Volume and Participating Media

Digital Image Synthesis

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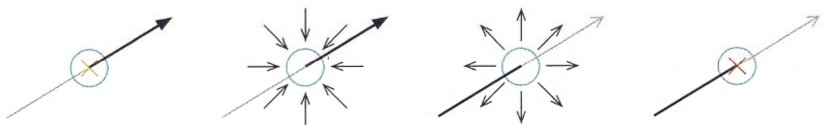
with slides by Pat Hanrahan and Torsten Moller

Participating media

- We have by far assumed that the scene is in a vacuum. Hence, radiance is constant along the ray. However, some real-world situations such as fog and smoke attenuate and scatter light. They participate in rendering.
- Natural phenomena
  - Fog, smoke, fire
  - Atmosphere haze
  - Beam of light through clouds
  - Subsurface scattering

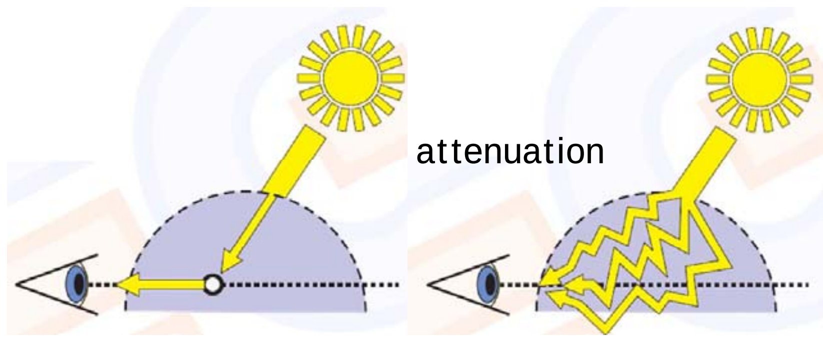
Volume scattering processes

- Absorption (conversion from light to other forms)
- Emission (contribution from luminous particles)
- Scattering (direction change of particles)
  - Out-scattering
  - In-scattering
  - Single scattering v.s. multiple scattering
- Homogeneous v.s. inhomogeneous(heterogeneous)



emission   in-scattering   out-scattering   absorption

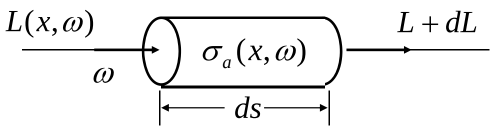
Single scattering and multiple scattering



single scattering   multiple scattering

Absorption

The reduction of energy due to conversion of light to another form of energy (e.g. heat)



$$dL(x,\omega) = -\sigma_a(x,\omega)L(x,\omega)ds$$

Absorption cross-section: $\sigma_a(x,\omega)$

Probability of being absorbed per unit length

Transmittance

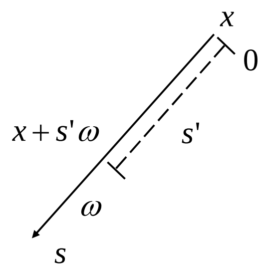
$$dL(x,\omega) = -\sigma_a(x,\omega)L(x,\omega)ds$$

$$\frac{dL(x,\omega)}{L(x,\omega)} = -\sigma_a(x,\omega)ds \longrightarrow \int_x^{x+s\omega} \frac{dL(x',\omega)}{L(x',\omega)} = -\int_0^s \sigma_a(x+s'\omega,\omega)ds'$$

$$\ln L(x+s\omega,\omega) - \ln L(x,\omega) = -\int_0^s \sigma_a(x+s'\omega,\omega)ds' = -\tau_\omega(s)$$

Optical distance or depth

$$\tau_\omega(s) = \int_0^s \sigma_a(x+s'\omega,\omega)ds'$$



Homogenous media: constant σ_a

$$\sigma_a \to \tau(s) = \sigma_a s$$

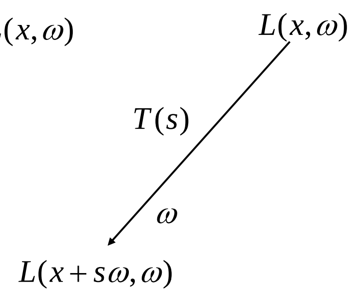
Transmittance and opacity

$$dL(x,\omega) = -\sigma_a(x,\omega)L(x,\omega)ds$$

$$\frac{dL(x,\omega)}{L(x,\omega)} = -\sigma_a(x,\omega)ds \longrightarrow \int_x^{x+s\omega} \frac{dL(x',\omega)}{L(x',\omega)} = -\int_0^s \sigma_a(x+s'\omega,\omega)ds'$$

$$\ln L(x+s\omega,\omega) - \ln L(x,\omega) = -\int_0^s \sigma_a(x+s'\omega,\omega)ds' = -\tau_\omega(s)$$

$$L(x+s\omega,\omega) = e^{-\tau_\omega(s)}L(x,\omega) = T_\omega(s)L(x,\omega)$$



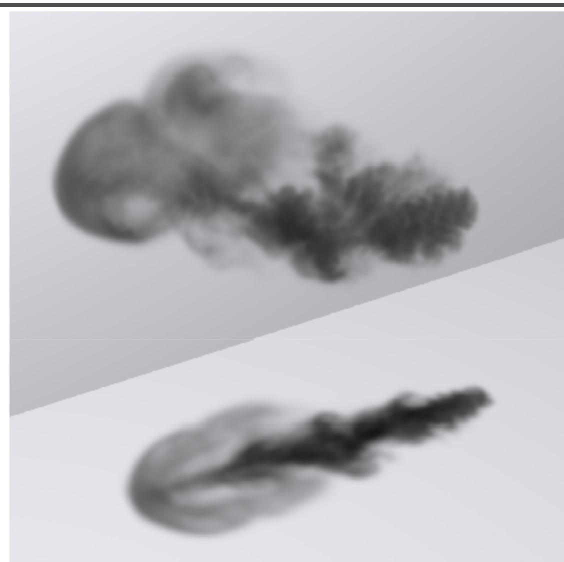
Transmittance

$$T_\omega(s) = e^{-\tau_\omega(s)}$$

Opacity

$$\alpha_\omega(s) = 1 - T_\omega(s)$$

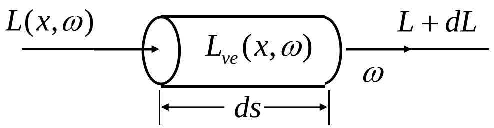
Absorption



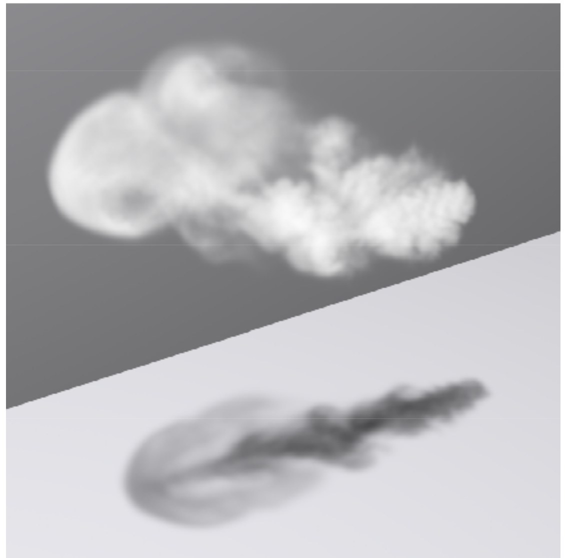
Emission

- Energy that is added to the environment from luminous particles due to chemical, thermal, or nuclear processes that convert energy to visible light.
- $L_{ve}(x,\omega)$: emitted radiance added to a ray per unit distance at a point x in direction ω

$$dL(x,\omega) = L_{ve}(x,\omega)ds$$

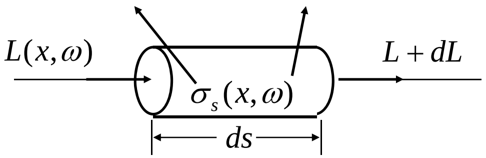


Emission



Out-scattering

Light heading in one direction is scattered to other directions due to collisions with particles

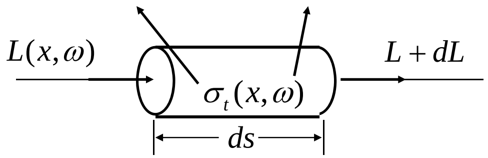


$$dL(x,\omega) = -\sigma_s(x,\omega)L(x,\omega)ds$$

Scattering cross-section: σ_s

Probability of being scattered per unit length

Extinction



$$dL(x,\omega) = -\sigma_t(x,\omega)L(x,\omega)ds$$

Total cross-section $\sigma_t = \sigma_a + \sigma_s$

Albedo $W = \dfrac{\sigma_s}{\sigma_t} = \dfrac{\sigma_s}{\sigma_a + \sigma_s}$

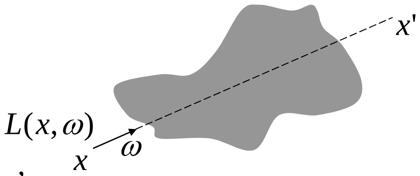
Attenuation due to both absorption and scattering

$$\tau_\omega(s) = \int_0^s \sigma_t(x + s'\omega, \omega)ds'$$

Extinction

- Beam transmittance

$$Tr(x \to x') = e^{-\int_0^s \sigma_t(x+s'\omega,\omega)ds'}$$



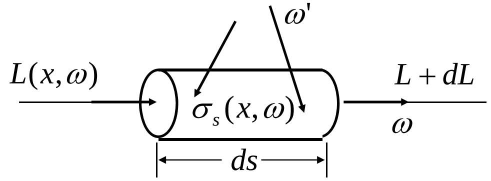
s: distance between x and x’

- Properties of Tr:
- In vaccum $\quad Tr(x \to x') = 1$
- Multiplicative $\quad Tr(x \to x'') = Tr(x \to x') \cdot Tr(x' \to x'')$
- Beer’s law (in homogeneous medium)

$$Tr(x \to x') = e^{-\sigma_t s}$$

In-scattering

Increased radiance due to scattering from other directions



$$dL(x,\omega) = \left[\sigma_s(x,\omega) \int_\Omega p(x,\omega' \to \omega) L(x,\omega') d\omega' \right] ds$$

Phase function $\quad p(\omega' \to \omega)$

Reciprocity

$$p(\omega \to \omega') = p(\omega' \to \omega)$$

Energy conserving

$$\int_{S^2} p(\omega' \to \omega)\, d\omega' = 1$$

Source term

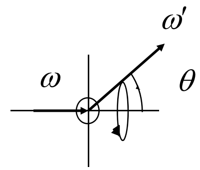
$$S(x,\omega) = L_{ve}(x,\omega) + \sigma_s(x,\omega) \int_\Omega p(x,\omega' \to \omega) L(x,\omega') d\omega'$$

$$dL(x,\omega) = S(x,\omega) ds$$

- S is determined by
  - Volume emission
  - Phase function which describes the angular distribution of scattered radiation (volume analog of BSDF for surfaces)

Phase functions

Phase angle $\cos\theta = \omega \bullet \omega'$



Phase functions
(from the phase of the moon)

1. Isotropic $\quad p(\cos\theta) = \frac{1}{4\pi}$

- simple

2. Rayleigh $\quad p(\cos\theta) = \frac{3}{4} \frac{1+\cos^2\theta}{\lambda^4}$

- Molecules (useful for very small particles whose radii smaller than wavelength of light)

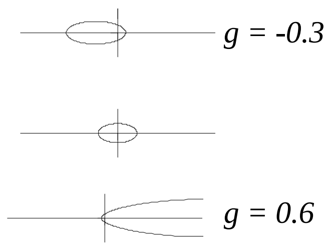
3. Mie scattering

- small spheres (based on Maxwell’s equations; good model for scattering in the atmosphere due to water droplets and fog)

Henyey-Greenstein phase function

Empirical phase function

$$p(\cos\theta) = \frac{1}{4\pi}\frac{1-g^2}{\left(1+g^2-2g\cos\theta\right)^{3/2}}$$



$$2\pi\int_0^\pi p(\cos\theta)\cos\theta\, d\theta = g$$

g: **average phase angle**

Henyey-Greenstein approximation

- Any phase function can be written in terms of a series of Legendre polynomials (typically, n<4)

$$p(\cos\theta) = \frac{1}{4\pi}\sum_{n=0}^{\infty}(2n+1)\boxed{b_n}\boxed{P_n(\cos\theta)}$$

$$b_n = \langle p(\cos\theta), P_n(\cos\theta)\rangle = \int_{-1}^{1} p(\cos\theta)P_n(\cos\theta)d\cos\theta$$

$$P_0(x) = 1$$
$$P_1(x) = x$$
$$P_2(x) = \frac{1}{2}(3x^2-1)$$
$$P_3(x) = \frac{1}{2}(5x^3-3x)$$
...

Schlick approximation

- Approximation to Henyey-Greenstein

$$p_{Schlick}(\cos\theta) = \frac{1}{4\pi}\frac{1-k^2}{(1-k\cos\theta)^2}$$

- K plays a similar role like g
  - 0: isotropic
  - -1: back scattering
  - Could use $k = 1.55g - 0.55g^2$

Importance sampling for HG

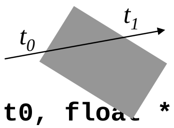
$$p(\cos\theta) = \frac{1}{4\pi}\frac{1-g^2}{\left(1+g^2-2g\cos\theta\right)^{3/2}}$$

$$\phi = 2\pi\xi$$

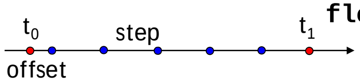
$$\cos\theta = \begin{cases} 1-2\xi & \text{if } g = 0 \\ -\dfrac{1}{|2g|}\left(1+g^2-\left(\dfrac{1-g^2}{1-g+2g\xi}\right)^2\right) & \text{otherwise} \end{cases}$$

Pbrt implementation

- `core/volume.*  volume/*`




```
class VolumeRegion {
public:
  bool IntersectP(Ray &ray, float *t0, float *t1);
  Spectrum sigma_a(Point &, Vector &);
  Spectrum sigma_s(Point &, Vector &);
  Spectrum Lve(Point &, Vector &);
  // phase functions: pbrt has isotropic, Rayleigh,
  // Mie, HG, Schlick
  virtual float p(Point &, Vector &, Vector &);
  // attenuation coefficient; s_a+s_s
  Spectrum sigma_t(Point &, Vector &);
  // calculate optical thickness by Monte Carlo or
  // closed-form solution
  Spectrum Tau(Ray &ray, float step=1.,
                float offset=0.5);
};
```



Homogenous volume

- Determined by (constant)
  - σ_s and σ_a
  - g in phase function
  - Emission L_{ve}
  - Spatial extent

```
Spectrum Tau(Ray &ray, float, float){
  float t0, t1;
  if (!IntersectP(ray,&t0,&t1))
    return 0.;
  return Distance(ray(t0),ray(t1)) * (sig_a + sig_s);
}
```

Homogenous volume

Varying-density volumes

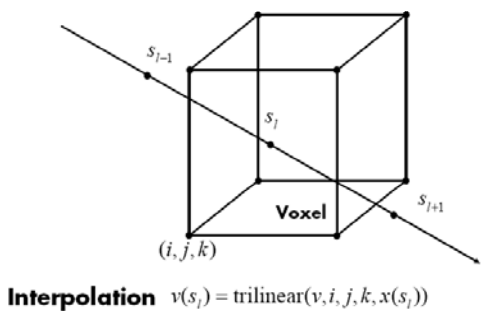
- Density is varying in the medium and the volume scattering properties at a point is the product of the density at that point and some baseline value.
- **`DensityRegion`**
  - 3D grid, **`VolumeGrid`**
  - Exponential density, **`ExponentialDensity`**

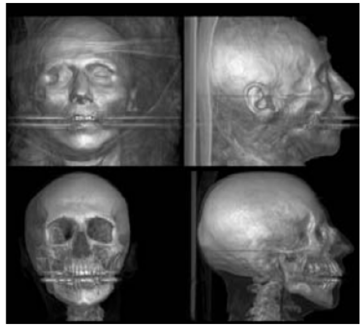
DensityRegion

```
class DensityRegion : public VolumeRegion {
public:
  DensityRegion(Spectrum &sig_a, Spectrum &sig_s,
    float g, Spectrum &Le, Transform &VolumeToWorld);
  float Density(Point &Pobj) const = 0;
  sigma_a(Point &p, Vector &) {
    return Density(WorldToVolume(p)) * sig_a; }
  Spectrum sigma_s(Point &p, Vector &) {
    return Density(WorldToVolume(p)) * sig_s; }
  Spectrum sigma_t(Point &p, Vector &) {
    return Density(WorldToVolume(p))*(sig_a+sig_s);}
  Spectrum Lve(Point &p, Vector &) {
    return Density(WorldToVolume(p)) * le; }
  ...
protected:
  Transform WorldToVolume;
  Spectrum sig_a, sig_s, le;
  float g;
};
```

VolumeGrid

- Standard form of given data
- Tri-linear interpolation of data to give continuous volume
- Often used in volume rendering





VolumeGrid

```
VolumeGrid(Spectrum &sa, Spectrum &ss, float gg,
           Spectrum &emit, BBox &e, Transform &v2w,
           int nx, int ny, int nz, const float *d);

float VolumeGrid::Density(const Point &Pobj) const {
  if (!extent.Inside(Pobj)) return 0;
  // Compute voxel coordinates and offsets
  float voxx = (Pobj.x - extent.pMin.x) /
      (extent.pMax.x - extent.pMin.x) * nx - .5f;
  float voxy = (Pobj.y - extent.pMin.y) /
      (extent.pMax.y - extent.pMin.y) * ny - .5f;
  float voxz = (Pobj.z - extent.pMin.z) /
      (extent.pMax.z - extent.pMin.z) * nz - .5f;
```

VolumeGrid

```
  int vx = Floor2Int(voxx);
  int vy = Floor2Int(voxy);
  int vz = Floor2Int(voxz);
  float dx = voxx - vx, dy = voxy - vy, dz = voxz - vz;
  // Trilinearly interpolate density values
  float d00 = Lerp(dx, D(vx, vy, vz),   D(vx+1, vy, vz));
  float d10 = Lerp(dx, D(vx, vy+1, vz), D(vx+1, vy+1, vz));
  float d01 = Lerp(dx, D(vx, vy, vz+1), D(vx+1, vy, vz+1));
  float d11 = Lerp(dx, D(vx, vy+1,vz+1),D(vx+1,vy+1,vz+1));
  float d0 = Lerp(dy, d00, d10);
  float d1 = Lerp(dy, d01, d11);
  return Lerp(dz, d0, d1);
}
```

```
float D(int x, int y, int z) {
  x = Clamp(x, 0, nx-1);
  y = Clamp(y, 0, ny-1);
  z = Clamp(z, 0, nz-1);
  return density[z*nx*ny+y*nx+x];
}
```

Exponential density

- Given by

$$d(h) = ae^{-bh}$$

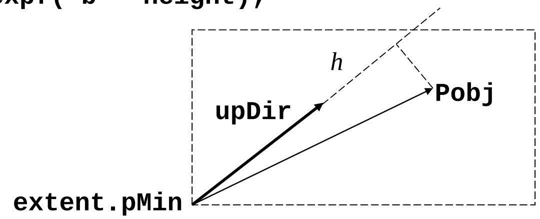
- Where h is the height in the direction of the up-vector

ExponentialDensity

ExponentialDensity

```
class ExponentialDensity : public DensityRegion {
public:
   ExponentialDensity(Spectrum &sa, Spectrum &ss,
     float g, Spectrum &emit, BBox &e, Transform &v2w,
     float aa, float bb, Vector &up)
     ...

   float Density(const Point &Pobj) const {
      if (!extent.Inside(Pobj)) return 0;
      float height = Dot(Pobj - extent.pMin, upDir);
      return a * expf(-b * height);
   }
private:
   BBox extent;
   float a, b;
   Vector upDir;
};
```



Light transport

- Emission + in-scattering (source term)

$$S(x,\omega) = L_{ve}(x,\omega) + \sigma_s(x,\omega)\int_{\Omega} p(x,\omega' \to \omega)L(x,\omega')d\omega'$$

$$dL(x,\omega) = S(x,\omega)ds$$

- Absorption + out-scattering (extinction)

$$dL(x,\omega) = -\sigma_t(x,\omega)L(x,\omega)ds$$

- Combined

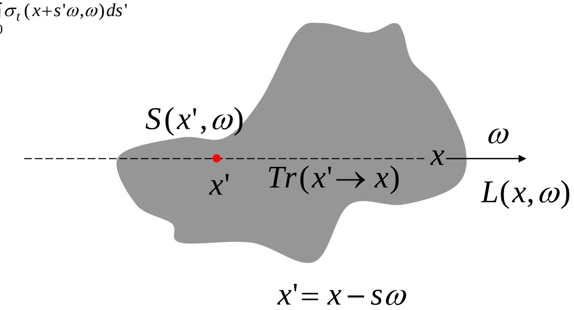
$$\frac{dL(x,\omega)}{ds} = -\sigma_t(x,\omega)L(x,\omega) + S(x,\omega)$$

Infinite length, no surface

- Assume that there is no surface and we have an infinite length, we have the solution

$$L(x,\omega) = \int_0^{\infty} Tr(x' \to x)S(x',\omega)ds$$

$$Tr(x' \to x) = e^{-\int_0^s \sigma_t(x+s'\omega,\omega)ds'}$$

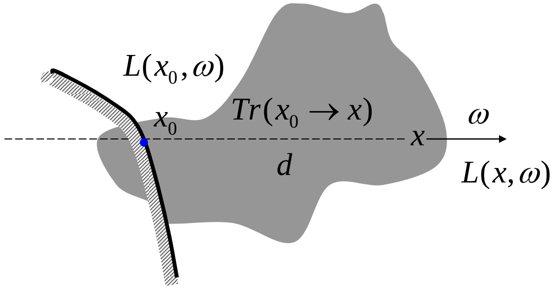


With surface

- The solution

$$L(x,\omega) = \boxed{Tr(x_0 \to x) L(x_0, -\omega)}$$

from the surface point x_0

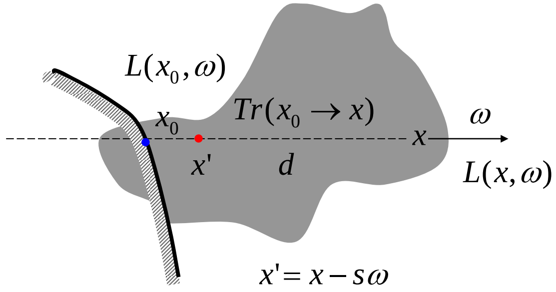


With surface

- The solution

$$L(x,\omega) = \boxed{Tr(x_0 \to x) L(x_0, -\omega)} + \boxed{\int_0^d Tr(x' \to x) S(x', -\omega)\, ds}$$

from the surface point x_0 from the participating media



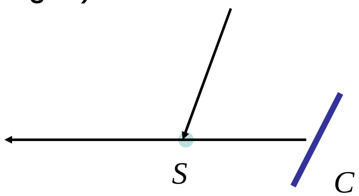
Simple atmosphere model

Assumptions

- Homogenous media
- Constant source term (airlight)

$$\frac{\partial L(s)}{\partial s} = -\sigma_t L(s) + S$$

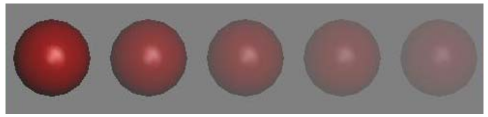
$$L(s) = \left(1 - e^{-\sigma_t s}\right) S \;\; + e^{-\sigma_t s} C$$



- Fog
- Haze

OpenGL fog model

$$C = f C_{in} + (1 - f) C_{fog}$$



GL_EXP

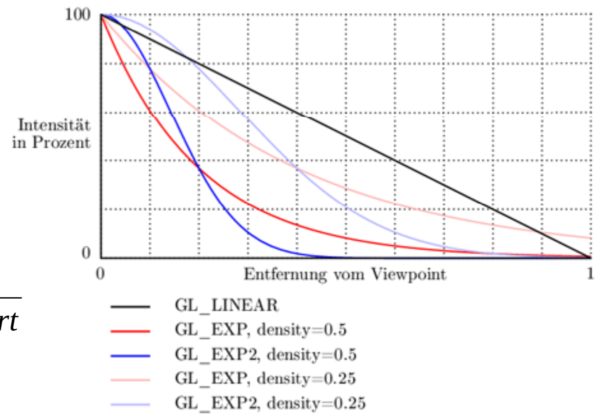
$$f(z) = e^{-(density \cdot z)}$$

GL_EXP2

$$f(z) = e^{-(density \cdot z)^2}$$

GL_LINEAR

$$f(z) = \frac{end - z}{end - start}$$



From http://wiki.delphigl.com/index.php/glFog

VolumeIntegrator

```
class VolumeIntegrator : public Integrator {
public:
  virtual Spectrum Transmittance(
      const Scene *scene,
      const Ray &ray,
      const Sample *sample,
      float *alpha) const = 0;
};
```

Beam transmittance for a given ray from mint to maxt

Pick up functions **Preprocess(), RequestSamples()** and **Li()** from **Integrator**.

Emission only

- Solution for the emission-only simplification

$$S(x',-\omega) = L_{ev}(x',-\omega)$$

$$L(x,\omega) = Tr(x_0 \to x)L(x_0,-\omega) + \int_0^d Tr(x' \to x)L_{ev}(x',-\omega)ds$$

- Monte Carlo estimator

$$\frac{1}{N}\sum_{i=1}^{N}\frac{Tr(x_i \to x)L_{ev}(x_i,\omega)}{p(x_i)} = \frac{t_1 - t_0}{N}\sum_{i=1}^{N} Tr(x_i \to x)L_{ev}(x_i,\omega)$$

Emission only

- Use multiplicativity of Tr

$$Tr(x_i \to x) = Tr(x_i \to x_{i-1}) \cdot Tr(x_{i-1} \to x)$$

- Break up integral and compute it incrementally by ray marching
- Tr can get small in a long ray
  - Early ray termination
  - Either use Russian Roulette or deterministically

EmissionIntegrator

```
class EmissionIntegrator : public VolumeIntegrator {
public:
  EmissionIntegrator(float ss) { stepSize = ss; }
  void RequestSamples(Sample *sample, const Scene
  *scene);
  Spectrum Transmittance(const Scene *, const Ray
  &ray, const Sample *sample, float *alpha) const;
  Spectrum Li(const Scene *, const RayDifferential
  &ray, const Sample *sample, float *alpha) const;
private:
  float stepSize;
  int tauSampleOffset, scatterSampleOffset;
};
```

single 1D sample for each

EmissionIntegrator::Transmittance

```
if (!scene->volumeRegion) return Spectrum(1);
float step =
  sample ? stepSize : 4.f * stepSize;
float offset =
  sample ? sample->oneD[tauSampleOffset][0] :
  RandomFloat();
Spectrum tau =
  scene->volumeRegion->Tau(ray,step,offset);
return Exp(-tau);
```

use larger steps for shadow and indirect rays for efficiency

$$\tau_\omega(s) = \int_0^s \sigma_a(x + s'\omega, \omega)ds' \qquad T_\omega(s) = e^{-\tau_\omega(s)}$$

EmissionIntegrator::Li

```
VolumeRegion *vr = scene->volumeRegion;
float t0, t1;
if (!vr || !vr->IntersectP(ray, &t0, &t1)) return 0;
// Do emission-only volume integration in vr
Spectrum Lv(0.);
// Prepare for volume integration stepping
int N = Ceil2Int((t1-t0) / stepSize);
float step = (t1 - t0) / N;
Spectrum Tr(1.f);
Point p = ray(t0), pPrev;
Vector w = -ray.d;
if (sample)
  t0 += sample->oneD[scatterSampleOffset][0]*step;
else
  t0 += RandomFloat() * step;
```

EmissionIntegrator::Li

```
for (int i = 0; i < N; ++i, t0 += step) {
  // Advance to sample at t0 and update T
  pPrev = p;
  p = ray(t0);
  Spectrum stepTau = vr->Tau(Ray(pPrev,p-pPrev,0,1),
            .5f * stepSize, RandomFloat());
  Tr *= Exp(-stepTau);
  // Possibly terminate if transmittance is small
  if (Tr.y() < 1e-3) {
    const float continueProb = .5f;
    if (RandomFloat() > continueProb) break;
    Tr /= continueProb;
  }
  // Compute emission-only source term at _p_
  Lv += Tr * vr->Lve(p, w);
}
return Lv * step;
```

$$Tr(x_i \to x) = Tr(x_i \to x_{i-1}) \cdot Tr(x_{i-1} \to x)$$

$$\frac{t_1 - t_0}{N} \sum_{i=1}^{N} Tr(x_i \to x) L_{ev}(x_i, \omega)$$

Emission only

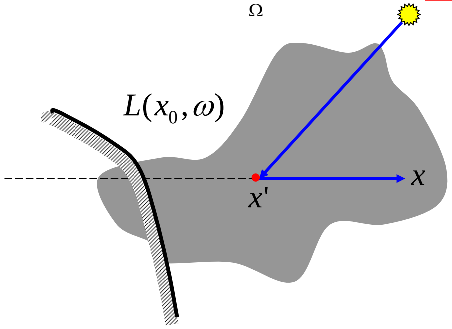
exponential density

Single scattering

- Consider incidence radiance due to direct illumination

$$L(x,\omega) = Tr(x_0 \to x)L(x_0,\omega) + \int_0^d Tr(x' \to x)S(x',\omega)ds$$

$$S(x,\omega) = L_{ve}(x,\omega) + \sigma_s(x,\omega)\int_\Omega p(x,\omega' \to \omega)\boxed{L_d(x,\omega')}d\omega'$$



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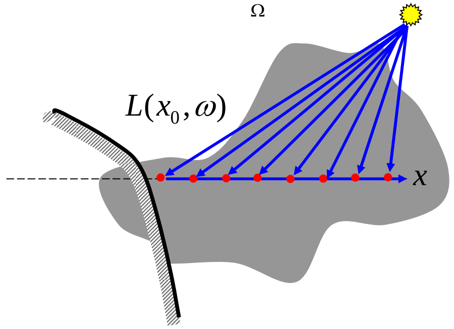

Single scattering

- L_d may be attenuated by participating media
- At each point of the integral, we could use multiple importance sampling to get

$$\sigma_s(x,\omega)\int_\Omega p(x,\omega' \to \omega)L_d(x,\omega')d\omega'$$

But, in practice, we can just pick up light source randomly.

Single scattering