Radiometry

- Radiometry: study of the propagation of electromagnetic radiation in an environment
- Four key quantities: flux, intensity, irradiance and radiance
- These radiometric quantities are described by their spectral power distribution (SPD)
- Human visible light ranges from 370nm to 730nm
Color

- Need a compact, efficient and accurate way to represent functions like these
- Find proper basis functions to map the infinite-dimensional space of all possible SPD functions to a low-dimensional space of coefficients
- For example, B(\lambda)=1 is a trivial but bad approximation

Human visual system

- Tristimulus theory: all visible SPDs S can be accurately represented for human observers with three values, x_\lambda, y_\lambda and z_\lambda.
- The basis are the spectral matching curves, X(\lambda), Y(\lambda) and Z(\lambda) determined by CIE (國際照明委員會).

\[
x_\lambda = \int_{\lambda} S(\lambda) X(\lambda) d\lambda
\]
\[
y_\lambda = \int_{\lambda} S(\lambda) Y(\lambda) d\lambda
\]
\[
z_\lambda = \int_{\lambda} S(\lambda) Z(\lambda) d\lambda
\]

Spectrum

- In core/color.*
- Not a plug-in, to use inline for performance
- Spectrum stores a fixed number of samples at a fixed set of wavelengths. Better for smooth functions.

Why is this possible? Human vision system

```cpp
#define COLOR_SAMPLES 3

class COREDLL Spectrum {
public:
    <arithmetic operations>
private:
    float c[COLOR_SAMPLES];
    ...
};
```

XYZ basis

pbrt has discrete versions (sampled every 1nm) of these bases in core/color.cpp

- --- X
- ---- Y
- ⋯ ⋯ ⋯ Z

360 400 450 500 550 600 650 700 830
Color matching experiment

- To avoid negative parameters
Human Photoreceptors

XYZ color

- Good for representing visible SPD to human observer, but not good for spectral computation.
- A product of two SPD's XYZ values is likely different from the XYZ values of the SPD which is the product of the two original SPDs.
- Hence, we often have to convert our samples (RGB) into XYZ

```c
void XYZ(float xyz[3]) const {
    for (int i = 0; i < COLOR_SAMPLES; ++i) {
        xyz[0] += XWeight[i] * c[i];
        xyz[1] += YWeight[i] * c[i];
        xyz[2] += ZWeight[i] * c[i];
    }
}
```

Conversion between XYZ and RGB

```c
float Spectrum::XWeight[COLOR_SAMPLES] = {
    0.412453f, 0.357580f, 0.180423f
};
float Spectrum::YWeight[COLOR_SAMPLES] = {
    0.212671f, 0.715160f, 0.072169f
};
float Spectrum::ZWeight[COLOR_SAMPLES] = {
    0.019334f, 0.119193f, 0.950227f
};
Spectrum FromXYZ(float x, float y, float z) {
    float c[3];
    c[0] =  3.240479f * x + -1.537150f * y + -0.498535f * z;
    c[1] = -0.969256f * x +  1.875992f * y +  0.041556f * z;
    c[2] =  0.055648f * x + -0.204043f * y +  1.057311f * z;
    return Spectrum(c);
}
```

Conversion between XYZ and RGB

vector sampled at several wavelengths such as (R,G,B) device dependent (R,G,B)

\[
\begin{pmatrix}
    x \lambda, y \lambda, z \lambda \\
\end{pmatrix} =
\int_S (x \lambda) X(\lambda) d\lambda
\int_S (y \lambda) Y(\lambda) d\lambda
\int_S (z \lambda) Z(\lambda) d\lambda
\]

\[
\begin{pmatrix}
    x_2 \\
    y_2 \\
    z_2 \\
\end{pmatrix} =
\begin{pmatrix}
    3.240479 & 0.357580 & 0.180423 \\
    0.212671 & 0.715160 & 0.072169 \\
    0.019334 & 0.119193 & 0.950227 \\
\end{pmatrix}
\begin{pmatrix}
    x_1, y_1, z_1 \\
\end{pmatrix}
\]
Basic radiometry

- pbrt is based on radiative transfer: study of the transfer of radiant energy based on radiometric principles and operates at the geometric optics level (light interacts with objects much larger than the light’s wavelength)
- It is based on the particle model. Hence, diffraction and interference can’t be easily accounted for.

Basic assumptions about light behavior

- **Linearity**: the combined effect of two inputs is equal to the sum of effects
- **Energy conservation**: scattering event can’t produce more energy than they started with
- **Steady state**: light is assumed to have reached equilibrium, so its radiance distribution isn’t changing over time.
- **No polarization**: we only care the frequency of light but not other properties (such as phases)
- **No fluorescence or phosphorescence**: behavior of light at a wavelength or time doesn’t affect the behavior of light at other wavelengths or time

Fluorescent materials

Basic quantities

<table>
<thead>
<tr>
<th>Flux: power, (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Irradiance: flux density per area, (W/m²)</td>
</tr>
<tr>
<td>Intensity: flux density per solid angle</td>
</tr>
<tr>
<td>Radiance: flux density per solid angle per area</td>
</tr>
</tbody>
</table>
Flux ($\Phi$)
- Radiant flux, power
- Total amount of energy passing through a surface per unit of time ($J/s, W$)

Irradiance ($E$)
- Area density of flux (W/m$^2$) $E = \frac{d\Phi}{dA}$
- Inverse square law $E = \frac{\Phi}{4\pi r^2}$
- Lambert’s law $E = \frac{\Phi}{A}$
  $E = \frac{\Phi \cos \theta}{A}$

Angles and Solid Angles
- Angle $\theta = \frac{I}{r}$
  $\Rightarrow$ circle has $2\pi$ radians
- Solid angle $\Omega = \frac{A}{R^2}$

Intensity ($I$)
- Flux density per solid angle $I = \frac{d\Phi}{d\omega}$
- Intensity describes the directional distribution of light

The solid angle subtended by a surface is defined as the surface area of a unit sphere covered by the surface's projection onto the sphere.

$\Rightarrow$ sphere has $4\pi$ steradians
Radiance (L)

- Flux density per unit area per solid angle
  \[ L = \frac{d\Phi}{d\omega dA} \]
- Most frequently used, remains constant along ray.
- All other quantities can be derived from radiance

\[ \Phi = \int d\omega L \]

Calculate irradiance from radiance

\[ E(x) = \frac{d\Phi}{dA} = \int_{\Omega} L(x, \omega) \cos \theta d\omega \]

Irradiance Environment Maps

Radiance Environment Map → Irradiance Environment Map

Differential solid angles

Goal: find out the relationship between \( d\omega \) and \( d\theta, d\phi \)

Why? In the integral,
\[
\int f(\omega)d\omega
\]
\( d\omega \) is uniformly divided.
To convert the integral to
\[
\int \int f(\theta, \phi)d\theta d\phi
\]
We have to find the relationship between \( d\omega \) and uniformly divided \( d\theta \) and \( d\phi \).
Differential solid angles

Goal: find out the relationship between $d\omega$ and $d\theta$, $d\phi$

By definition, we know that
\[
d\omega = \frac{dA}{r^2}
\]

\[dA = (r d\theta)(r \sin \theta d\phi) = r^2 \sin \theta d\theta d\phi\]

\[d\omega = \frac{dA}{r^2} = \sin \theta d\theta d\phi = -d \cos \theta d\phi\]

We can prove that $\Omega = \int d\omega = 4\pi$

Isotropic point source

If the total flux of the light source is $\Phi$, what is the intensity?
Isotropic point source

If the total flux of the light source is $\Phi$, what is the intensity?

$$\Phi = \int_S I \, d\omega$$

$$= 4\pi I$$

$$I = \frac{\Phi}{4\pi}$$

Warn’s spotlight

If the total flux is $\Phi$, what is the intensity?

$$I(\omega) \propto \cos^S \theta$$

Warn’s spotlight

If the total flux is $\Phi$, what is the intensity?

$$I(\omega) = \begin{cases} c \cos^S \theta & \theta < \frac{\pi}{2} \\ 0 & \text{otherwise} \end{cases}$$

$$\Phi = c \int_0^{2\pi} \int_0^1 \cos^S \theta \, d\theta \, d\phi = 2\pi c \int_0^1 \cos^S \theta \, d\theta$$

$$= 2\pi c \frac{y^{S+1}}{S+1} \bigg|_{y=1}^{y=0} = 2\pi c \frac{S+1}{S+1} \Rightarrow c = \frac{S+1}{2\pi} \Phi$$

Irradiance: isotropic point source

What is the irradiance for this point?
Irradiance: isotropic point source

\[ r = \frac{h}{\cos \theta} \]

\[ I = \frac{d\Phi}{d\omega} = \frac{\Phi}{4\pi} \]

\[ E = \frac{d\Phi}{dA} = \frac{Id\omega}{dA} = \frac{\Phi}{4\pi} \frac{d\omega}{dA} = \frac{\Phi}{4\pi} \frac{\cos \theta}{r^2} = \frac{\Phi \cos^3 \theta}{4\pi h^2} \]