y \end{bmatrix} \right) \approx f\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix} \right) + \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \frac{1}{2} \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} \frac{\partial^2 f}{\partial x \partial x} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial y \partial y} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$f(\mathbf{x}) \approx f(\mathbf{0}) + \frac{\partial f}{\partial \mathbf{x}}^T \mathbf{x} + \frac{1}{2} \mathbf{x}^T \frac{\partial^2 f}{\partial \mathbf{x}^2} \mathbf{x}$$

Accurate keypoint localization

- Taylor expansion in in matrix form, $\mathbf{x}$ is a vector, f maps $\mathbf{x}$ to a scalar

$$f(\mathbf{x}) = f + \frac{\partial f}{\partial \mathbf{x}}^T \mathbf{x} + \frac{1}{2}\mathbf{x}^T \frac{\partial^2 f}{\partial \mathbf{x}^2}\mathbf{x}$$

Hessian matrix (often symmetric)

gradient
$$\begin{pmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_1} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{pmatrix}$$

$$\begin{pmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_n} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \cdots & \frac{\partial^2 f}{\partial x_2 \partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial x_n \partial x_1} & \frac{\partial^2 f}{\partial x_n \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_n^2} \end{pmatrix}$$

2D illustration

$$f(\mathbf{x}) = f + \frac{\partial f}{\partial \mathbf{x}}^T \mathbf{x} + \frac{1}{2}\mathbf{x}^T \frac{\partial^2 f}{\partial \mathbf{x}^2}\mathbf{x}$$

$f_{-1,1}$	$f_{0,1}$	$f_{1,1}$
$f_{-1,0}$	$f_{0,0}$	$f_{1,0}$
$f_{-1,-1}$	$f_{0,-1}$	$f_{1,-1}$

$$\frac{\partial f}{\partial x} = (f_{1,0} - f_{-1,0})/2$$

$$\frac{\partial f}{\partial y} = (f_{0,1} - f_{0,-1})/2$$

$$\frac{\partial^2 f}{\partial x^2} = f_{1,0} - 2f_{0,0} + f_{-1,0}$$

$$\frac{\partial^2 f}{\partial y^2} = f_{0,1} - 2f_{0,0} + f_{0,-1}$$

$$\frac{\partial^2 f}{\partial x \partial y} = (f_{-1,-1} - f_{-1,1} - f_{1,-1} + f_{1,1})/4$$

2D example

$$f(\mathbf{x}) = f + \frac{\partial f}{\partial \mathbf{x}}^T \mathbf{x} + \frac{1}{2}\mathbf{x}^T \frac{\partial^2 f}{\partial \mathbf{x}^2}\mathbf{x}$$

-17	-1	-1
-9	7	7
-9	7	7

Derivation of matrix form

$$f(\mathbf{x}) = f + \frac{\partial f}{\partial \mathbf{x}}^T \mathbf{x} + \frac{1}{2}\mathbf{x}^T \frac{\partial^2 f}{\partial \mathbf{x}^2}\mathbf{x}$$

$$h(\mathbf{x}) = \mathbf{g}^\mathbf{T}\mathbf{x}$$

$$= \begin{pmatrix} g_1 & \cdots & g_n \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

$$= \sum_{i=1}^{n} g_i x_i$$

$$\frac{\partial h}{\partial \mathbf{x}} = \begin{pmatrix} \frac{\partial h}{\partial x_1} \\ \vdots \\ \frac{\partial h}{\partial x_n} \end{pmatrix} = \begin{pmatrix} g_1 \\ \vdots \\ g_n \end{pmatrix} = \mathbf{g}$$

Derivation of matrix form

$$f(\mathbf{x}) = f + \frac{\partial f}{\partial \mathbf{x}}^T \mathbf{x} + \frac{1}{2}\mathbf{x}^T \frac{\partial^2 f}{\partial \mathbf{x}^2}\mathbf{x}$$

$$h(\mathbf{x}) = \mathbf{x}^{\mathrm{T}}\mathbf{A}\mathbf{x} = \begin{pmatrix} x_1 & \cdots & x_n \end{pmatrix}^T \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} x_i x_j$$

$$\frac{\partial h}{\partial \mathbf{x}} = \begin{pmatrix} \dfrac{\partial h}{\partial x_1} \\ \vdots \\ \dfrac{\partial h}{\partial x_n} \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{n} a_{i1} x_i + \sum_{j=1}^{n} a_{1j} x_j \\ \vdots \\ \sum_{i=1}^{n} a_{in} x_i + \sum_{j=1}^{n} a_{nj} x_j \end{pmatrix} = \mathbf{A}^{\mathrm{T}}\mathbf{x} + \mathbf{A}\mathbf{x}$$

$$= (\mathbf{A}^{\mathrm{T}} + \mathbf{A})\mathbf{x}$$

Derivation of matrix form

$$f(\mathbf{x}) = f + \frac{\partial f}{\partial \mathbf{x}}^T \mathbf{x} + \frac{1}{2}\mathbf{x}^T \frac{\partial^2 f}{\partial \mathbf{x}^2}\mathbf{x}$$

$$\frac{\partial h}{\partial \mathbf{x}} = \frac{\partial f}{\partial \mathbf{x}}^T + \frac{1}{2}\left(\frac{\partial^2 f}{\partial \mathbf{x}^2} + \frac{\partial^2 f}{\partial \mathbf{x}^2}^T \right) x = \frac{\partial f}{\partial \mathbf{x}}^T + \frac{\partial^2 f}{\partial \mathbf{x}^2} x$$

$$\mathbf{x}_m = -\frac{\partial^2 f}{\partial \mathbf{x}^2}^{-1} \frac{\partial f}{\partial \mathbf{x}}$$

Accurate keypoint localization

$$f(\mathbf{x}) = f + \frac{\partial f}{\partial \mathbf{x}}^T \mathbf{x} + \frac{1}{2}\mathbf{x}^T \frac{\partial^2 f}{\partial \mathbf{x}^2}\mathbf{x}$$

- $\mathbf{x}$ is a 3-vector
- Change sample point if offset is larger than 0.5
- Throw out low contrast (<0.03)

Accurate keypoint localization

DigiVFX

- Throw out low contrast $|D(\hat{\mathbf{x}})| < 0.03$

$$D(\hat{\mathbf{x}}) = D + \frac{\partial D}{\partial \mathbf{x}}^T \hat{\mathbf{x}} + \frac{1}{2}\hat{\mathbf{x}}^T \frac{\partial^2 D}{\partial \mathbf{x}^2}\hat{\mathbf{x}}$$

$$= D + \frac{\partial D}{\partial \mathbf{x}}^T \hat{\mathbf{x}} + \frac{1}{2}\left(-\frac{\partial^2 D}{\partial \mathbf{x}^2}^{-1}\frac{\partial D}{\partial \mathbf{x}}\right)^T \frac{\partial^2 D}{\partial \mathbf{x}^2}\left(-\frac{\partial^2 D}{\partial \mathbf{x}^2}^{-1}\frac{\partial D}{\partial \mathbf{x}}\right)$$

$$= D + \frac{\partial D}{\partial \mathbf{x}}^T \hat{\mathbf{x}} + \frac{1}{2}\frac{\partial D}{\partial \mathbf{x}}^T \frac{\partial^2 D}{\partial \mathbf{x}^2}^{-T} \frac{\partial^2 D}{\partial \mathbf{x}^2}\frac{\partial^2 D}{\partial \mathbf{x}^2}^{-1}\frac{\partial D}{\partial \mathbf{x}}$$

$$= D + \frac{\partial D}{\partial \mathbf{x}}^T \hat{\mathbf{x}} + \frac{1}{2}\frac{\partial D}{\partial \mathbf{x}}^T \frac{\partial^2 D}{\partial \mathbf{x}^2}^{-1}\frac{\partial D}{\partial \mathbf{x}}$$

$$= D + \frac{\partial D}{\partial \mathbf{x}}^T \hat{\mathbf{x}} + \frac{1}{2}\frac{\partial D}{\partial \mathbf{x}}^T (-\hat{\mathbf{x}})$$

$$= D + \frac{1}{2}\frac{\partial D}{\partial \mathbf{x}}^T \hat{\mathbf{x}}$$

Eliminating edge responses

$$\mathbf{H} = \begin{bmatrix} D_{xx} & D_{xy} \\ D_{xy} & D_{yy} \end{bmatrix}$$

Hessian matrix at keypoint location

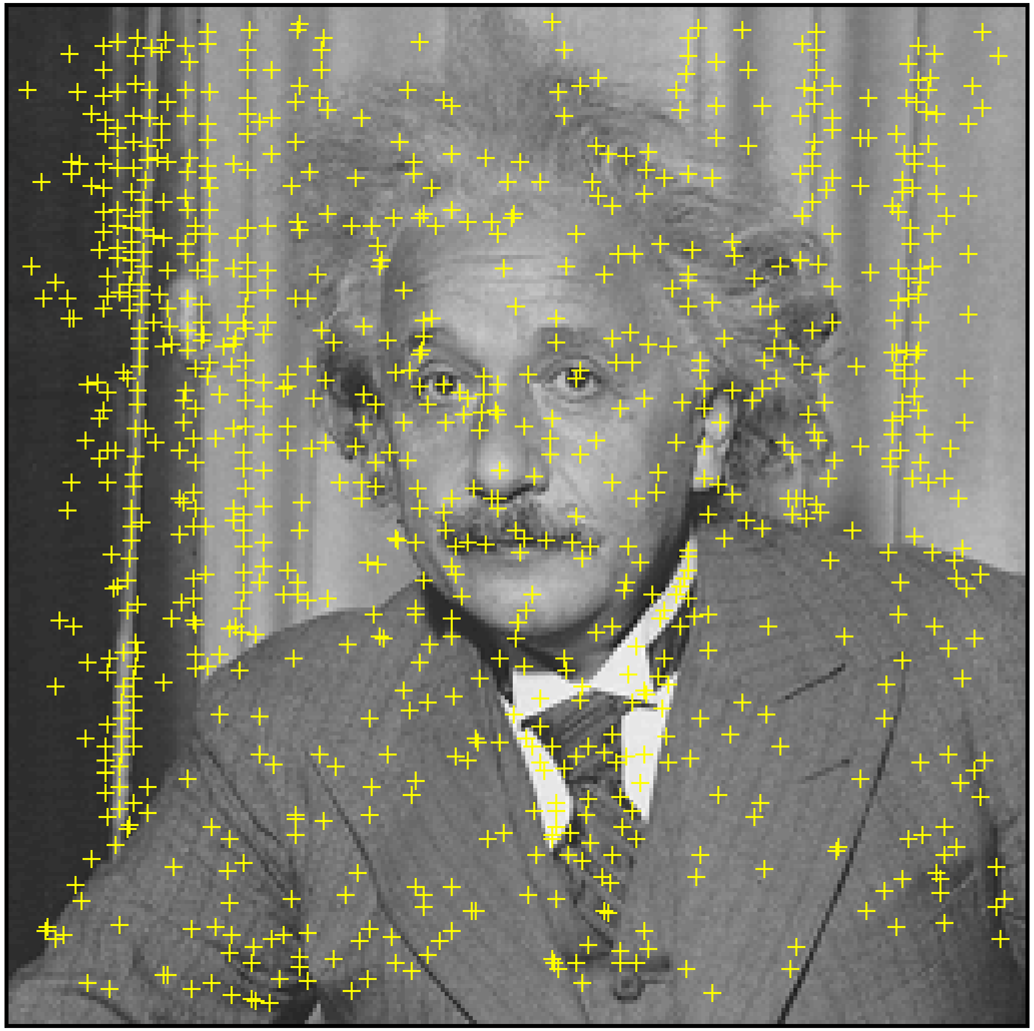
$$\mathrm{Tr}(\mathbf{H}) = D_{xx} + D_{yy} = \alpha + \beta,$$

$$\mathrm{Det}(\mathbf{H}) = D_{xx}D_{yy} - (D_{xy})^2 = \alpha\beta.$$

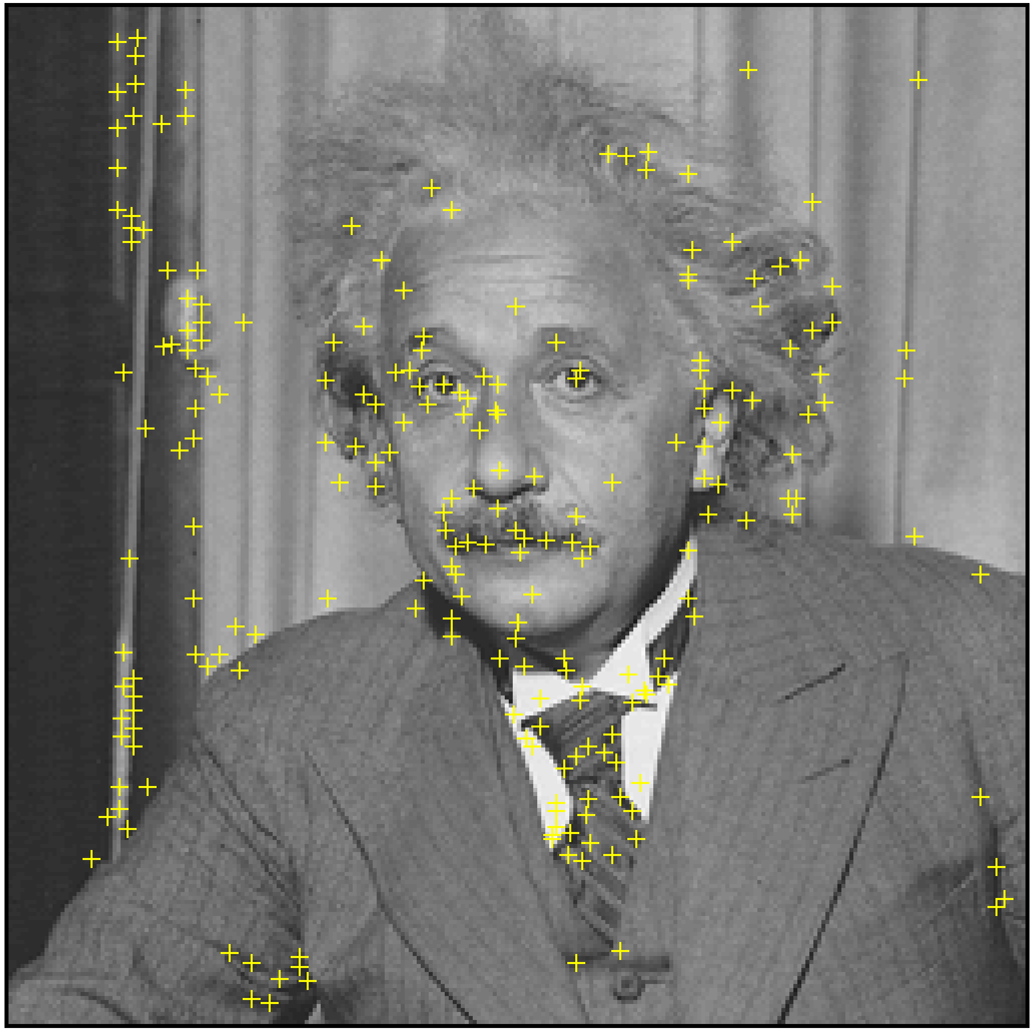
Let $\alpha = r\beta$ $\quad \frac{\mathrm{Tr}(\mathbf{H})^2}{\mathrm{Det}(\mathbf{H})} = \frac{(\alpha+\beta)^2}{\alpha\beta} = \frac{(r\beta+\beta)^2}{r\beta^2} = \frac{(r+1)^2}{r}$

Keep the points with $\frac{\mathrm{Tr}(\mathbf{H})^2}{\mathrm{Det}(\mathbf{H})} < \frac{(r+1)^2}{r}.$ r=10

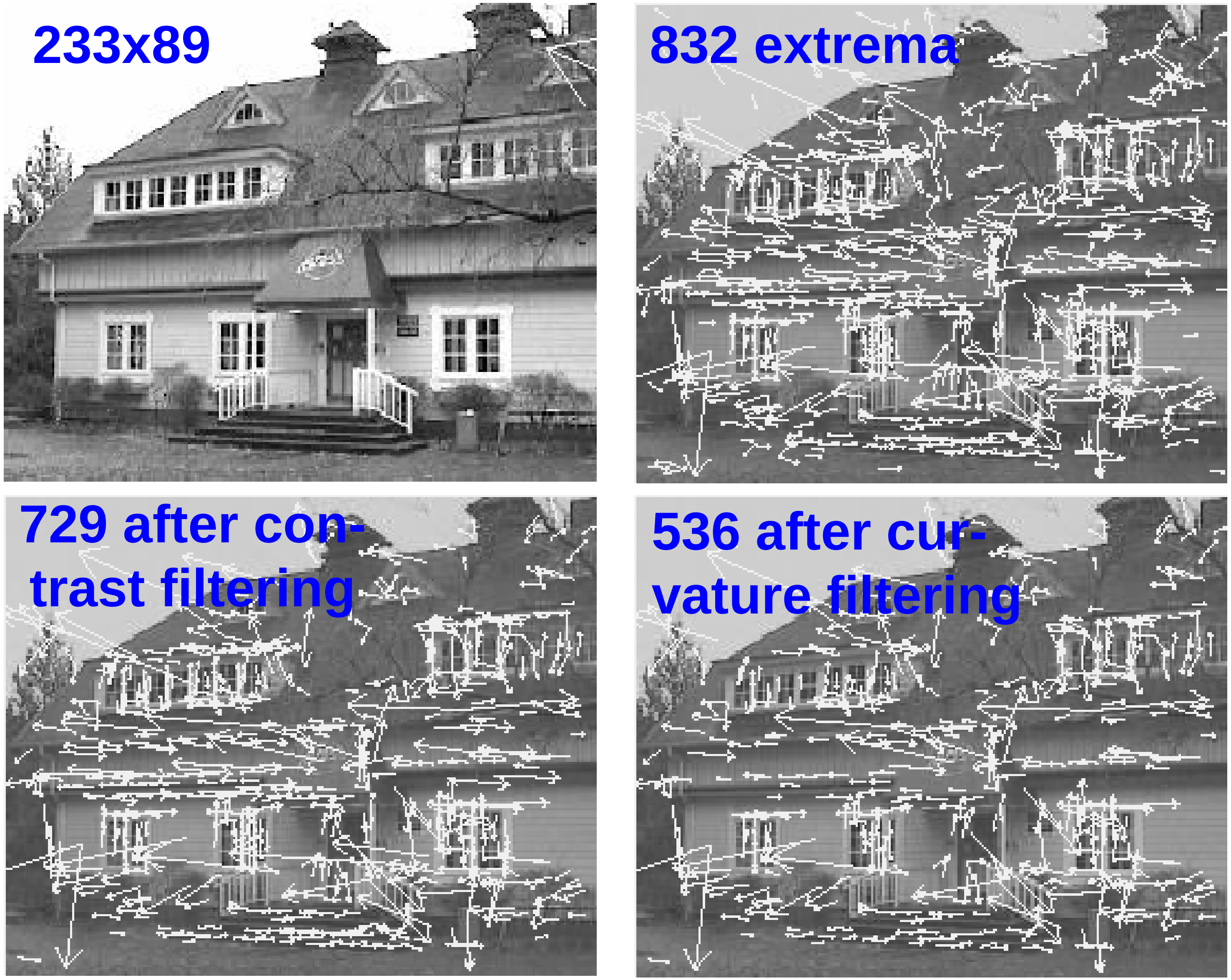
Maxima in D



Remove low contrast and edges



Keypoint detector

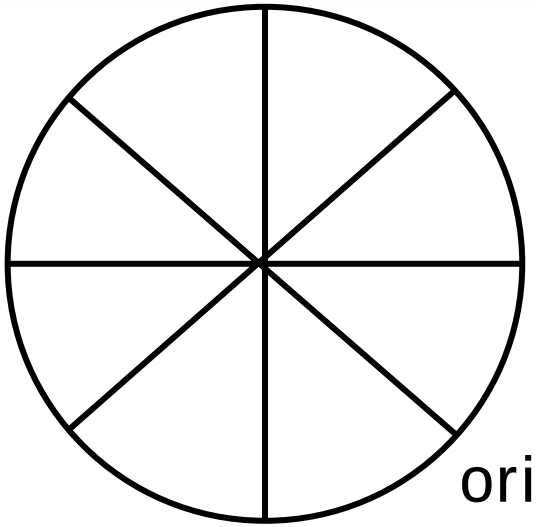


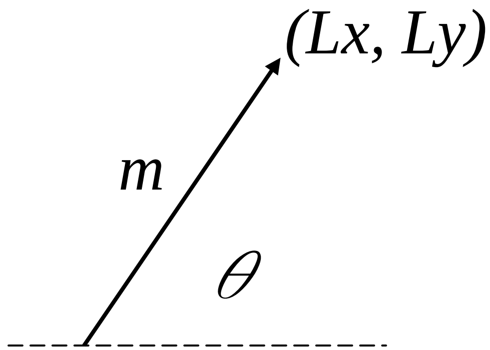
3. Orientation assignment

- By assigning a consistent orientation, the keypoint descriptor can be orientation invariant.
- For a keypoint, L is the Gaussian-smoothed image with the closest scale,

$$m(x,y) = \sqrt{(L(x+1,y) - L(x-1,y))^2 + (L(x,y+1) - L(x,y-1))^2}$$

$$\theta(x,y) = \tan^{-1}((L(x,y+1) - L(x,y-1))/(L(x+1,y) - L(x-1,y)))$$

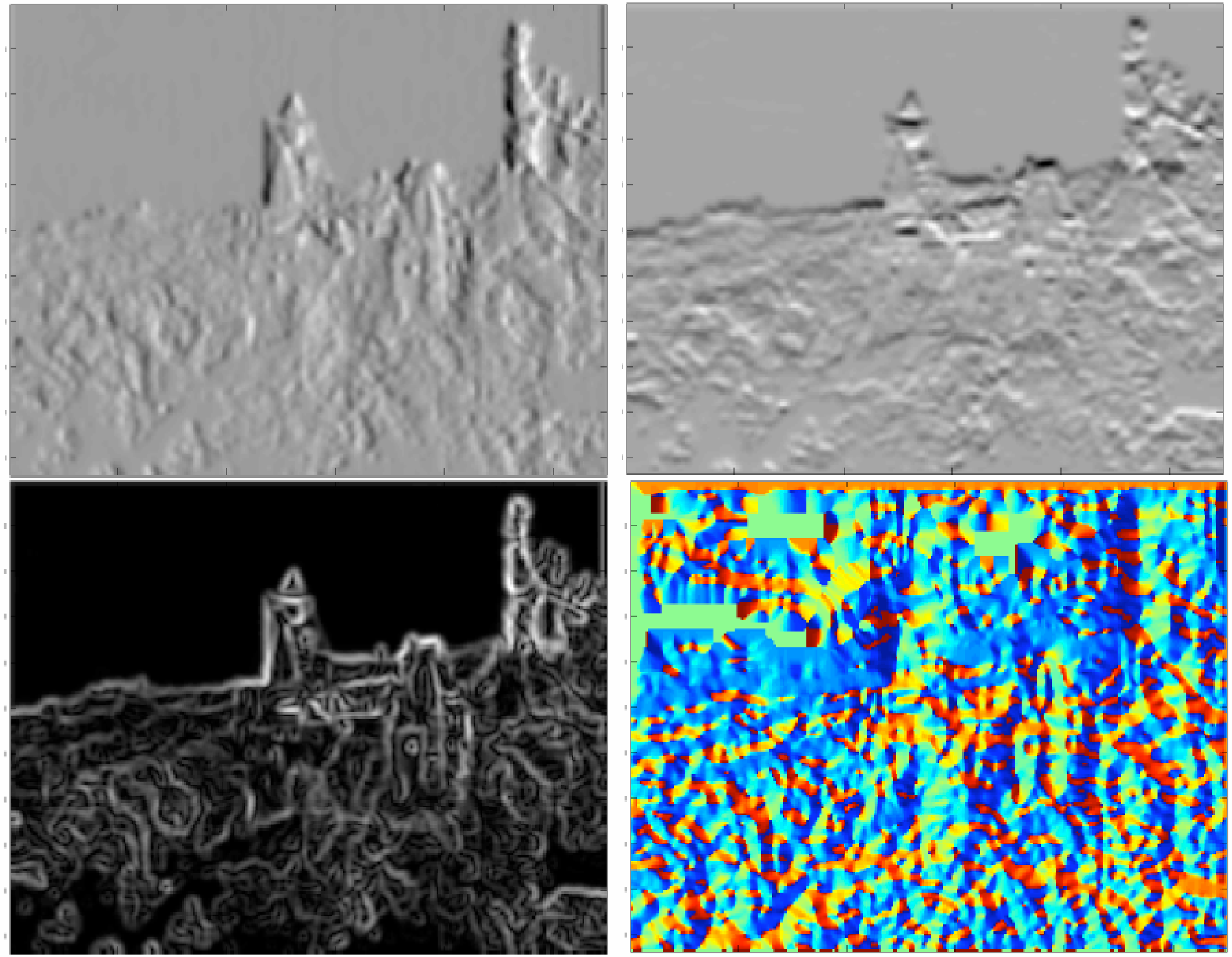




orientation histogram (36 bins)

Orientation assignment

Dx

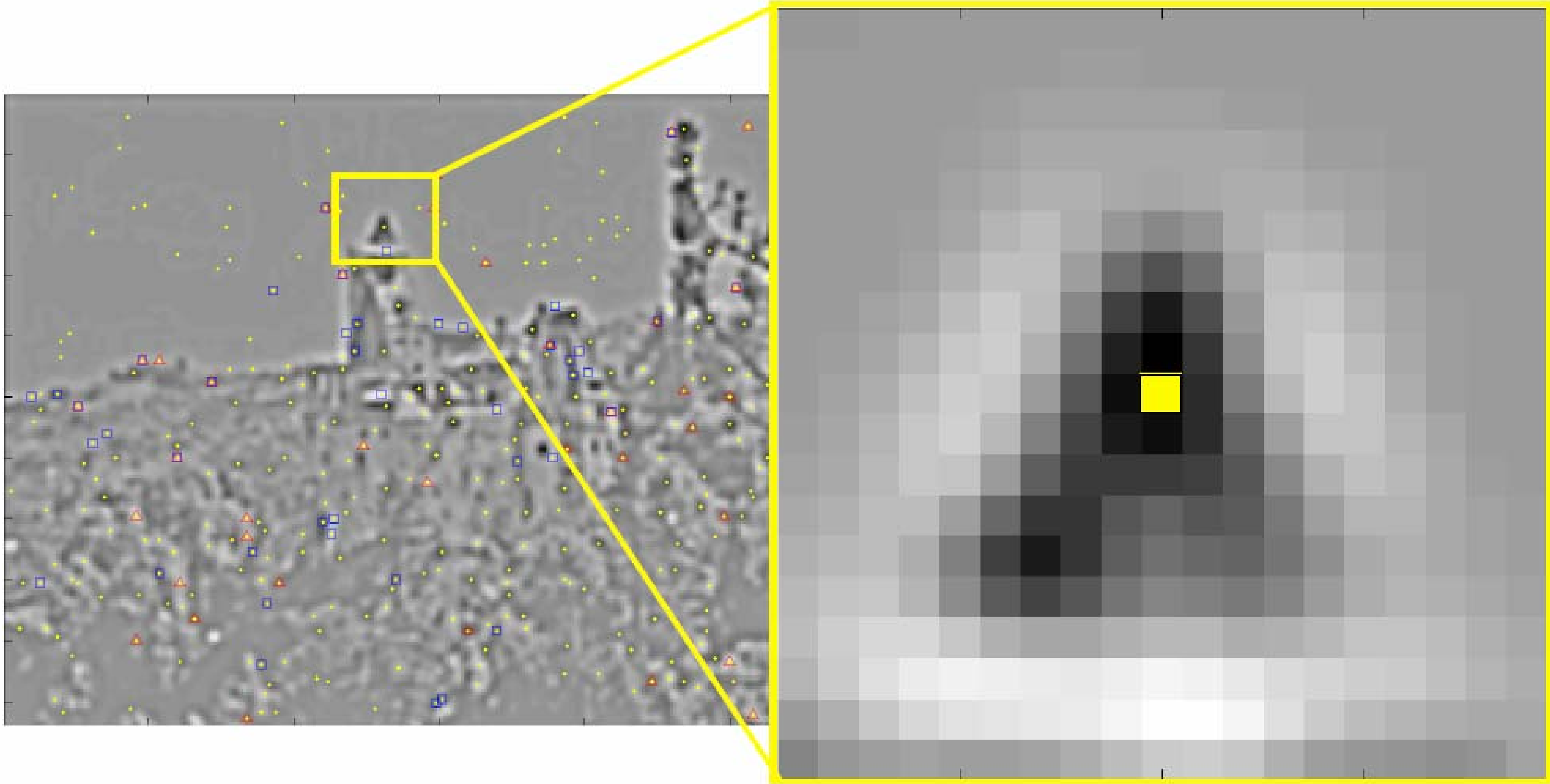


Dy

M

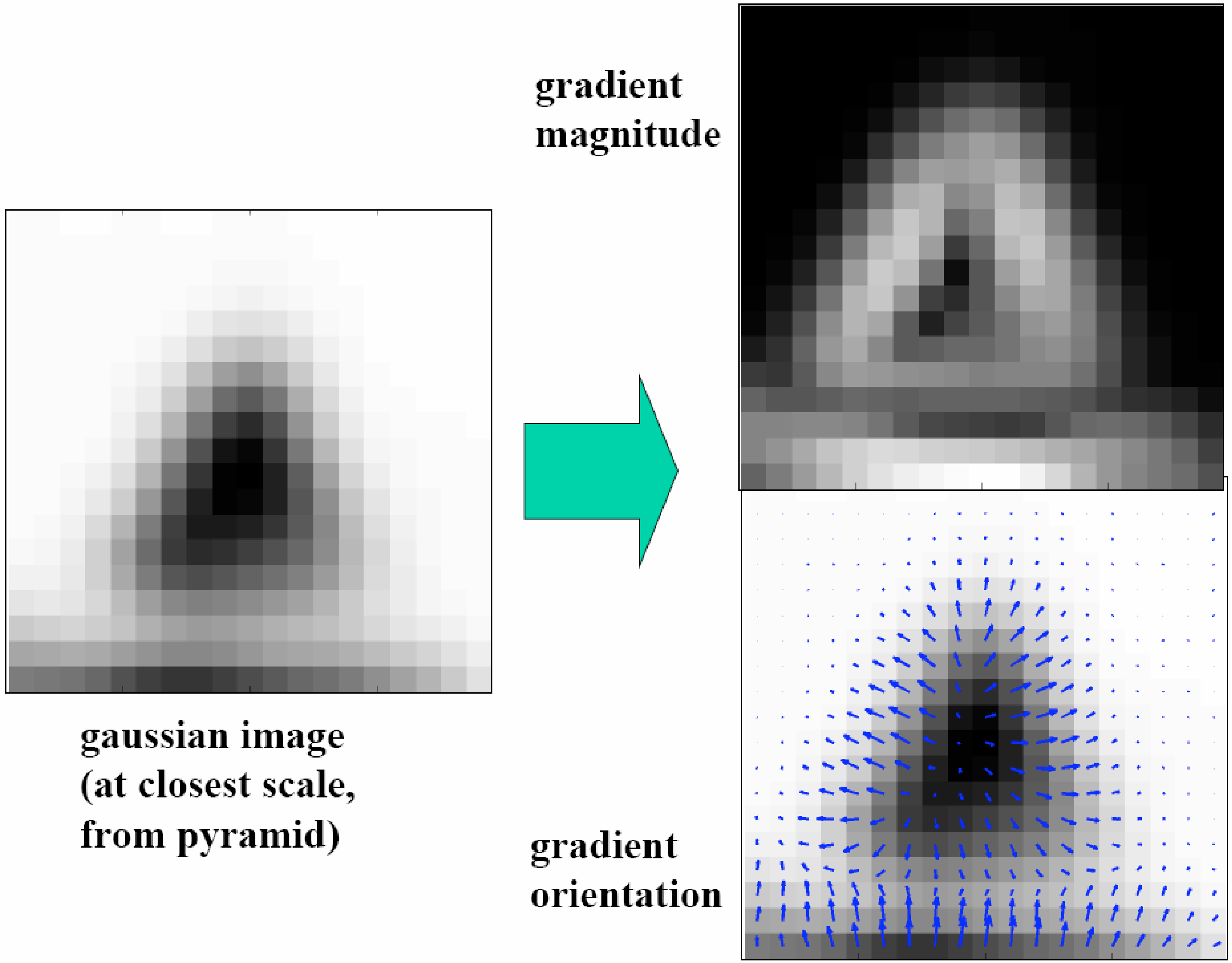
Θ

Orientation assignment

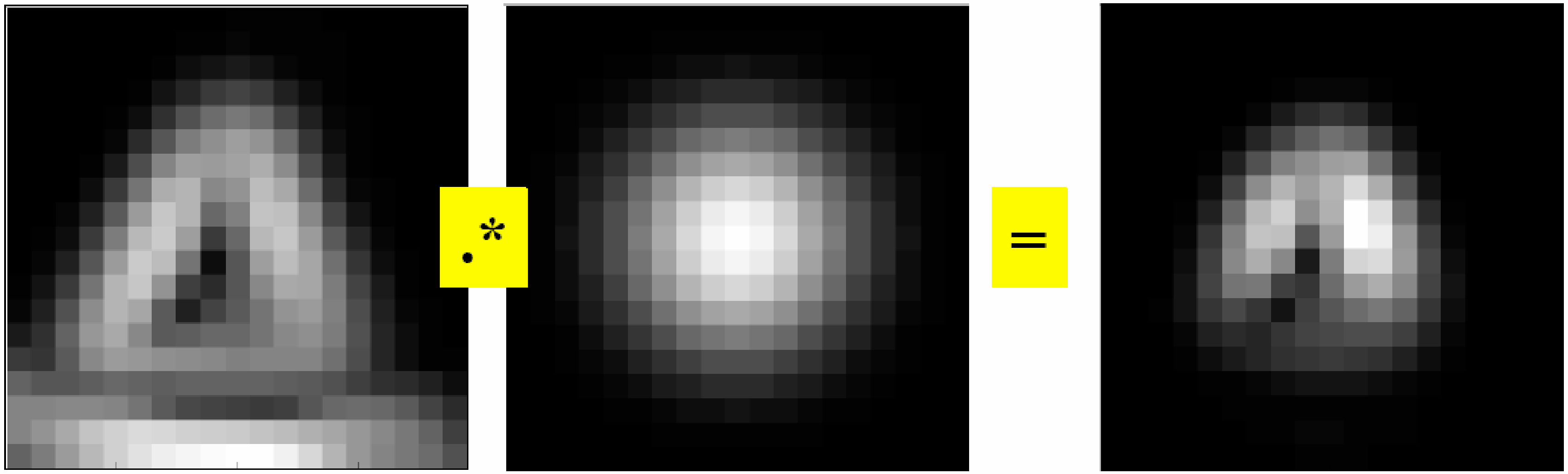


•Keypoint location = extrema location
•Keypoint scale is scale of the DOG image

Orientation assignment



Orientation assignment



gradient magnitude

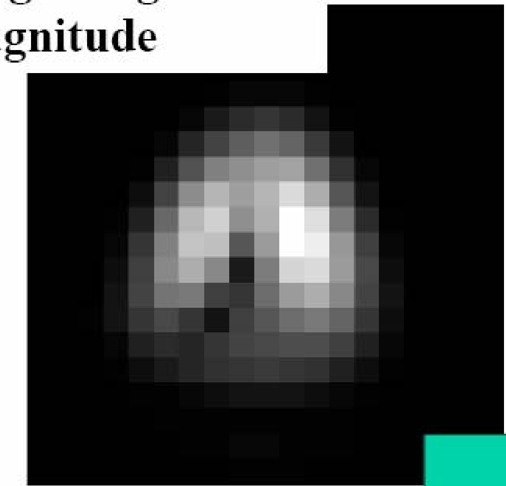
weighted by 2D gaussian kernel

weighted gradient magnitude

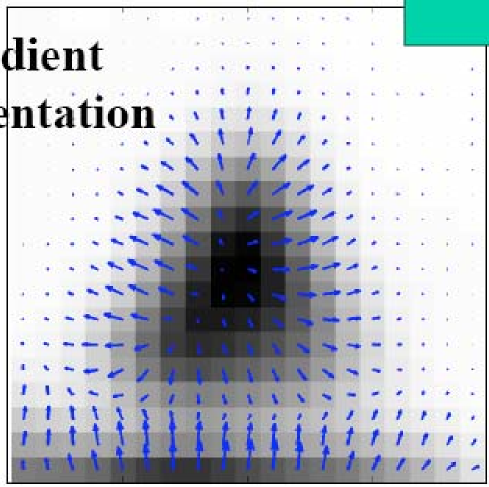
σ =1.5*scale of the keypoint

Orientation assignment

weighted gradient magnitude

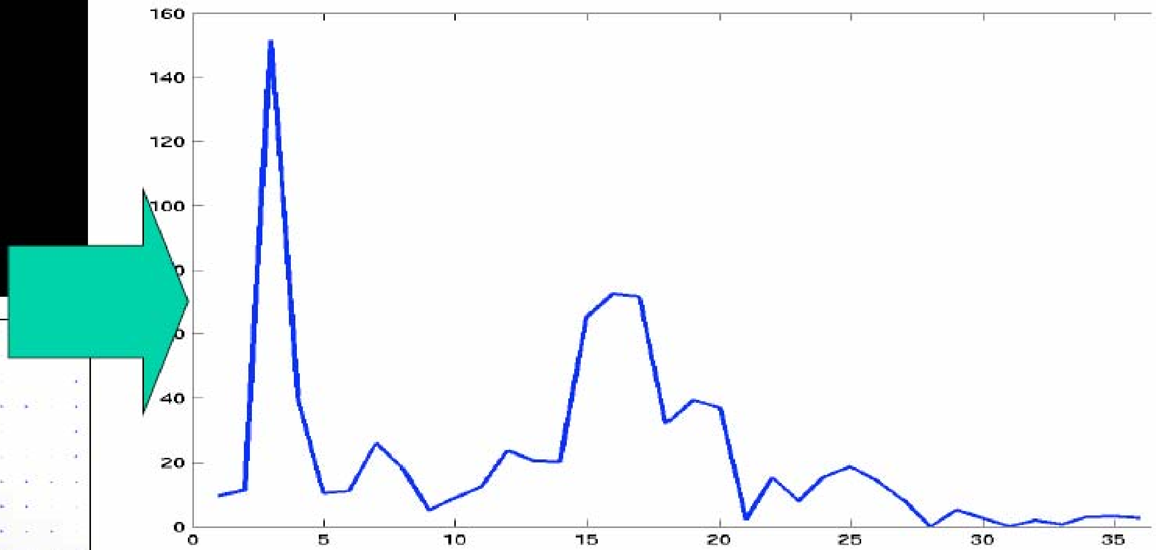


gradient orientation



weighted orientation histogram.

Each bucket contains sum of weighted gradient magnitudes corresponding to angles that fall within that bucket.

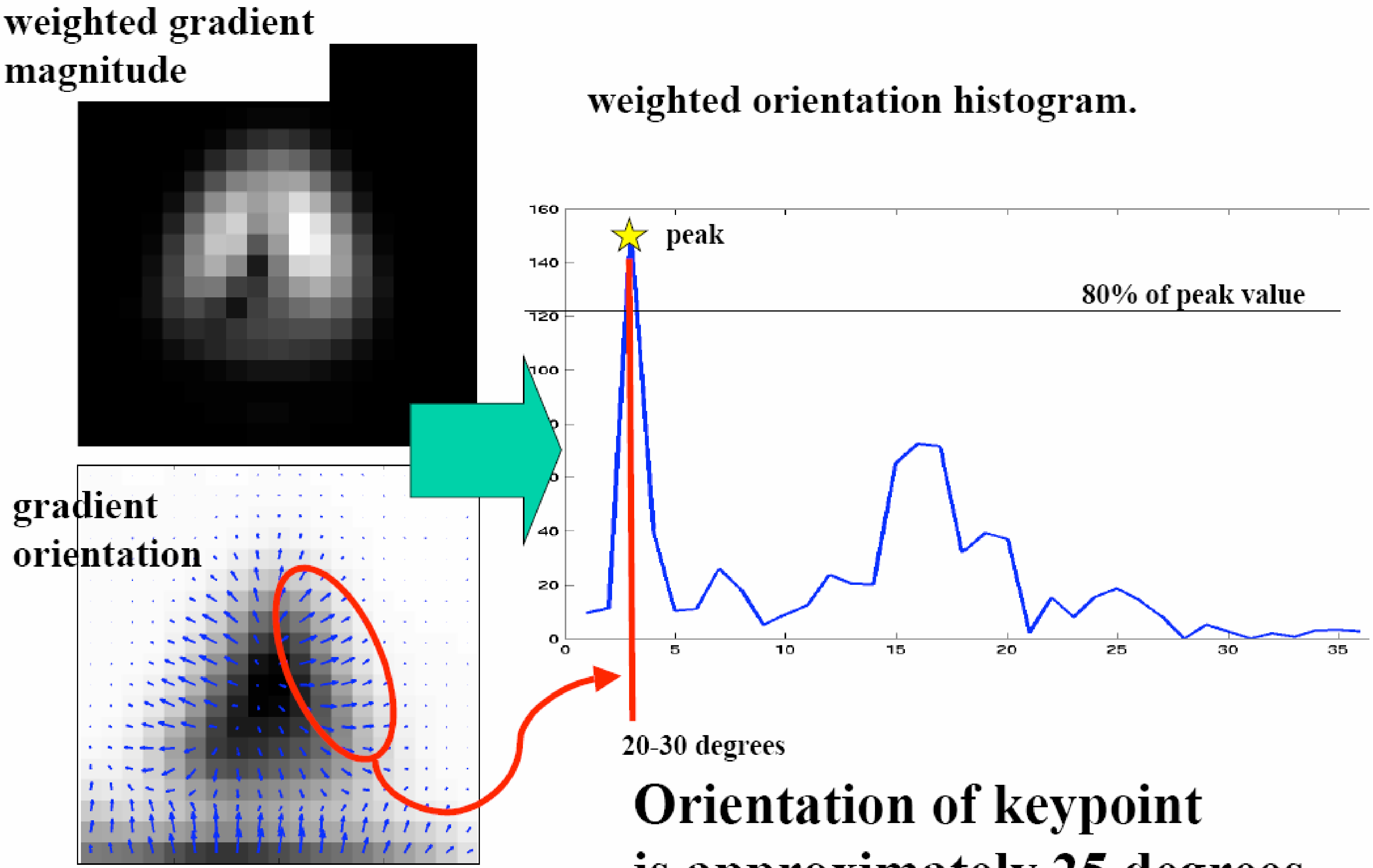


36 buckets

10 degree range of angles in each bucket, i.e.

- **0 <=ang<10 : bucket 1**
- **10<=ang<20 : bucket 2**
- **20<=ang<30 : bucket 3 …**

Orientation assignment



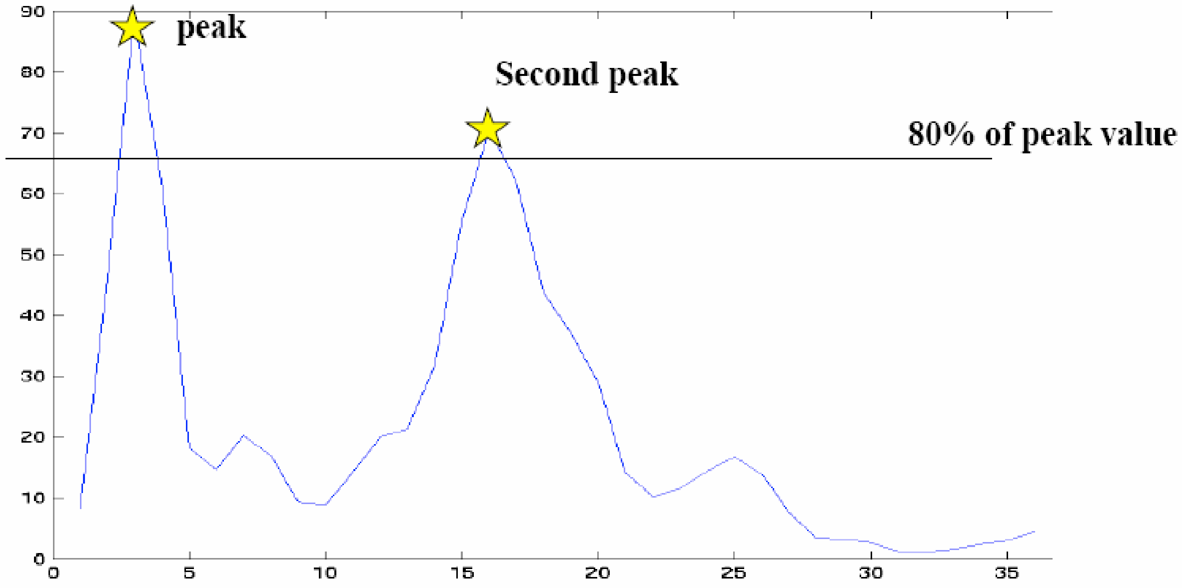
Orientation of keypoint is approximately 25 degrees

Orientation assignment

DigiVFX

There may be multiple orientations.

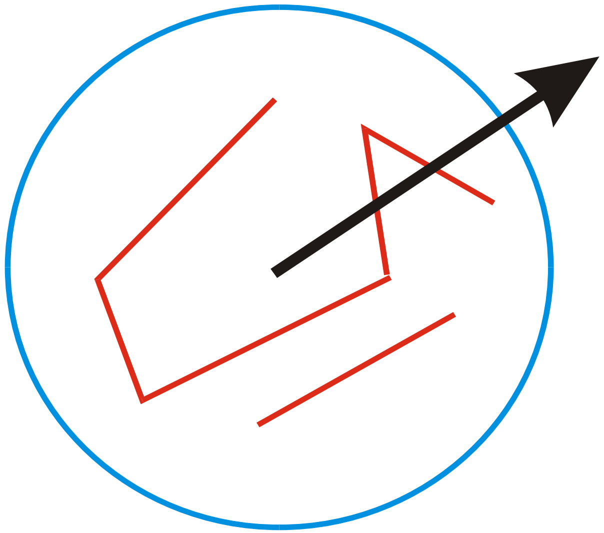
accurate peak position is determined by fitting

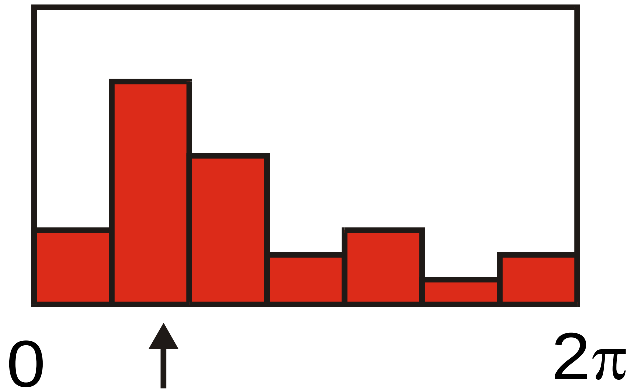


In this case, generate duplicate keypoints, one with orientation at 25 degrees, one at 155 degrees.

Design decision: you may want to limit number of possible multiple peaks to two.

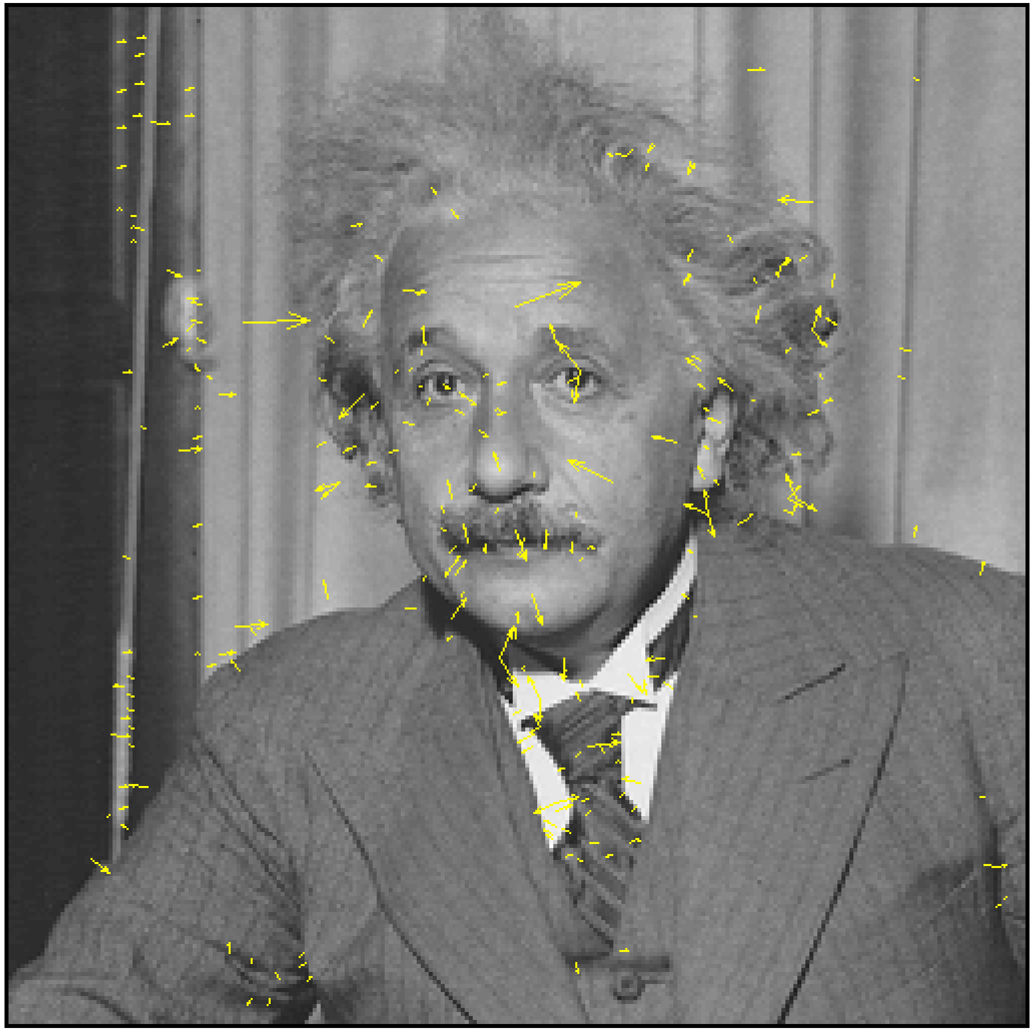
Orientation assignment





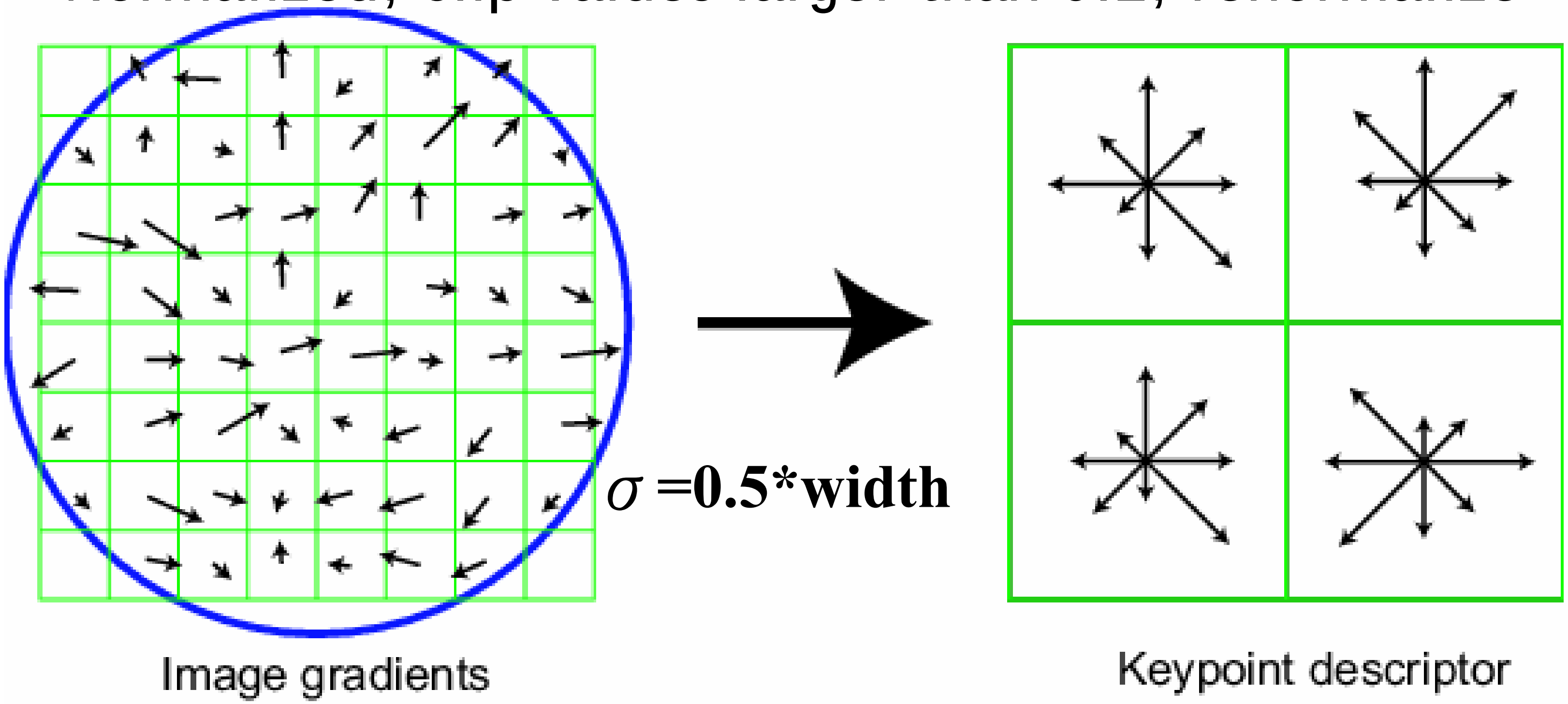
- 36-bin orientation histogram over 360°, weighted by m and 1.5*scale falloff
- Peak is the orientation
- Local peak within 80% creates multiple orientations
- About 15% has multiple orientations and they contribute a lot to stability

SIFT descriptor

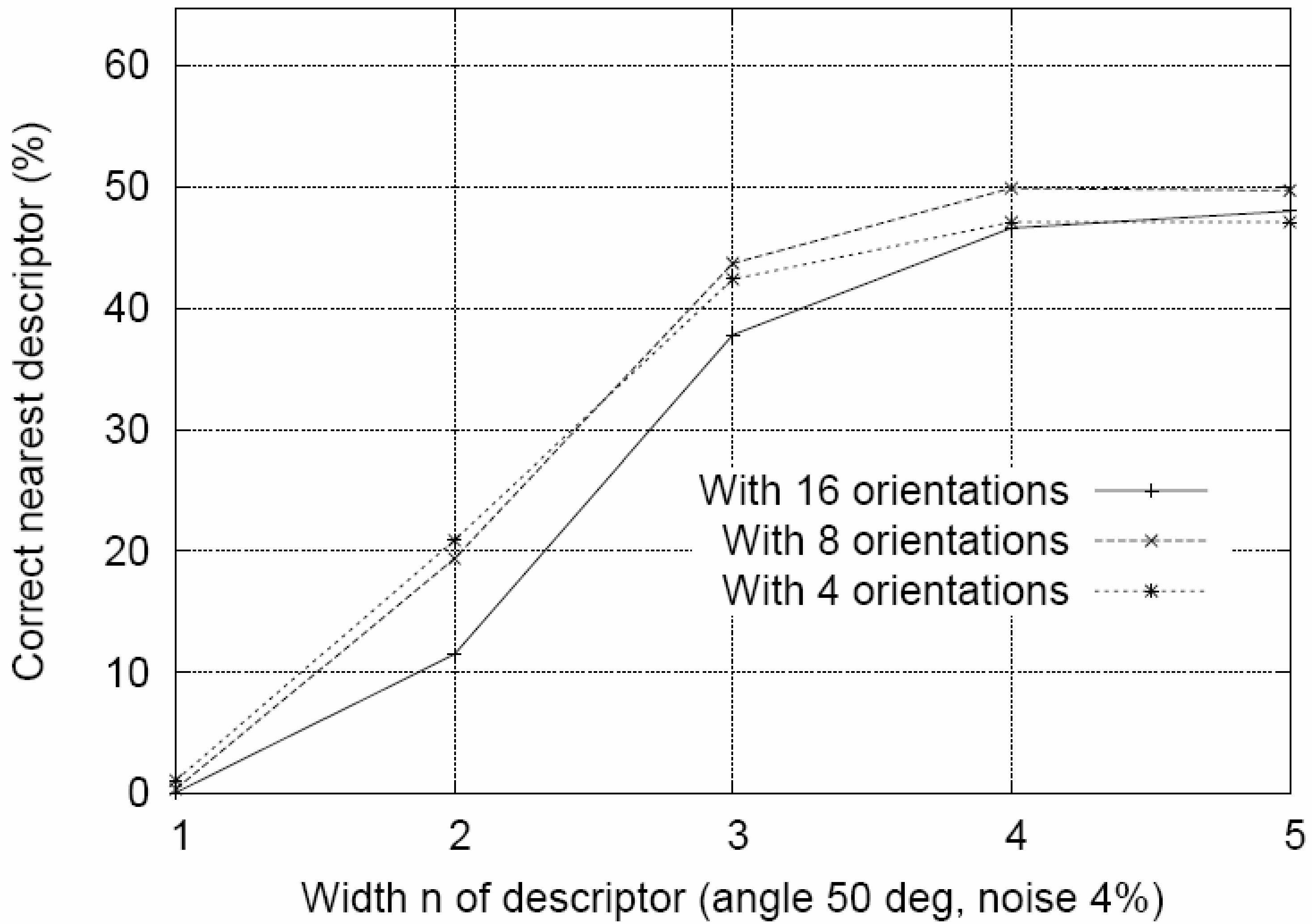


4. Local image descriptor

- Thresholded image gradients are sampled over 16x16 array of locations in scale space
- Create array of orientation histograms (w.r.t. key orientation)
- 8 orientations x 4x4 histogram array = 128 dimensions
- Normalized, clip values larger than 0.2, renormalize

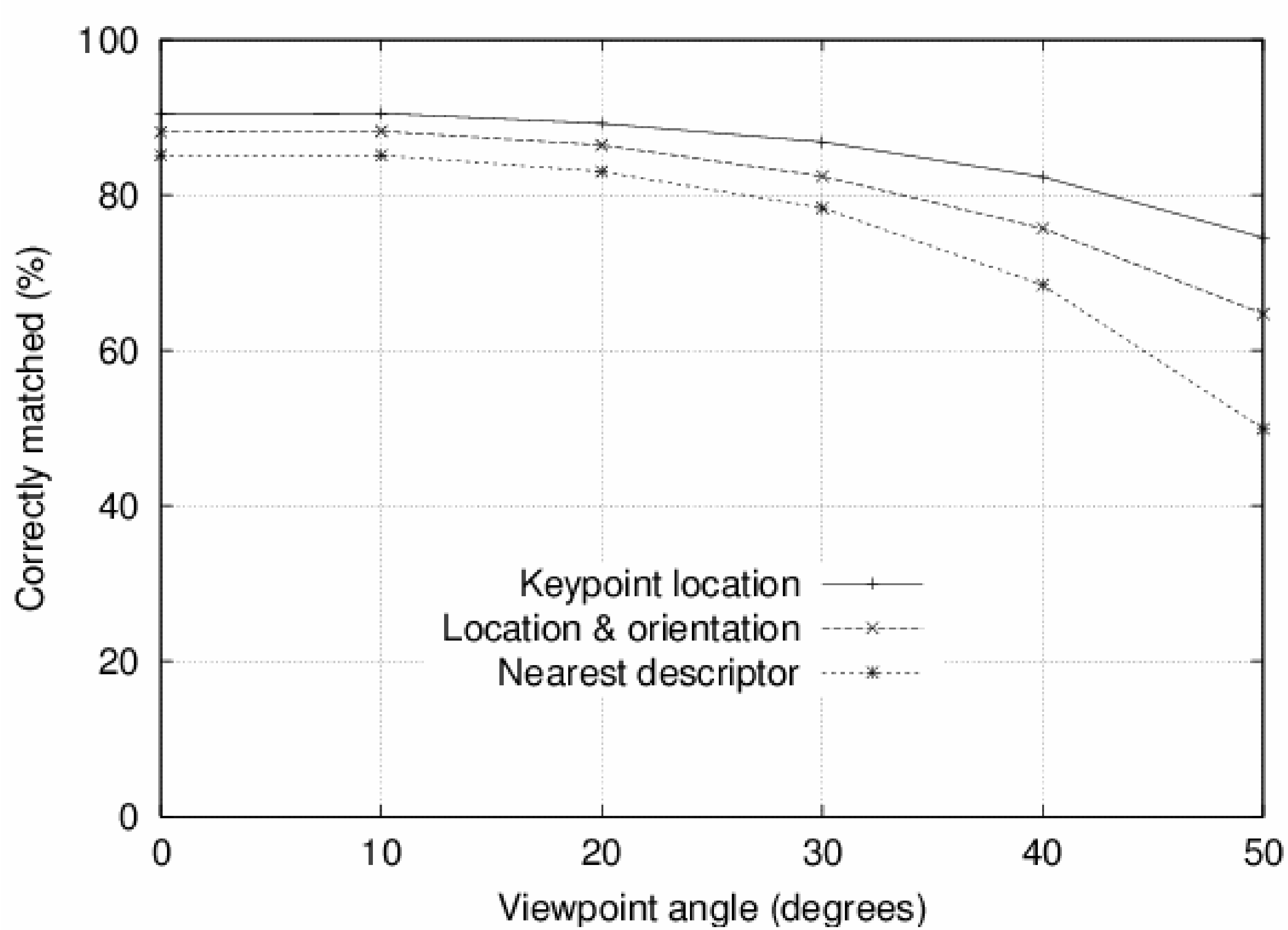


Why 4x4x8?



60
50
40
30
20
10
0
1
2
3
4
5
Correct nearest descriptor (%)
Width n of descriptor (angle 50 deg, noise 4%)
With 16 orientations
With 8 orientations
With 4 orientations

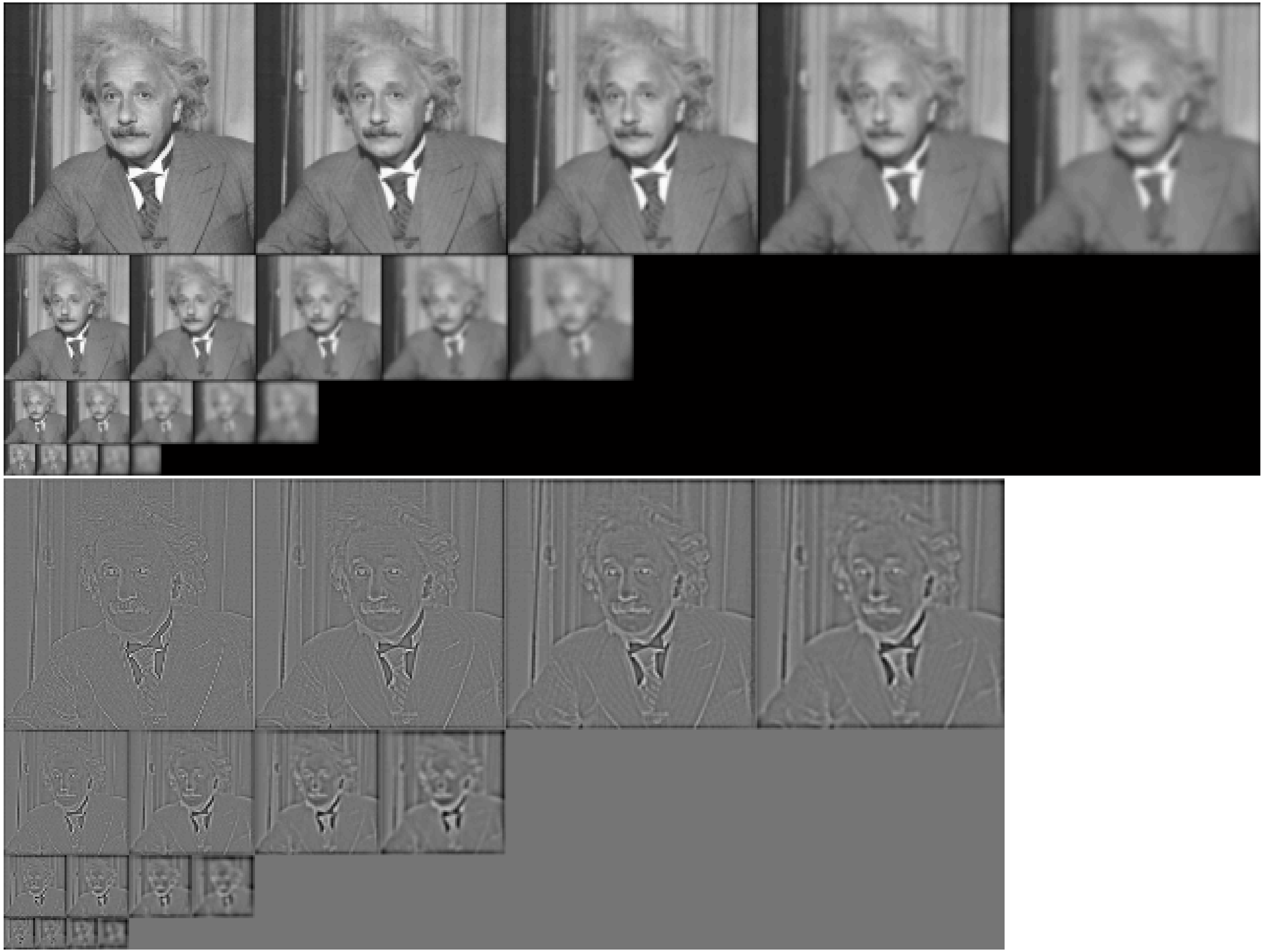
Sensitivity to affine change



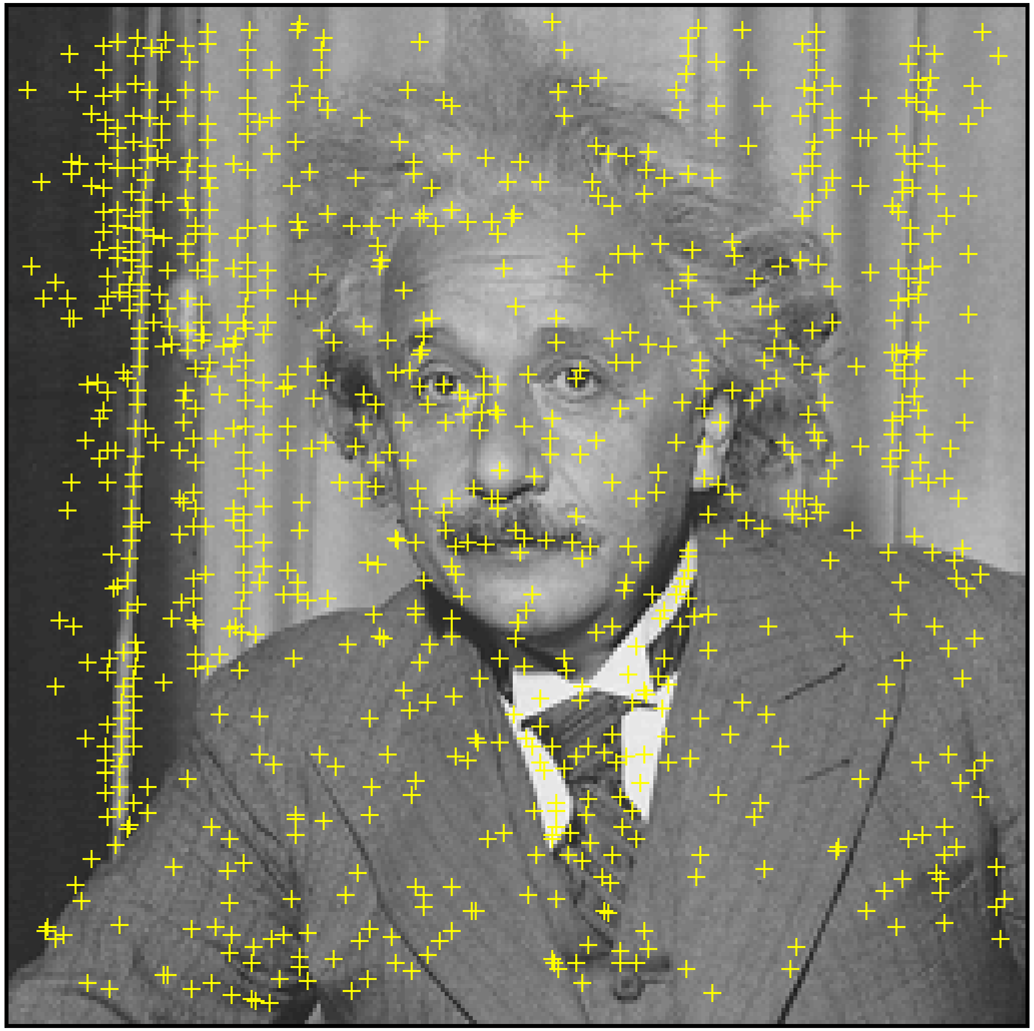
Feature matching

- for a feature x, he found the closest feature x_1 and the second closest feature x_2. If the distance ratio of $d(x, x_1)$ and $d(x, x_1)$ is smaller than 0.8, then it is accepted as a match.

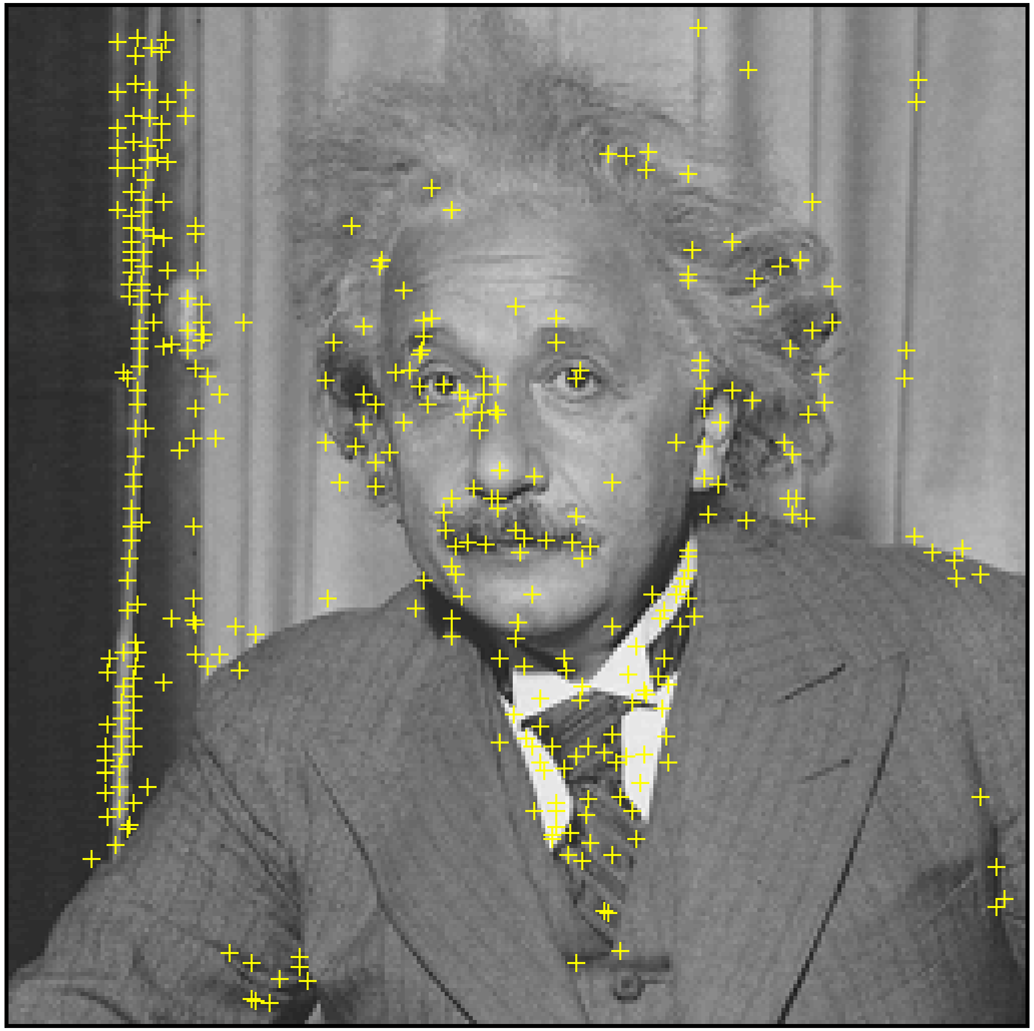
SIFT flow



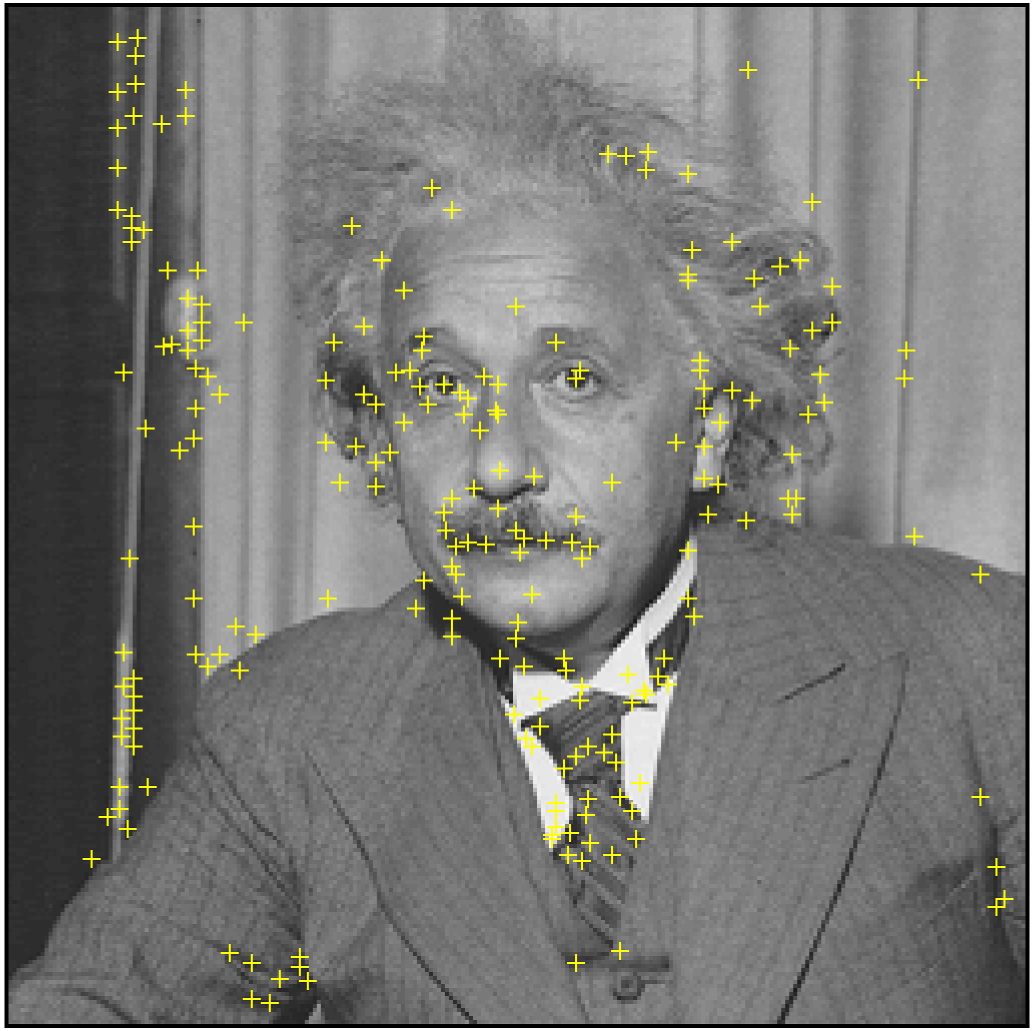
Maxima in D



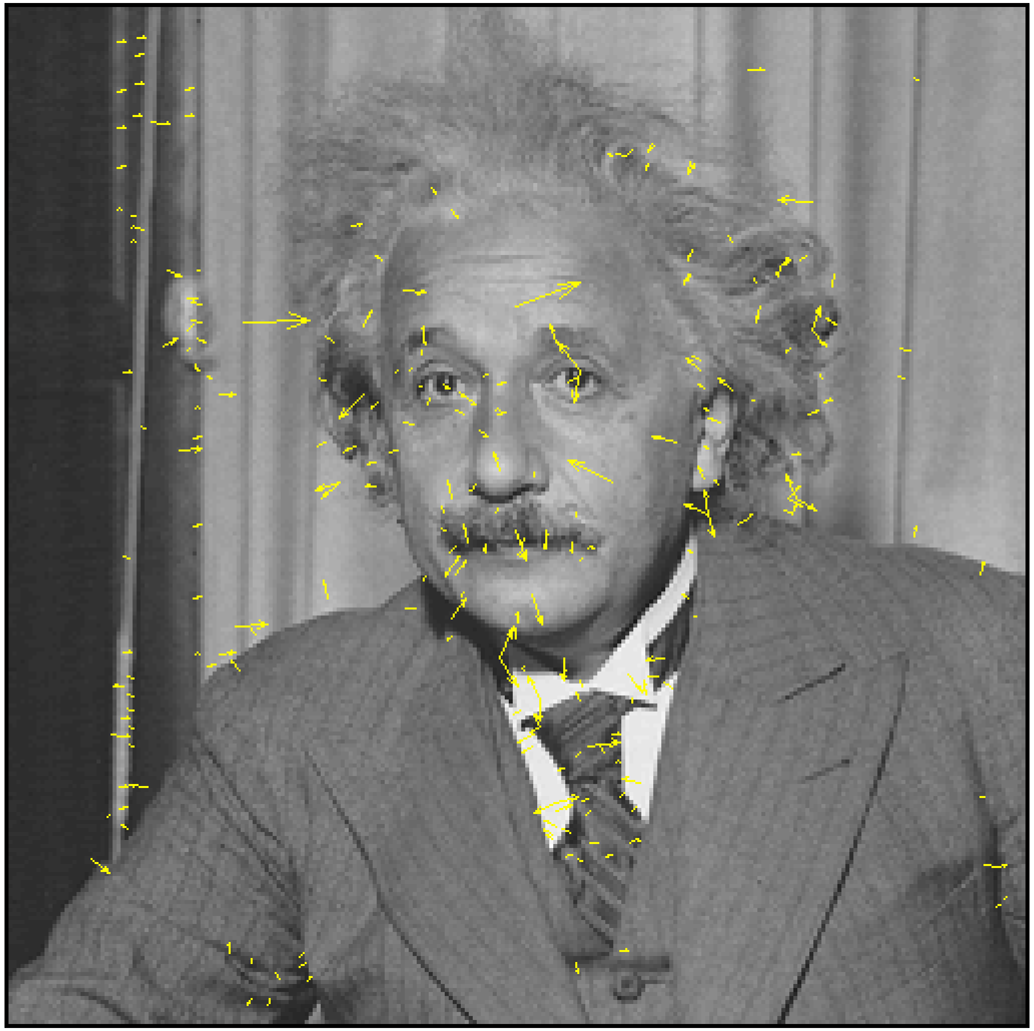
Remove low contrast



Remove edges



SIFT descriptor



DigiVFX

Estimated rotation

- Computed affine transformation from rotated image to original image:

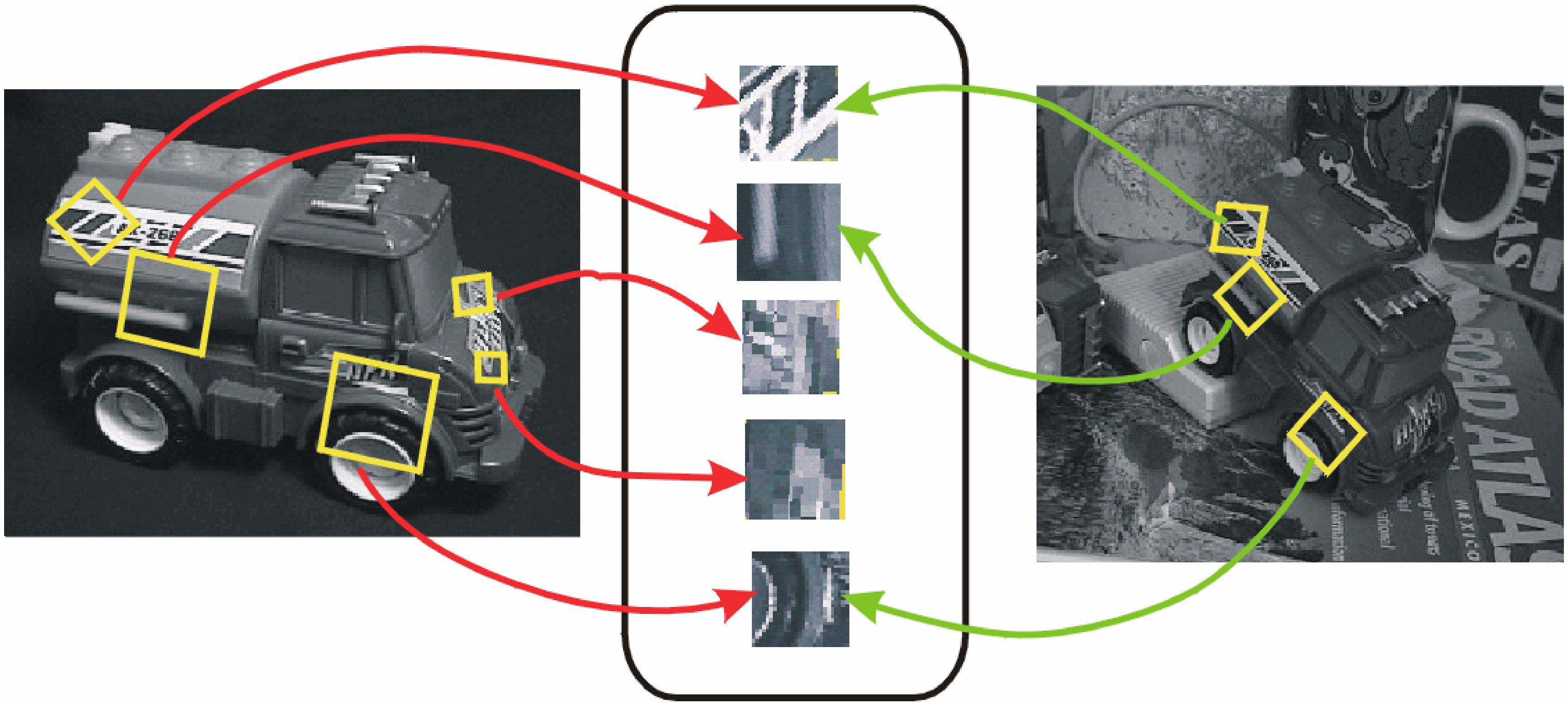
0.7060	-0.7052	128.4230
0.7057	0.7100	-128.9491
0	0	1.0000

- Actual transformation from rotated image to original image:

0.7071	-0.7071	128.6934
0.7071	0.7071	-128.6934
0	0	1.0000

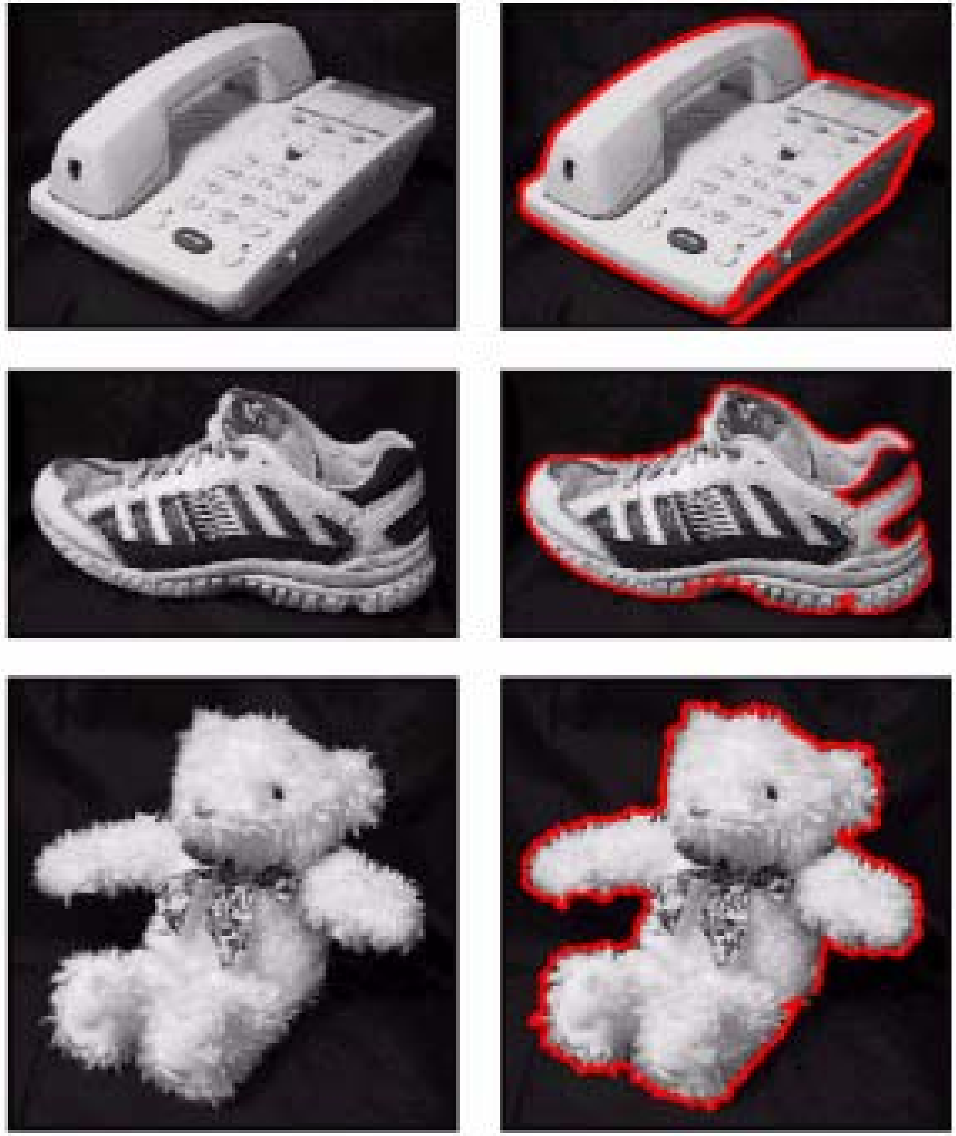
Applications

Recognition

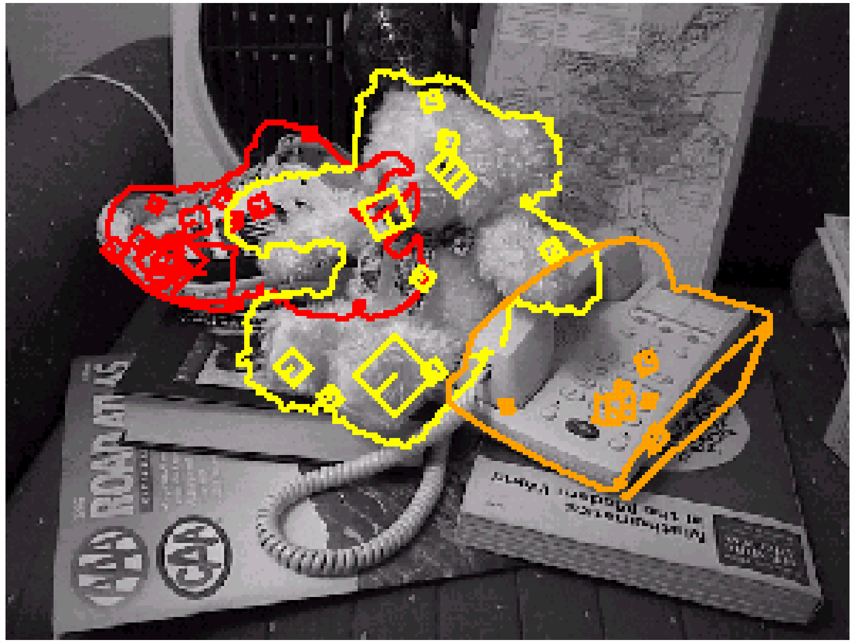


SIFT Features

3D object recognition



3D object recognition



Office of the past

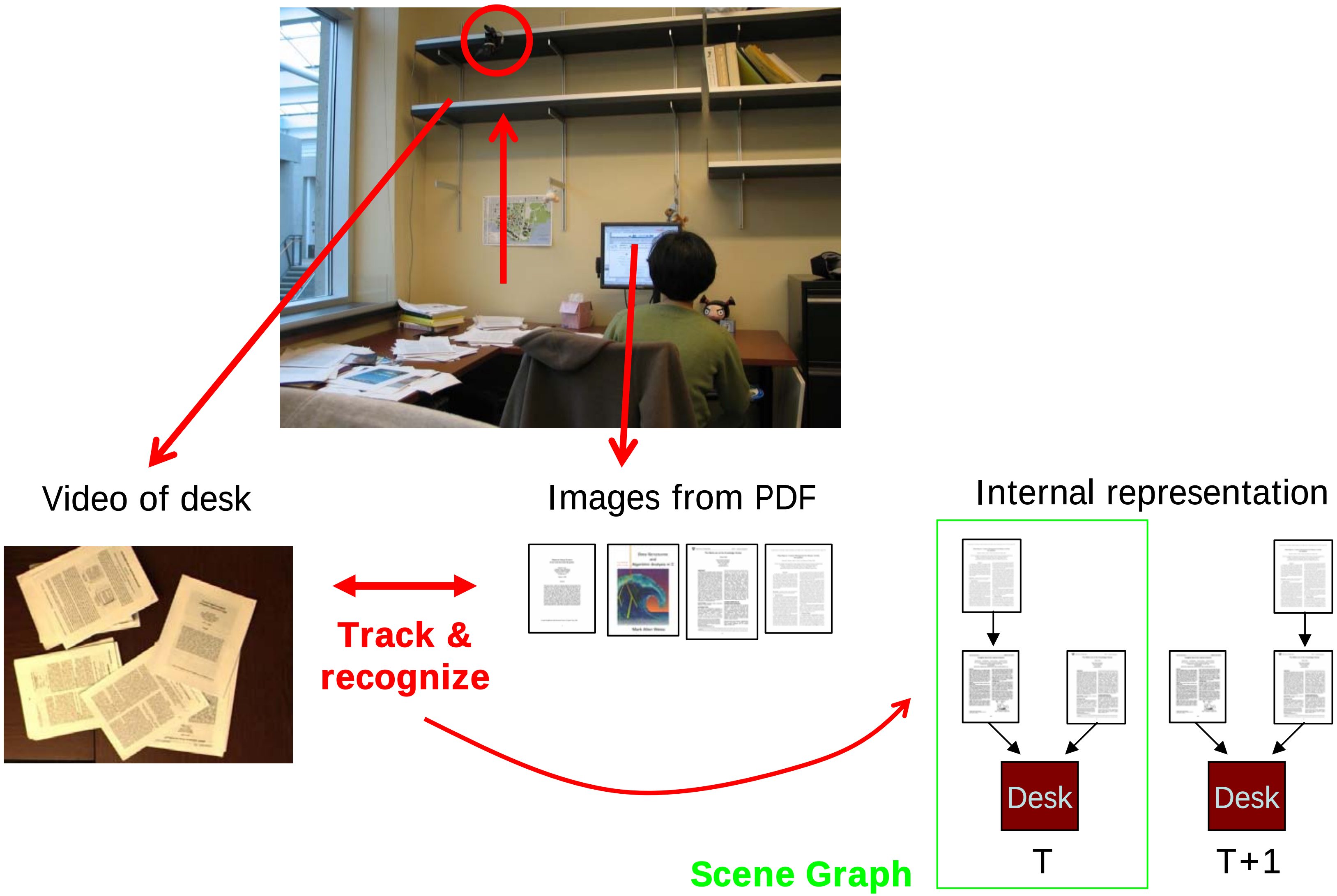
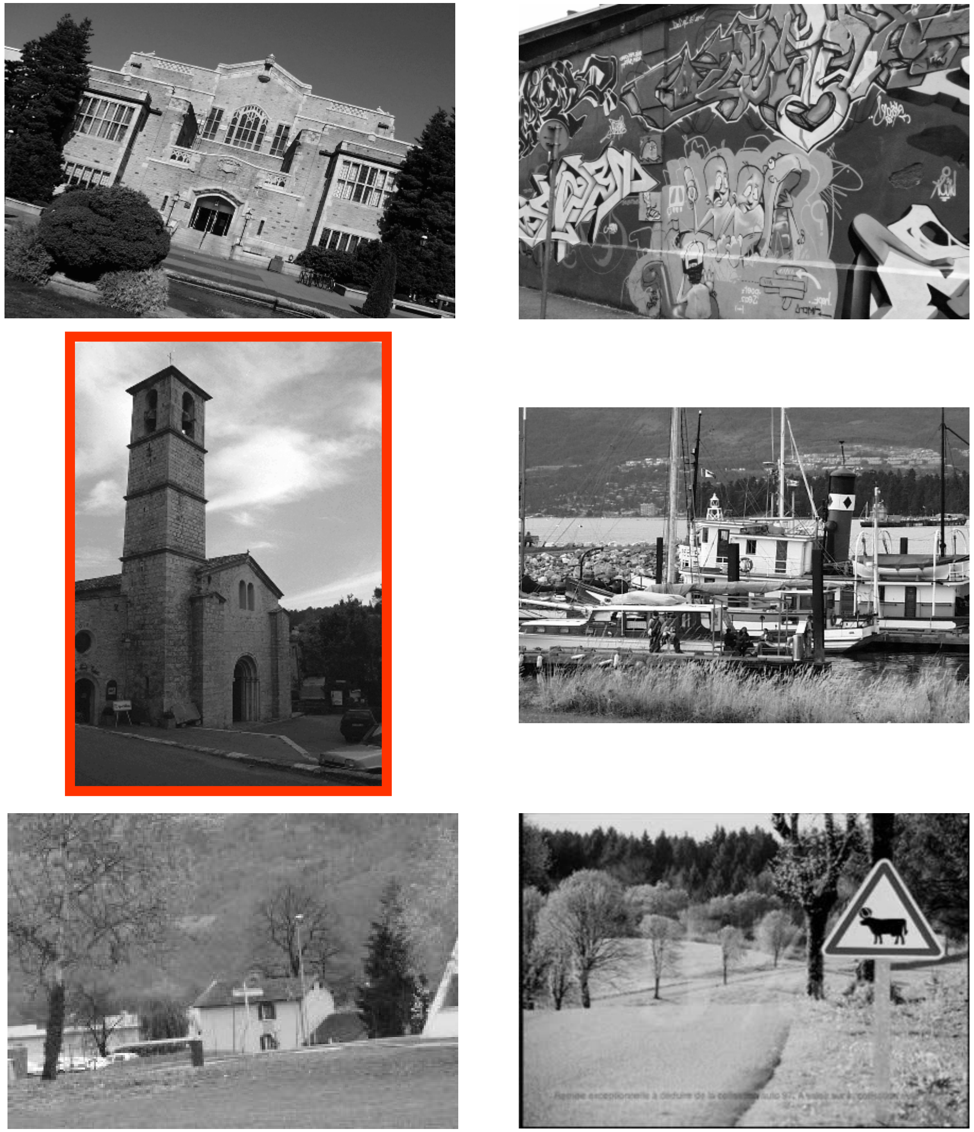


Image retrieval

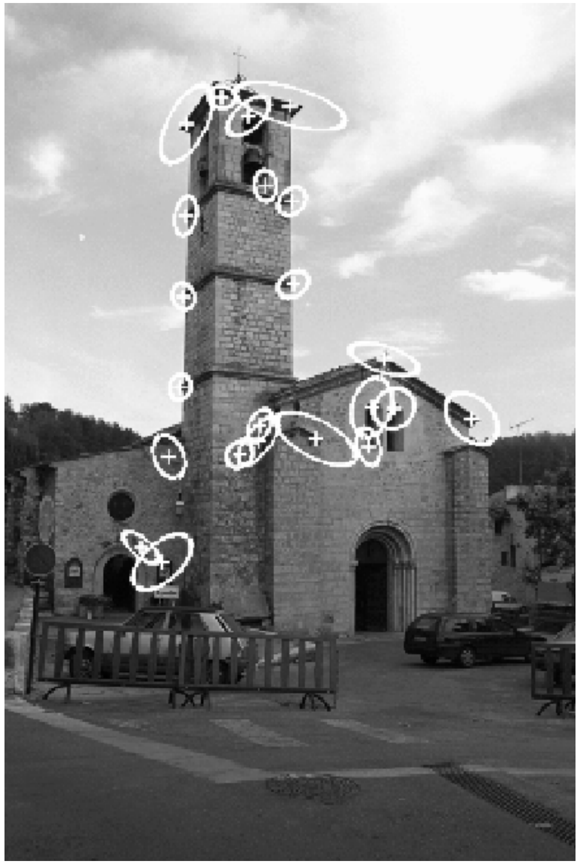
change in viewing angle

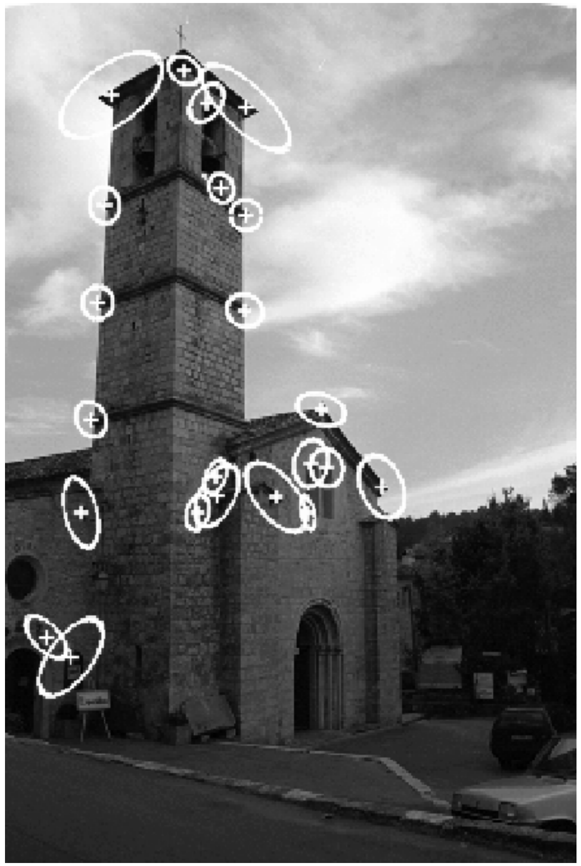


• • •

> 5000 images

Image retrieval





22 correct matches

Image retrieval

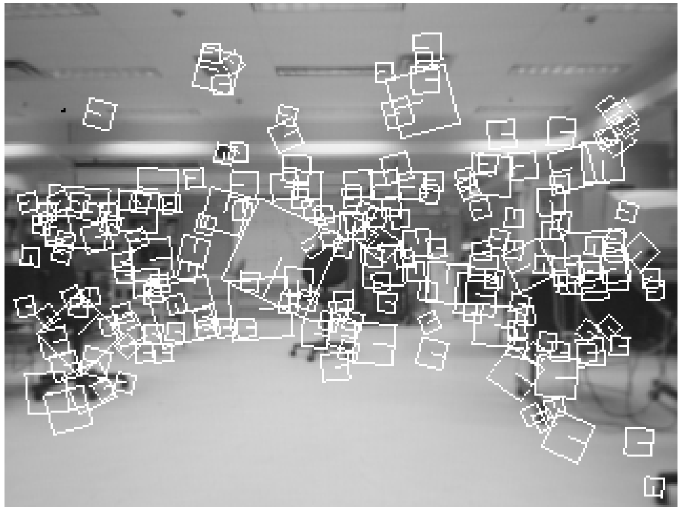
change in viewing angle

+ scale change

• • •

> 5000 images

Robot location



Robotics: Sony Aibo

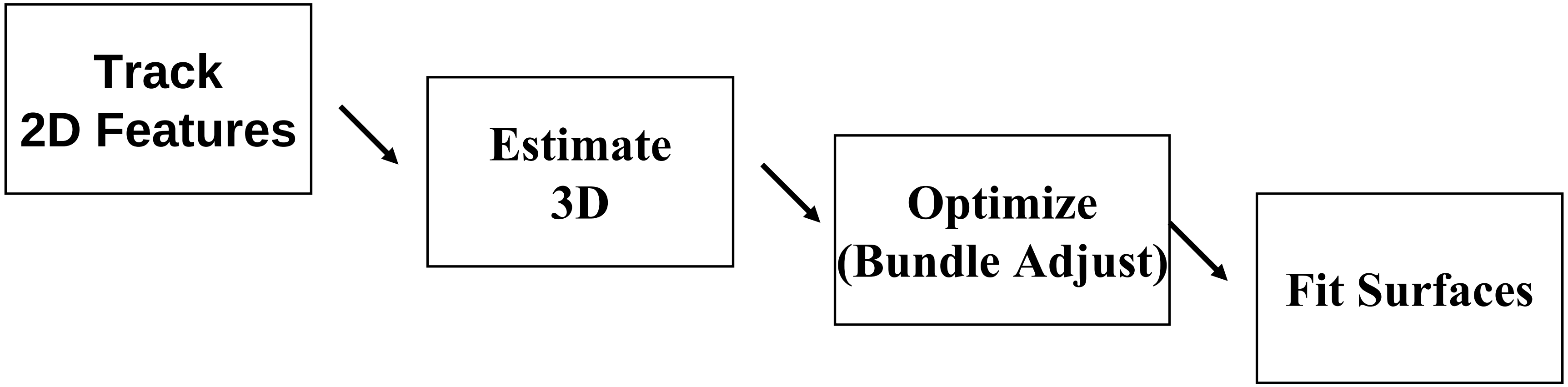
SIFT is used for

- Recognizing charging station
- Communicating with visual cards
- Teaching object recognition

- soccer

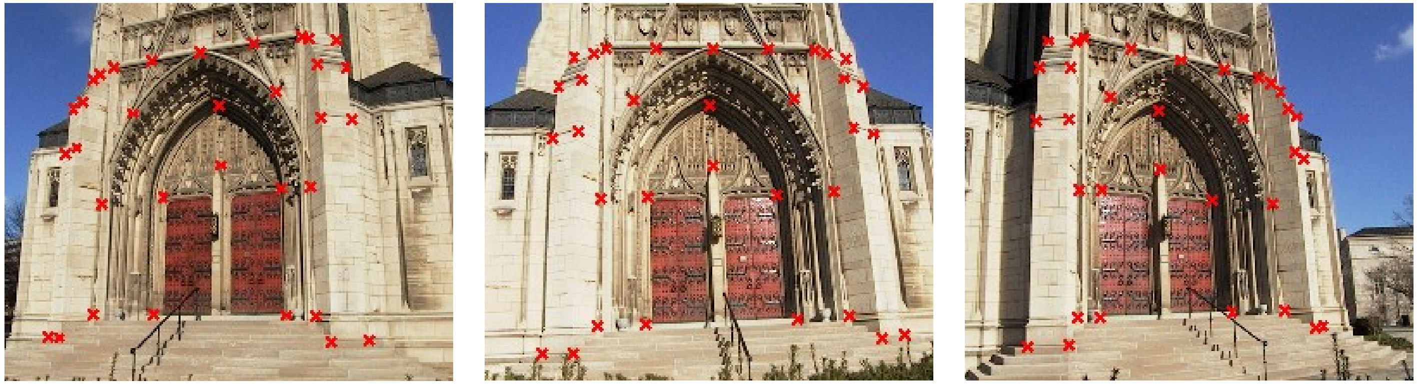
Structure from Motion

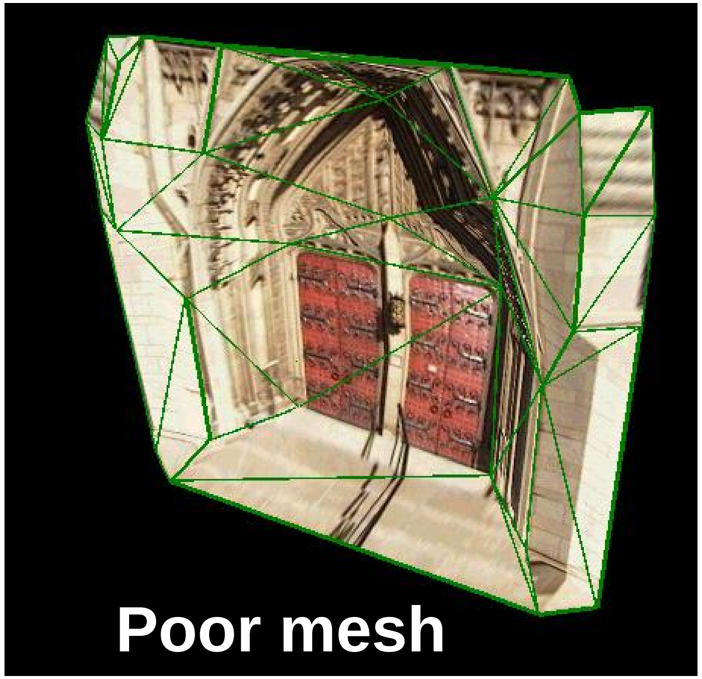
- The SFM Problem
  - Reconstruct scene geometry and camera motion from two or more images

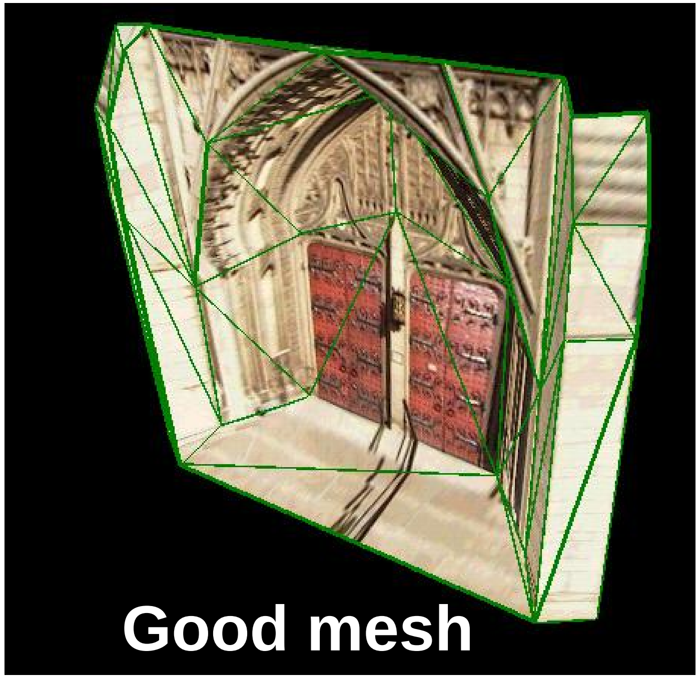


SFM Pipeline

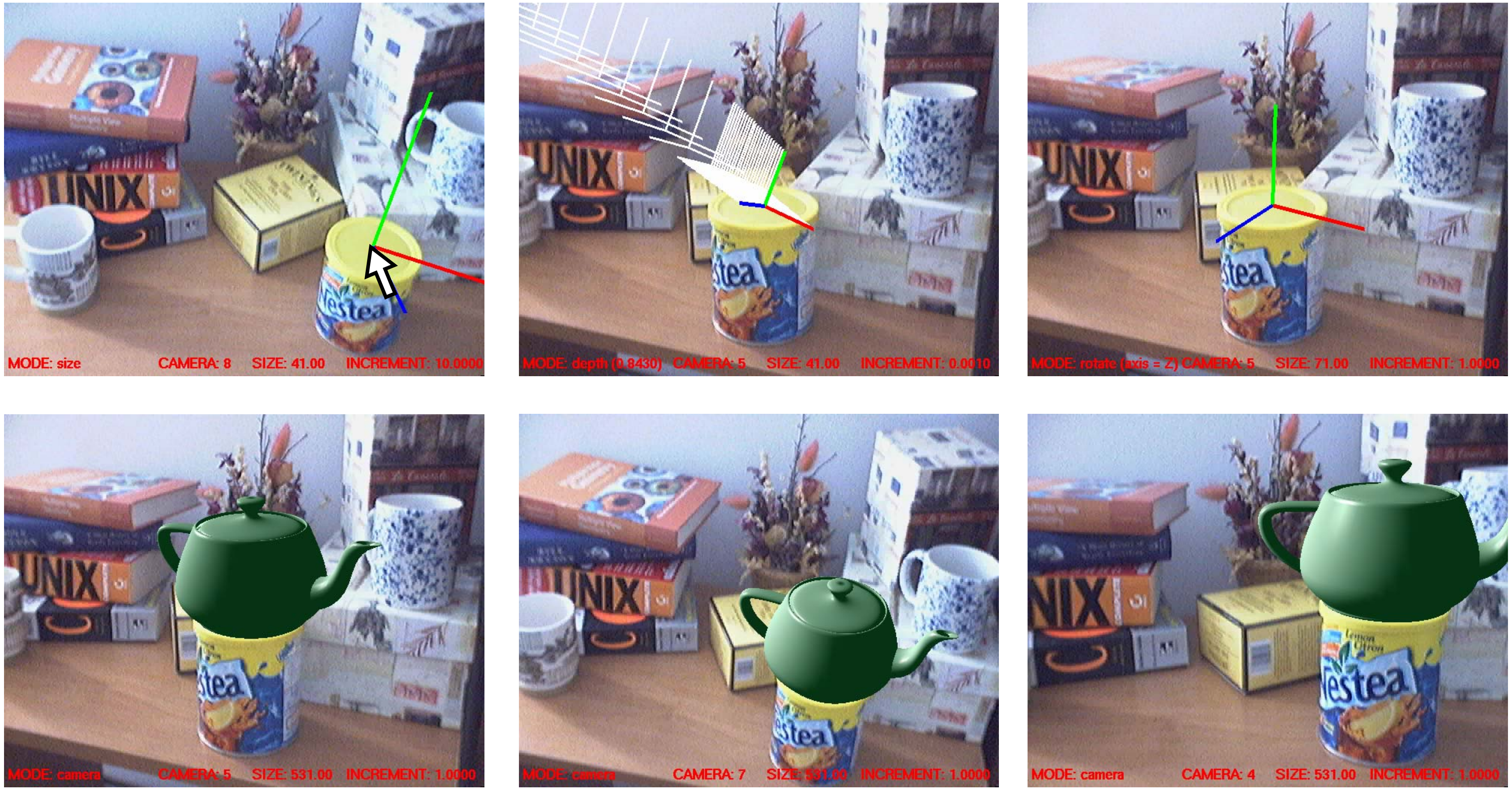
Structure from Motion



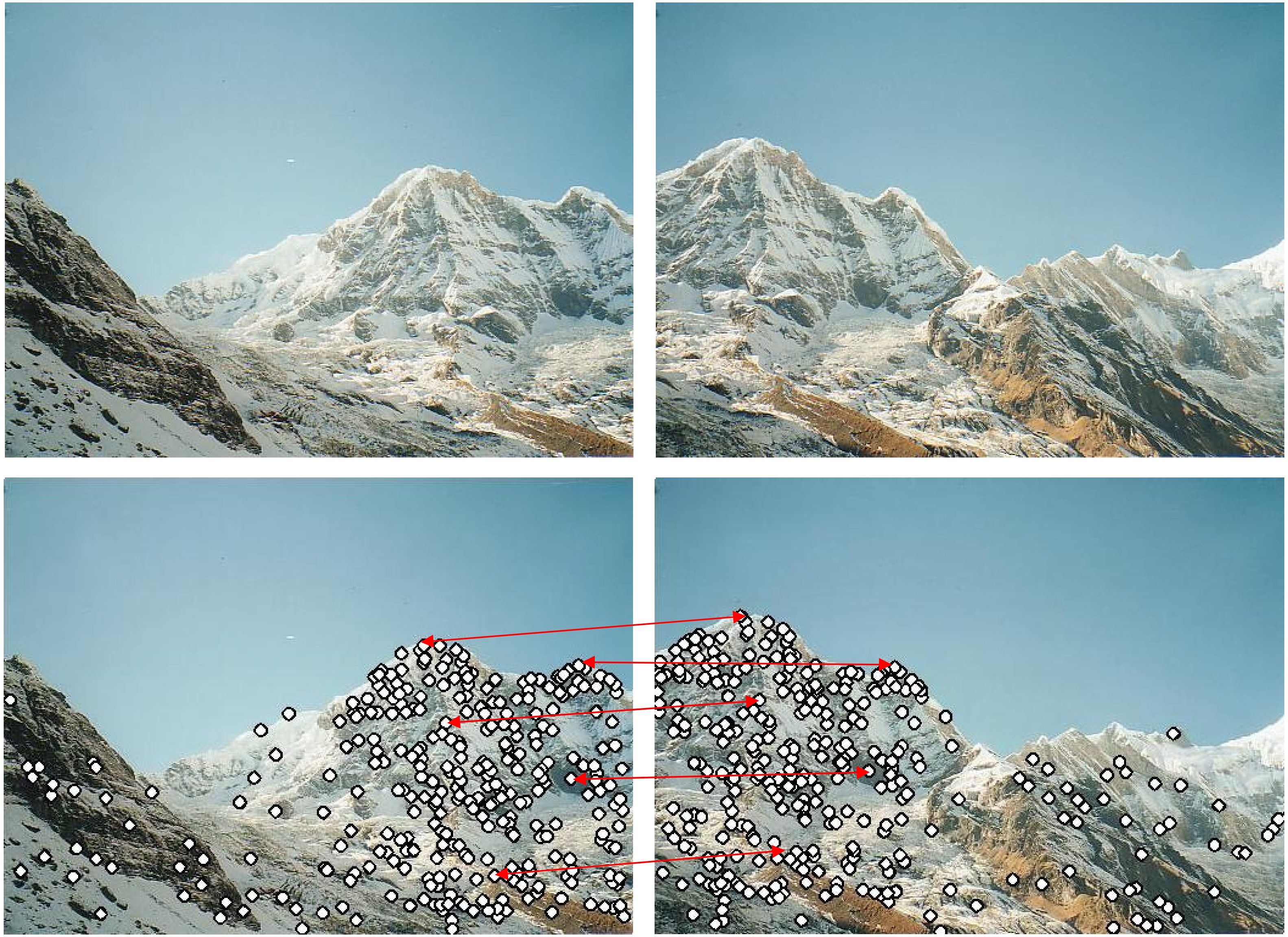




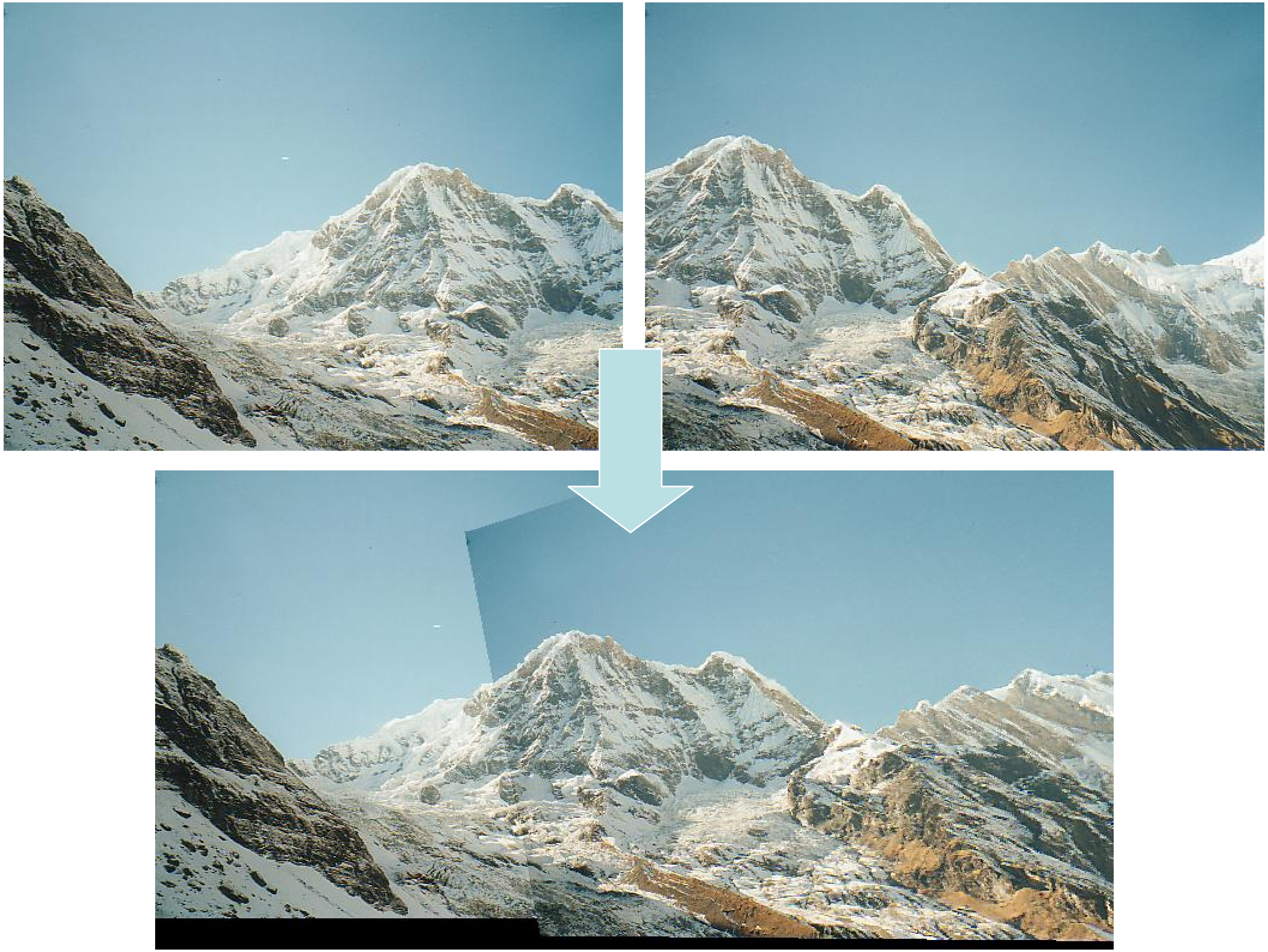
Augmented reality



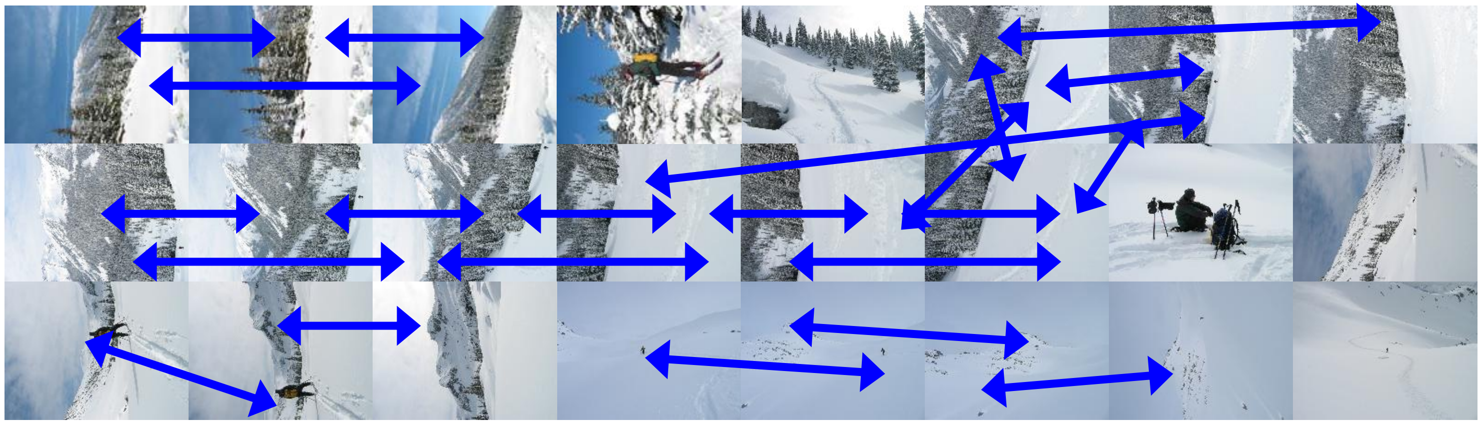
Automatic image stitching



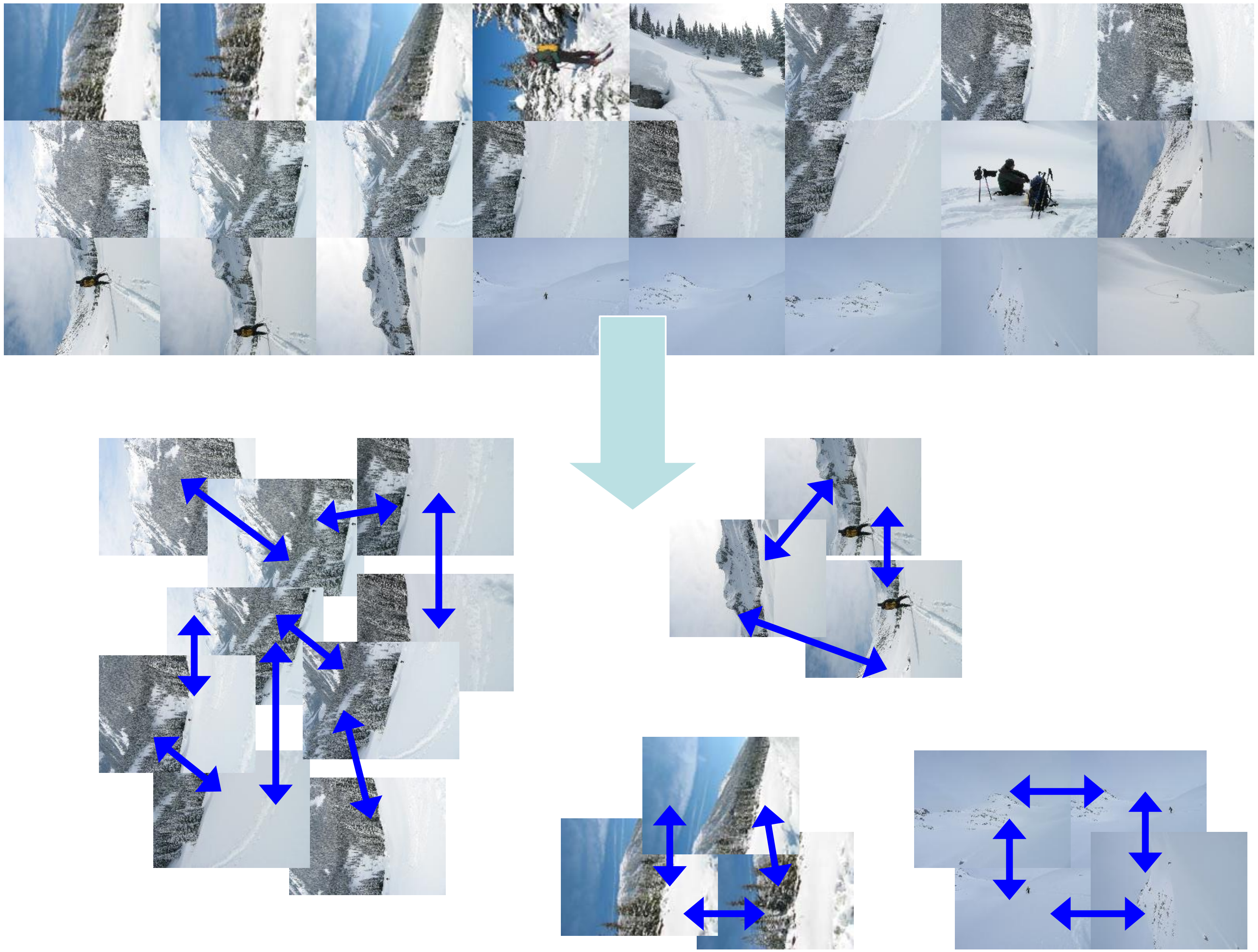
Automatic image stitching



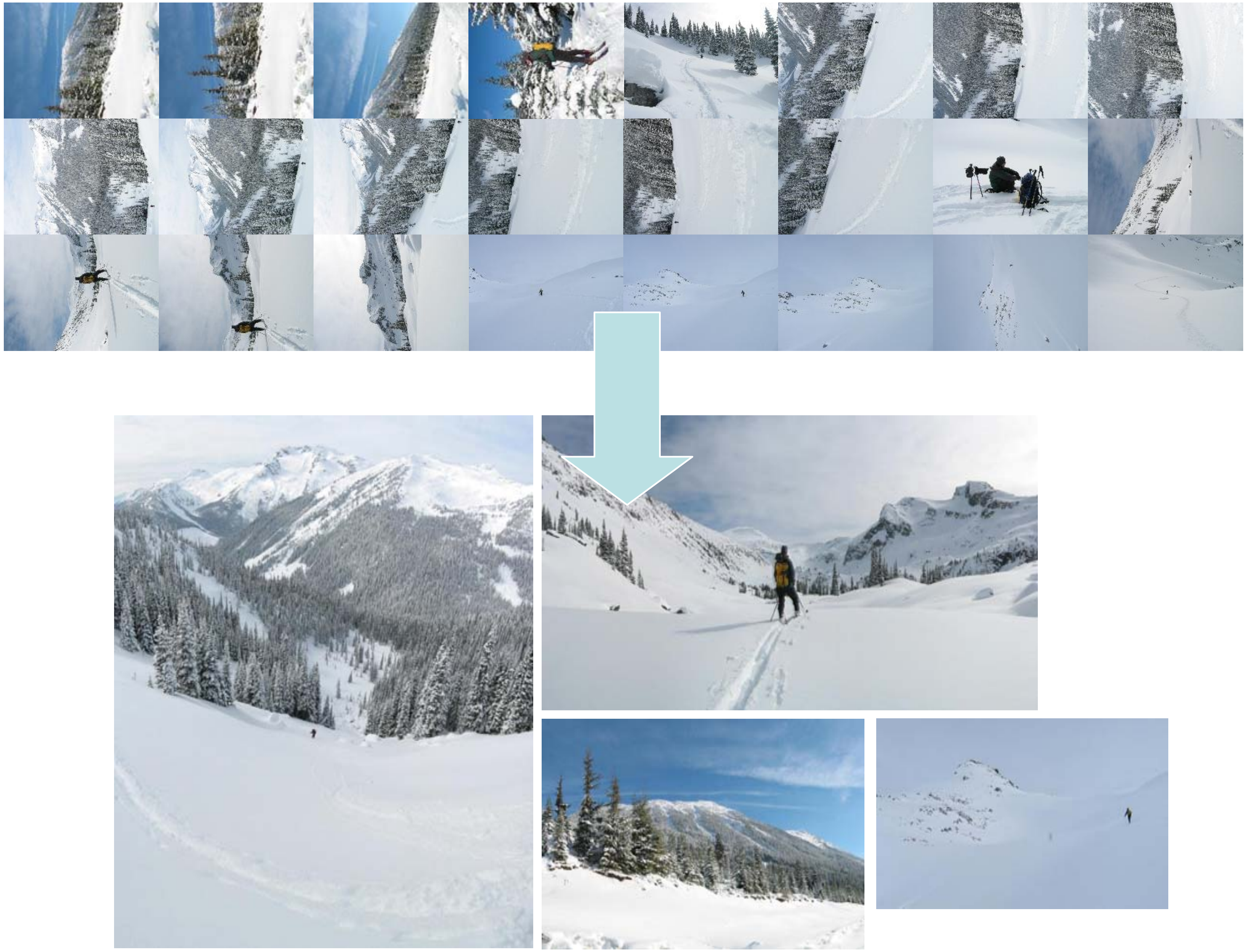
Automatic image stitching



Automatic image stitching



Automatic image stitching

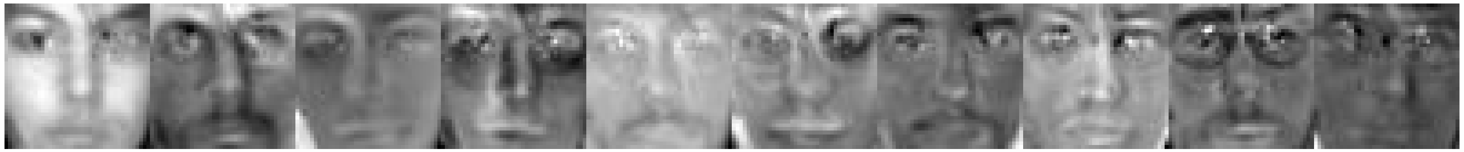


SIFT extensions

PCA

Average face:

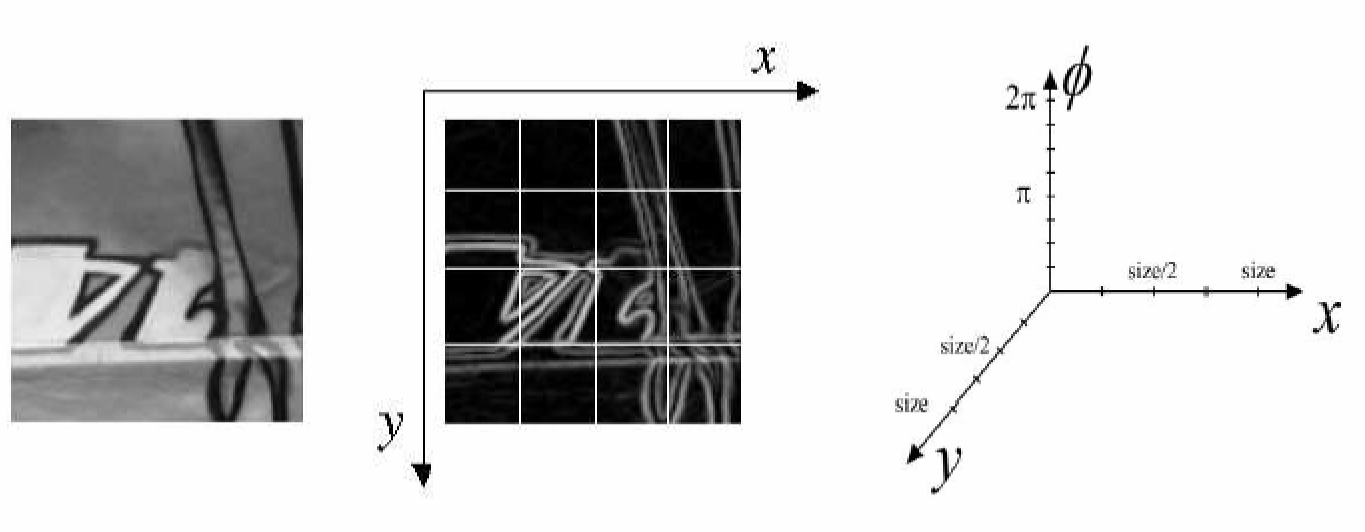
Top ten eigenfaces (left = highest eigenvalue, right = lowest eigenvalue):



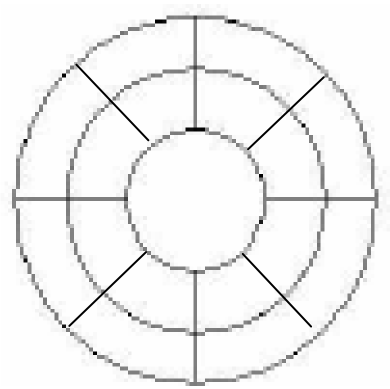
PCA-SIFT

- Only change step 4
- Pre-compute an eigen-space for local gradient patches of size 41x41
- 2x39x39=3042 elements
- Only keep 20 components
- A more compact descriptor

GLOH (Gradient location-orientation histogram)



SIFT



17 location bins
16 orientation bins
Analyze the 17x16=272-d
eigen-space, keep 128 components

SIFT is still considered the best.

Multi-Scale Oriented Patches

- Simpler than SIFT. Designed for image matching. [Brown, Szeliski, Winder, CVPR'2005]
- Feature detector
  - Multi-scale Harris corners
  - Orientation from blurred gradient
  - Geometrically invariant to rotation
- Feature descriptor
  - Bias/gain normalized sampling of local patch (8x8)
  - Photometrically invariant to affine changes in intensity

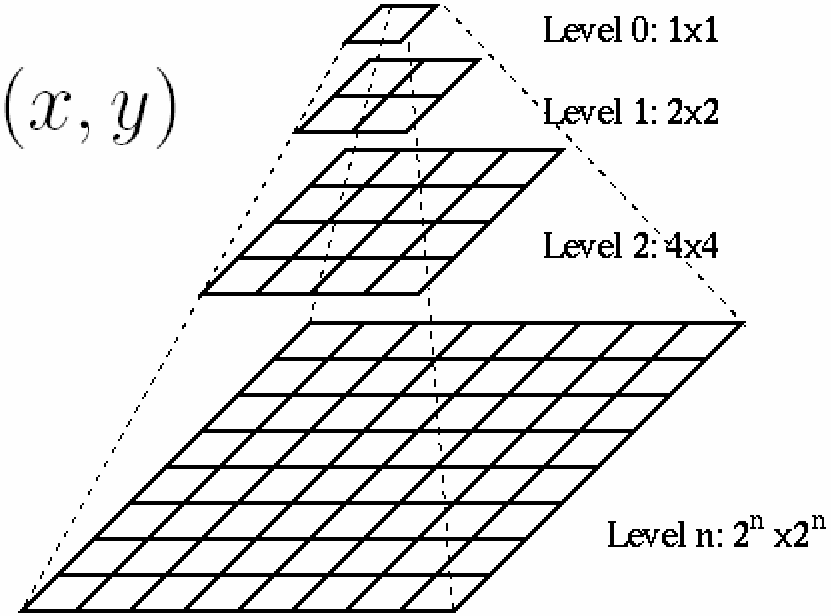
Multi-Scale Harris corner detector

$$P_0(x, y) = I(x, y)$$

$$P'_l(x, y) = P_l(x, y) * g_{\sigma_p}(x, y)$$

$$P_{l+1}(x, y) = P'_l(sx, sy)$$

$$s = 2 \qquad \sigma_p = 1.0$$



- Image stitching is mostly concerned with matching images that have the same scale, so sub-octave pyramid might not be necessary.

Multi-Scale Harris corner detector

$$\mathbf{H}_l(x,y) = \nabla_{\sigma_d} P_l(x,y) \nabla_{\sigma_d} P_l(x,y)^T * g_{\sigma_i}(x,y)$$

$$\nabla_\sigma f(x,y) \triangleq \nabla f(x,y) * g_\sigma(x,y)$$

smoother version of gradients

$$\sigma_i = 1.5 \qquad \sigma_d = 1.0$$

Corner detection function:

$$f_{HM}(x,y) = \frac{\det \mathbf{H}_l(x,y)}{\operatorname{tr} \mathbf{H}_l(x,y)} = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$$

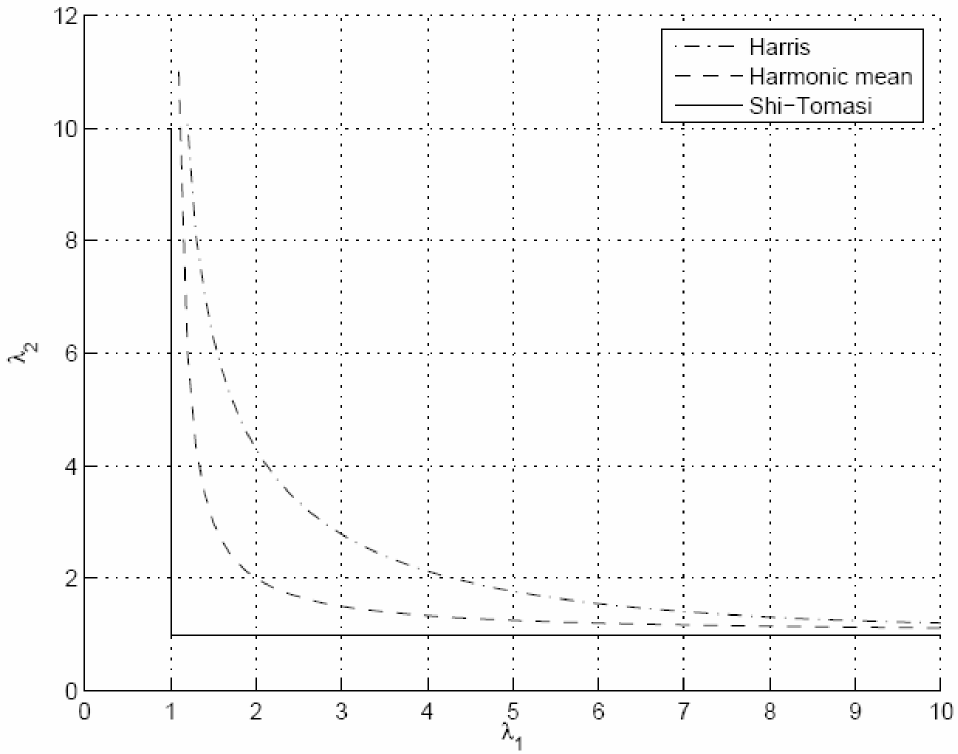
Pick local maxima of 3x3 and larger than 10

Keypoint detection function

$$\text{Harris} \quad f_H = \lambda_1\lambda_2 - 0.04(\lambda_1+\lambda_2)^2 = \det \mathbf{H} - 0.04(\text{tr } \mathbf{H})^2$$

$$\text{Harmonic mean} \quad f_{HM} = \lambda_1\lambda_2/(\lambda_1+\lambda_2) = \det \mathbf{H}/\text{tr } \mathbf{H}$$

$$\text{Shi-Tomasi} \quad f_{ST} = \min(\lambda_1, \lambda_2)$$

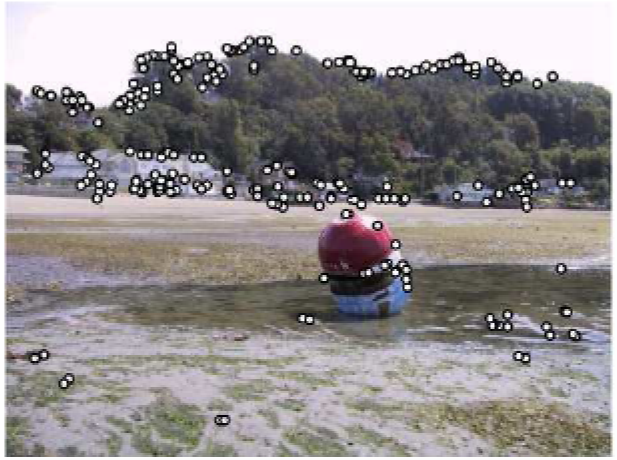


Experiments show roughly the same performance.

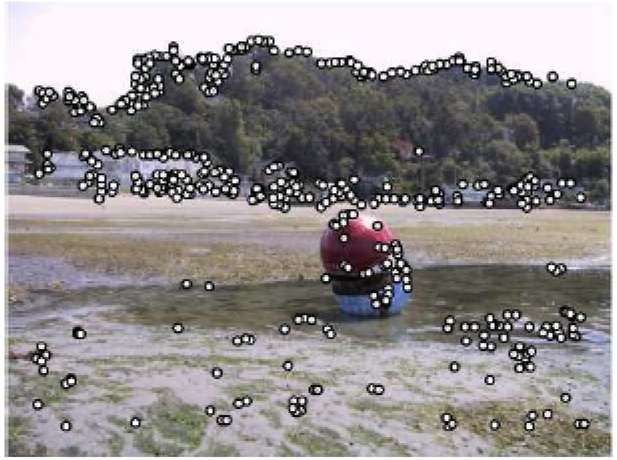
Non-maximal suppression

- Restrict the maximal number of interest points, but also want them spatially well distributed
- Only retain maximums in a neighborhood of radius r.
- Sort them by strength, decreasing r from infinity until the number of keypoints (500) is satisfied.

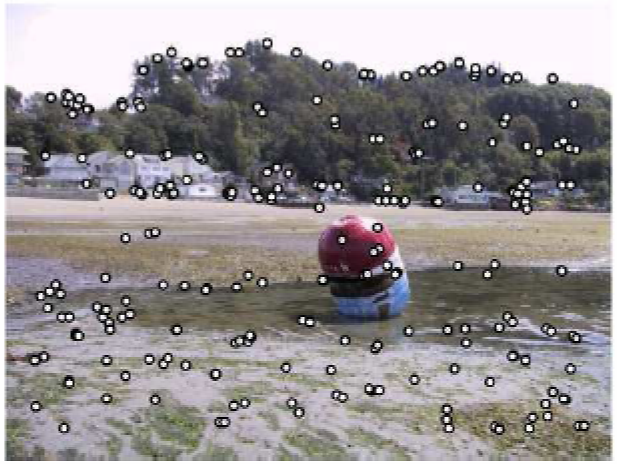
Non-maximal suppression



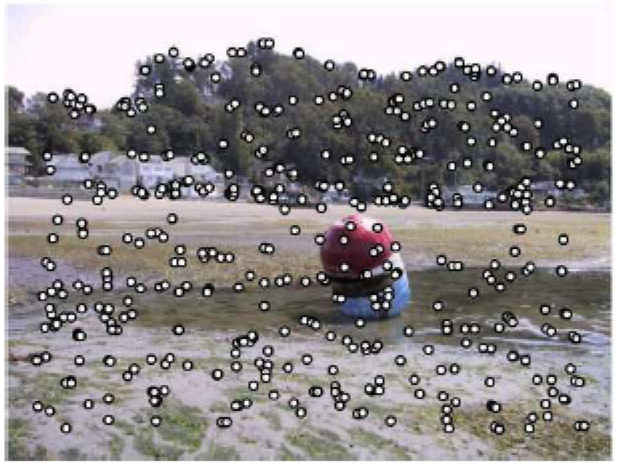
(a) Strongest 250



(b) Strongest 500



(c) ANMS 250, $r = 24$



(d) ANMS 500, $r = 16$

Sub-pixel refinement

$$f(\mathbf{x}) = f + \frac{\partial f}{\partial \mathbf{x}}^T \mathbf{x} + \frac{1}{2}\mathbf{x}^T \frac{\partial^2 f}{\partial \mathbf{x}^2}\mathbf{x}$$

$$\mathbf{x}_m = -\frac{\partial^2 f}{\partial \mathbf{x}^2}^{-1} \frac{\partial f}{\partial \mathbf{x}}$$

$f_{-1,1}$	$f_{0,1}$	$f_{1,1}$
$f_{-1,0}$	$f_{0,0}$	$f_{1,0}$
$f_{-1,-1}$	$f_{0,-1}$	$f_{1,-1}$

$$\begin{aligned}
\frac{\partial f}{\partial x} &= (f_{1,0} - f_{-1,0})/2 \\
\frac{\partial f}{\partial y} &= (f_{0,1} - f_{0,-1})/2 \\
\frac{\partial^2 f}{\partial x^2} &= f_{1,0} - 2f_{0,0} + f_{-1,0} \\
\frac{\partial^2 f}{\partial y^2} &= f_{0,1} - 2f_{0,0} + f_{0,-1} \\
\frac{\partial^2 f}{\partial x \partial y} &= (f_{-1,-1} - f_{-1,1} - f_{1,-1} + f_{1,1})/4
\end{aligned}$$

Orientation assignment

- Orientation = blurred gradient

$$\mathbf{u}_l(x, y) = \nabla_{\sigma_o} P_l(x, y)$$

$$\sigma_o = 4.5$$

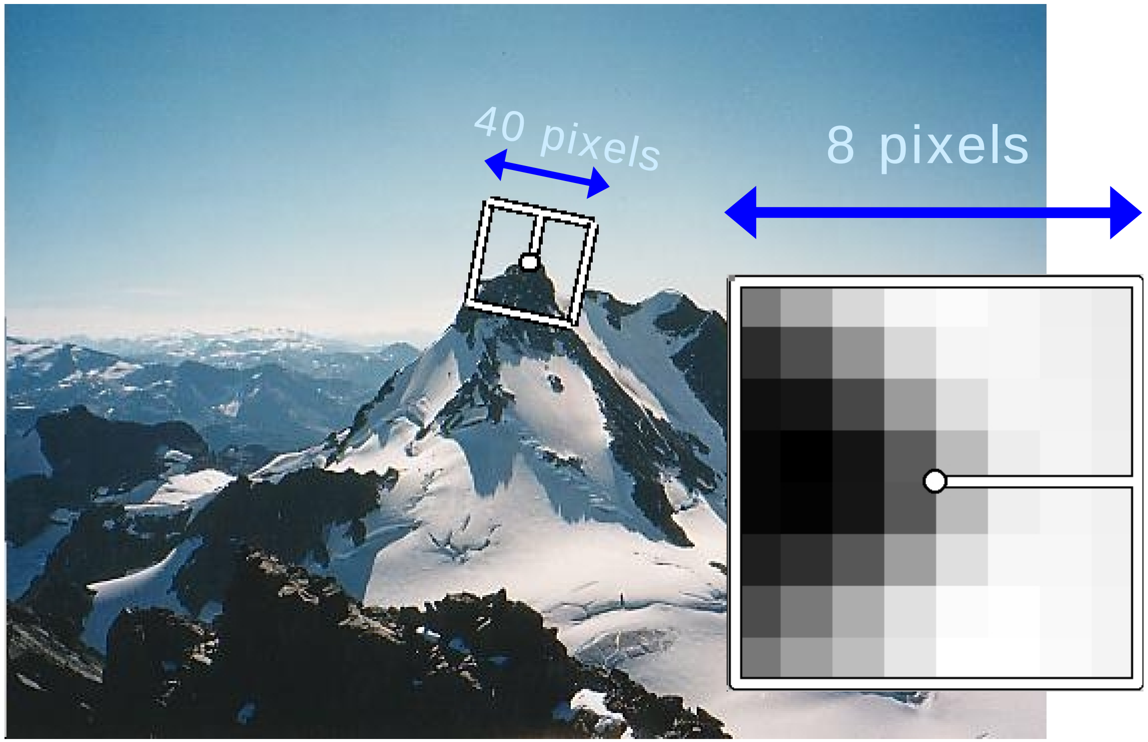
$$[\cos\theta, \sin\theta] = \mathbf{u}/|\mathbf{u}|$$

Descriptor Vector

- Rotation Invariant Frame
  - Scale-space position (x, y, s) + orientation (θ)

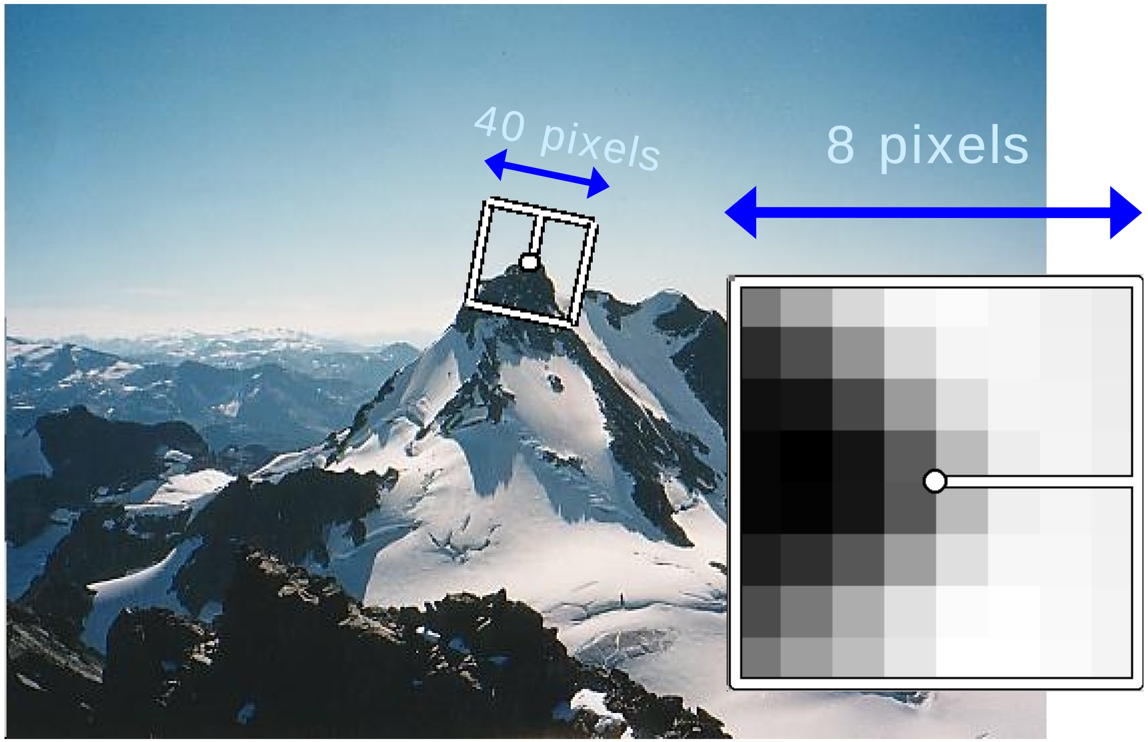
MOPS descriptor vector

- 8x8 oriented patch sampled at 5 x scale. See TR for details.
- Sampled from $P_l(x,y) * g_{2\times\sigma_p}(x,y)$ with spacing=5



MOPS descriptor vector

- 8x8 oriented patch sampled at 5 x scale. See TR for details.
- Bias/gain normalisation: $I' = (I - \mu)/\sigma$
- Wavelet transform



Detections at multiple scales

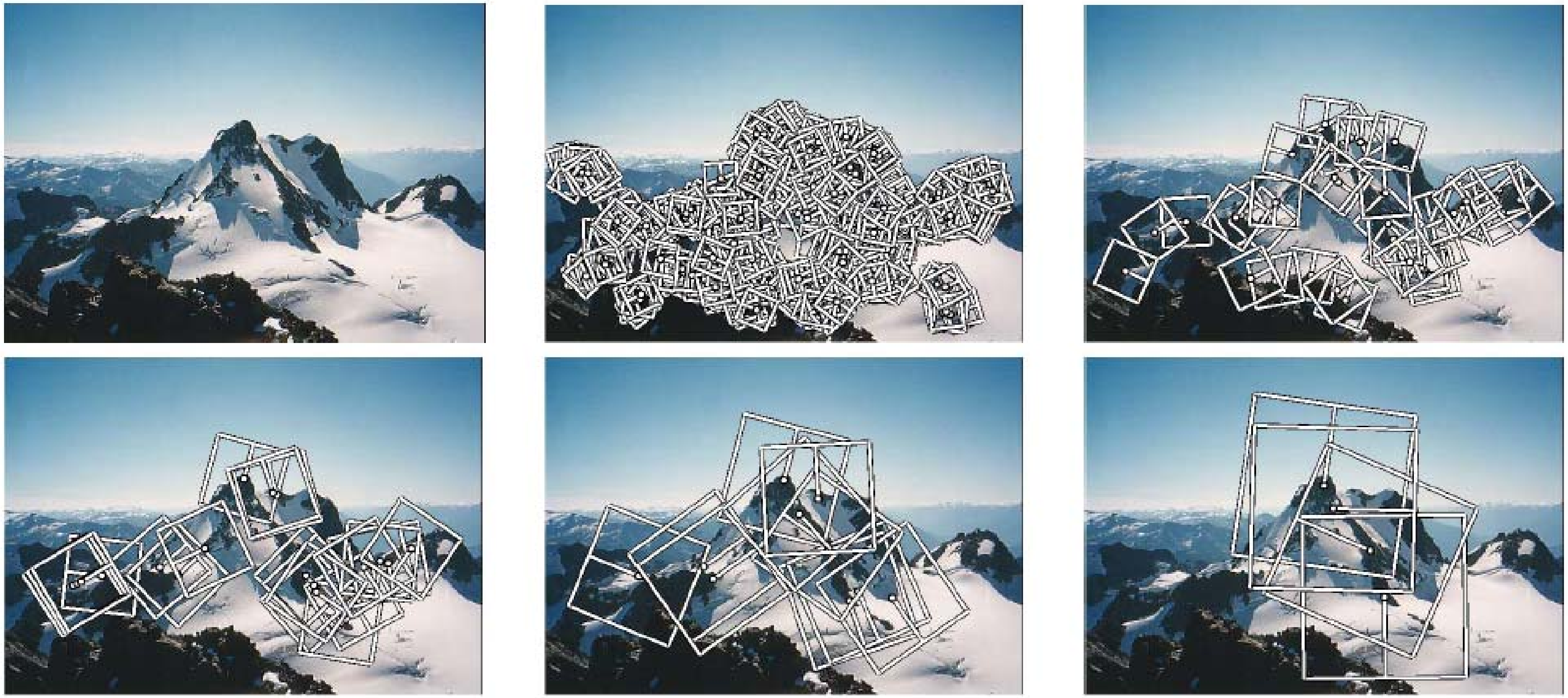


Figure 1. Multi-scale Oriented Patches (MOPS) extracted at five pyramid levels from one of the Matier images. The boxes show the feature orientation and the region from which the descriptor vector is sampled.

Summary

- Multi-scale Harris corner detector
- Sub-pixel refinement
- Orientation assignment by gradients
- Blurred intensity patch as descriptor

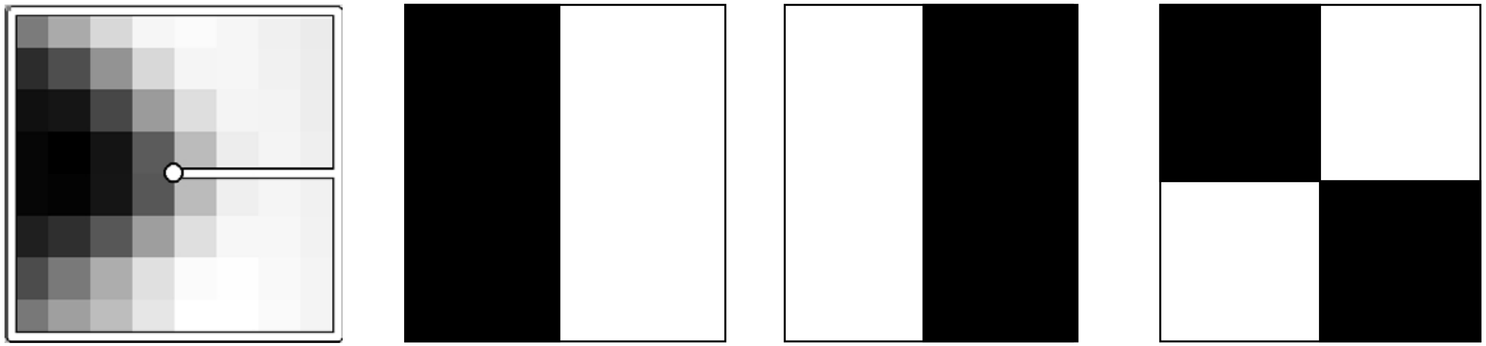
Feature matching

DigiVFX

- Exhaustive search
  - for each feature in one image, look at *all* the other features in the other image(s)
- Hashing
  - compute a short descriptor from each feature vector, or hash longer descriptors (randomly)
- Nearest neighbor techniques
  - *k*-trees and their variants (Best Bin First)

Wavelet-based hashing

- Compute a short (3-vector) descriptor from an 8x8 patch using a Haar “wavelet”



- Quantize each value into 10 (overlapping) bins (10^3 total entries)
- [Brown, Szeliski, Winder, CVPR’2005]

Nearest neighbor techniques

- *k*-D tree and
- Best Bin First (BBF)

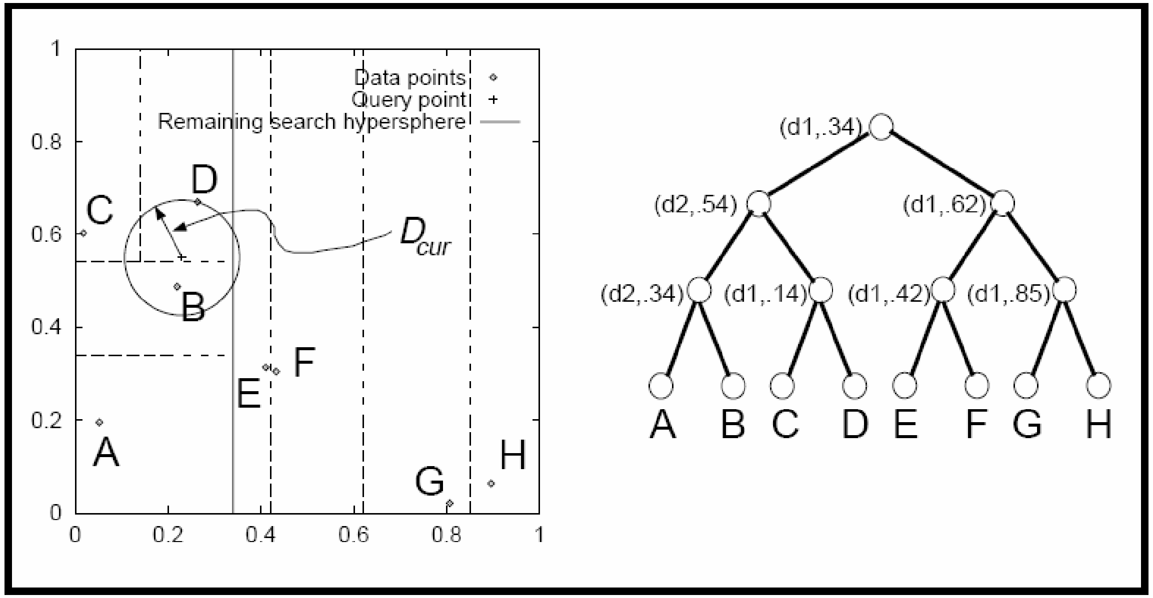


Figure 6: kd-tree with 8 data points labelled A-H, dimension of space k=2. On the right is the full tree, the leaf nodes containing the data points. Internal node information consists of the dimension of the cut plane and the value of the cut in that dimension. On the left is the 2D feature space carved into various sizes and shapes of bin, according to the distribution of the data points. The two representations are isomorphic. The situation shown on the left is after initial tree traversal to locate the bin for query point "+" (contains point D). In standard search, the closest nodes in the tree are examined first (starting at C). In BBF search, the closest bins to query point q are examined first (starting at B). The latter is more likely to maximize the overlap of (i) the hypersphere centered on q with radius D_{cur}, and (ii) the hyperrectangle of the bin to be searched. In this case, BBF search reduces the number of leaves to examine, since once point B is discovered, all other branches can be pruned.

Indexing Without Invariants in 3D Object Recognition, Beis and Lowe, PAMI'99

Project #2 Image stitching

- Assigned: 3/27
- Checkpoint: 11:59pm 4/15
- Due: 11:59am 4/24
- Work in pairs

Reference software

- Autostitch

http://www.cs.ubc.ca/~mbrown/autostitch/autostitch.html

- Many others are available online.

Tips for taking pictures

- Common focal point
- Rotate your camera to increase vertical FOV
- Tripod
- Fixed exposure?

Bells & whistles

- Recognizing panorama
- Bundle adjustment
- Handle dynamic objects
- Better blending techniques

Artifacts

- Take your own pictures and generate a stitched image, be creative.
- http://www.cs.washington.edu/education/courses/cse590ss/01wi/projects/project1/students/allen/index.html

Submission

- You have to turn in your complete source, the executable, a html report and an artifact.
- Report page contains:

  description of the project, what do you learn, algorithm, implementation details, results, bells and whistles…
- Artifacts must be made using your own program.

Reference

- Chris Harris, Mike Stephens, A Combined Corner and Edge Detector, 4th Alvey Vision Conference, 1988, pp147-151.
- David G. Lowe, Distinctive Image Features from Scale-Invariant Keypoints, International Journal of Computer Vision, 60(2), 2004, pp91-110.
- Yan Ke, Rahul Sukthankar, PCA-SIFT: A More Distinctive Representation for Local Image Descriptors, CVPR 2004.
- Krystian Mikolajczyk, Cordelia Schmid, A performance evaluation of local descriptors, Submitted to PAMI, 2004.
- SIFT Keypoint Detector, David Lowe.
- Matlab SIFT Tutorial, University of Toronto.