

## Real-time environment map lighting

Digital Image Synthesis

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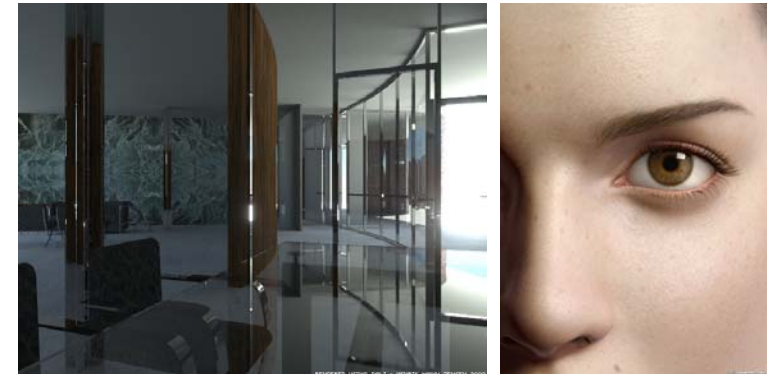
12/28/2006

*with slides by Ravi Ramamoorthi and Robin Green*

## Realistic rendering



- So far, we have talked photorealistic rendering including complex materials, complex geometry and complex lighting. They are slow.



## Real-time rendering



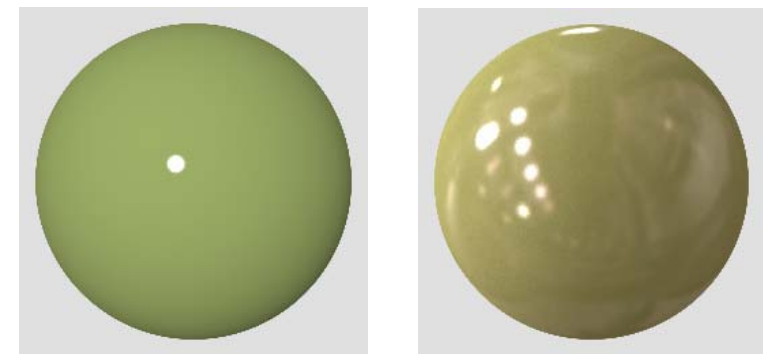
- Goal is to achieve interactive rendering with reasonable quality. It's important in many applications such as games, visualization, computer-aided design, ...



## Natural illumination



People perceive materials more easily under natural illumination than simplified illumination.

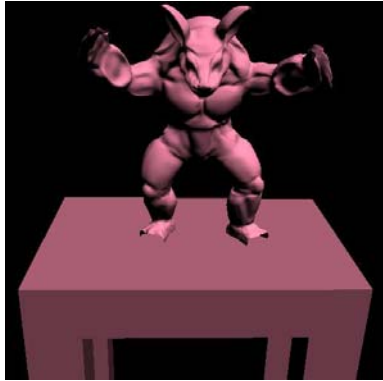


Images courtesy Ron Dror and Ted Adelson

## Natural illumination



Classically, rendering with natural illumination is very expensive compared to using simplified illumination

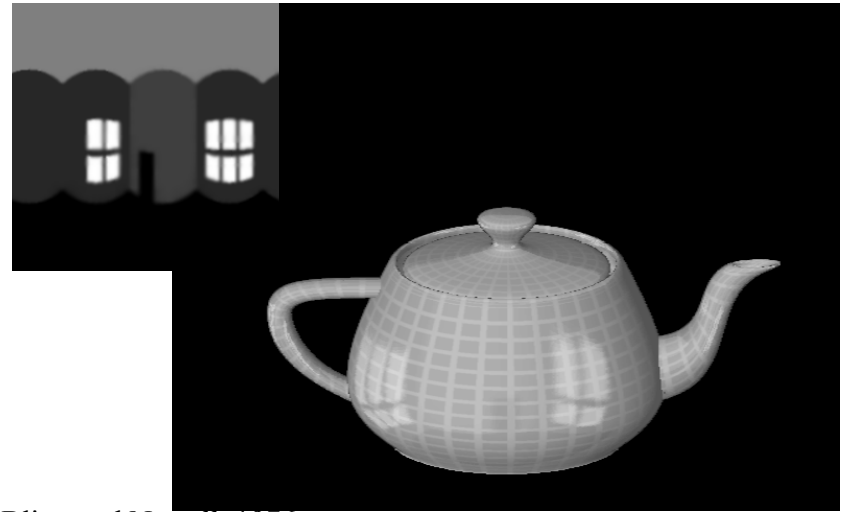


directional source



natural illumination

## Reflection maps



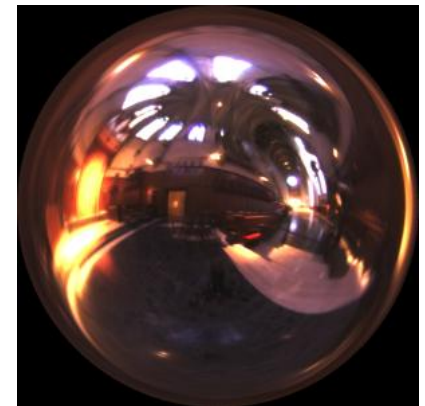
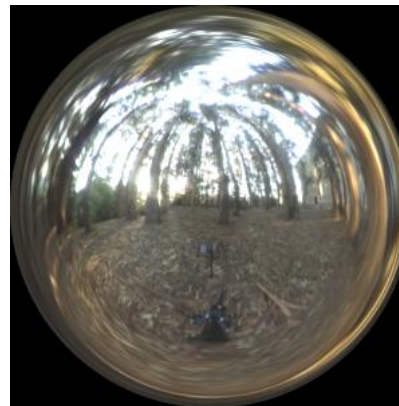
Blinn and Newell, 1976

## Environment maps



Miller and Hoffman, 1984

## HDR lighting



## Examples



## Examples



## Examples



## Complex illumination



$$L_o(\mathbf{p}, \omega_o) = L_e(\mathbf{p}, \omega_o) + \int_{s^2} f(\mathbf{p}, \omega_o, \omega_i) L_i(\mathbf{p}, \omega_i) |\cos \theta_i| d\omega_i$$
$$B(\mathbf{p}, \omega_o) = \int_{s^2} f(\mathbf{p}, \omega_o, \omega_i) L_d(\mathbf{p}, \omega_i) |\cos \theta_i| d\omega_i$$

## Function approximation



- $G(x)$  ... function to approximate
- $B_1(x), B_2(x), \dots, B_n(x)$  ... basis functions
- We want

$$G(x) = \sum_{i=1}^n c_i B_i(x)$$

- Storing a finite number of coefficients  $c_i$  gives an approximation of  $G(x)$

## Function approximation



- How to find coefficients  $c_i$ ?
  - Minimize an error measure
- What error measure?
  - $L_2$  error

$$E_{L_2} = \int_I [G(x) - \sum_i c_i B_i(x)]^2$$

## Function approximation



- Orthonormal basis

$$\langle F | G \rangle = \int_I F(x)G(x)dx \quad \langle B_i | B_j \rangle = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

- If basis is orthonormal then

$$\begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} = \begin{bmatrix} \langle G | B_1 \rangle \\ \langle G | B_2 \rangle \\ \vdots \\ \langle G | B_n \rangle \end{bmatrix}$$

- $\rightarrow$  we want our bases to be orthonormal

## Orthogonal basis functions



- Orthogonal Basis Functions
  - These are families of functions with special properties

$$\int B_i(x)B_j(x)dx = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

- Intuitively, it's like functions don't overlap each other's footprint
  - A bit like the way a Fourier transform breaks a functions into component sine waves

## Integral of product



$$I = \int F(x)G(x) dx$$

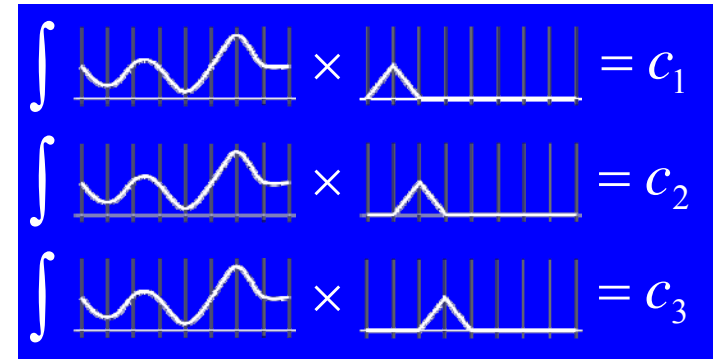
$$F(x) = \sum_i f_i B_i(x) \quad G(x) = \sum_j g_j B_j(x)$$

$$\begin{aligned} \int F(x)G(x) dx &= \int \left( \sum_i f_i B_i(x) \sum_j g_j B_j(x) \right) dx \\ &= \int \sum_i \sum_j f_i g_j B_i(x) B_j(x) dx = \int \sum_i f_i g_i dx = \hat{F} \cdot \hat{G} \end{aligned}$$

## Function approximation



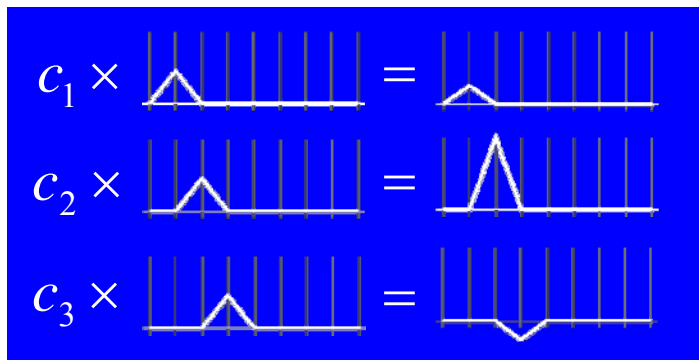
- Basis Functions are pieces of signal that can be used to produce approximations to a function



## Function approximation



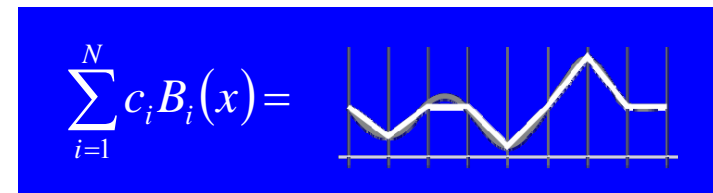
- We can then use these coefficients to reconstruct an approximation to the original signal



## Function approximation



- We can then use these coefficients to reconstruct an approximation to the original signal



## Basis functions



- Transform data to a space in which we can capture the essence of the data better
- Here, we use spherical harmonics, similar to Fourier transform in spherical domain

## Harr wavelets



- Scaling functions ( $\mathbf{V}^j$ )

$$\phi(x) := \begin{cases} 1 & \text{for } 0 \leq x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

$$\phi_i^j(x) := \phi(2^j x - i), \quad i = 0, \dots, 2^j - 1$$

- Wavelet functions ( $\mathbf{W}^j$ )

$$\psi(x) := \begin{cases} 1 & \text{for } 0 \leq x < 1/2 \\ -1 & \text{for } 1/2 \leq x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

$$\psi_i^j(x) := \psi(2^j x - i), \quad i = 0, \dots, 2^j - 1$$

- The set of scaling functions and wavelet functions forms an orthogonal basis

## Harr wavelets



Figure 2 The box basis for  $V^2$ .

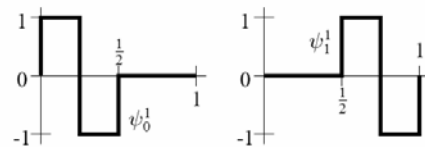


Figure 3 The Haar wavelets for  $W^1$ .

## Example for wavelet transform



- Delta functions,  $f=(9,7,3,5)$  in  $V^2$

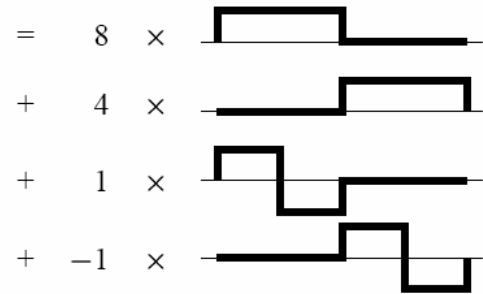
$$\begin{aligned} \mathcal{I}(x) &= 9 \times \text{[Step function: 1 on } [0, 1/2], 0 \text{ elsewhere]} \\ &+ 7 \times \text{[Step function: 0 on } [0, 1/2], 1 \text{ on } [1/2, 3/4], 0 \text{ elsewhere]} \\ &+ 3 \times \text{[Step function: 0 on } [0, 1/4], 1 \text{ on } [1/4, 3/4], 0 \text{ elsewhere]} \\ &+ 5 \times \text{[Step function: 0 on } [0, 3/4], 1 \text{ on } [3/4, 1], 0 \text{ elsewhere]} \end{aligned}$$

## Wavelet transform



### • $V^1, W^1$

$$\mathcal{I}(x) = c_0^1 \phi_0^1(x) + c_1^1 \phi_1^1(x) + d_0^1 \psi_0^1(x) + d_1^1 \psi_1^1(x)$$

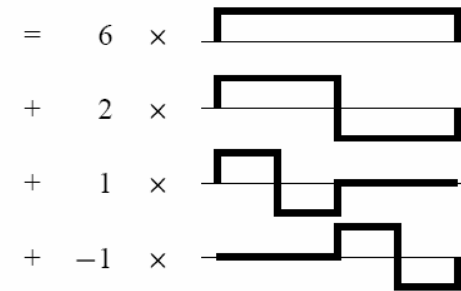


## Example for wavelet transform



### • $V^0, W^0, W^1$

$$\mathcal{I}(x) = c_0^0 \phi_0^0(x) + d_0^0 \psi_0^0(x) + d_0^1 \psi_0^1(x) + d_1^1 \psi_1^1(x)$$

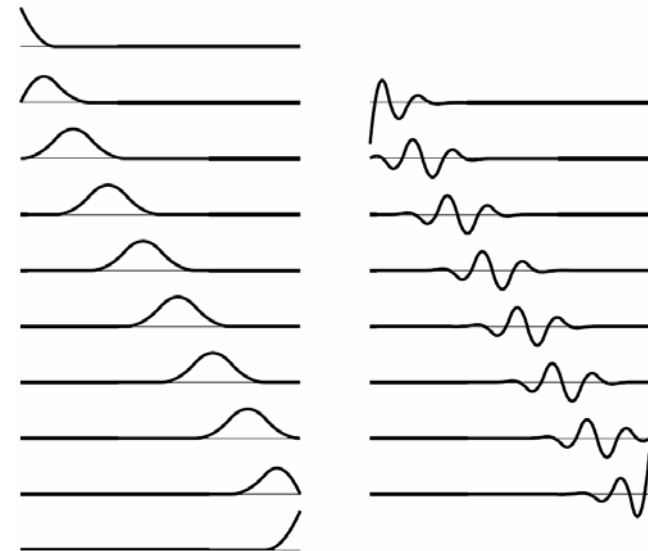


## Example for wavelet transform

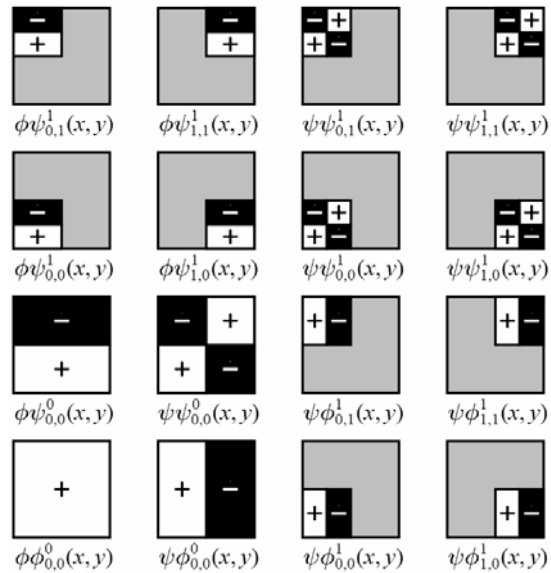


Resolution	Averages	Detail coefficients
4	[ 9 7 3 5 ]	
2	[ 8 4 ]	[ 1 -1 ]
1	[ 6 ]	[ 2 ]

## Quadratic B-spline scaling and wavelets



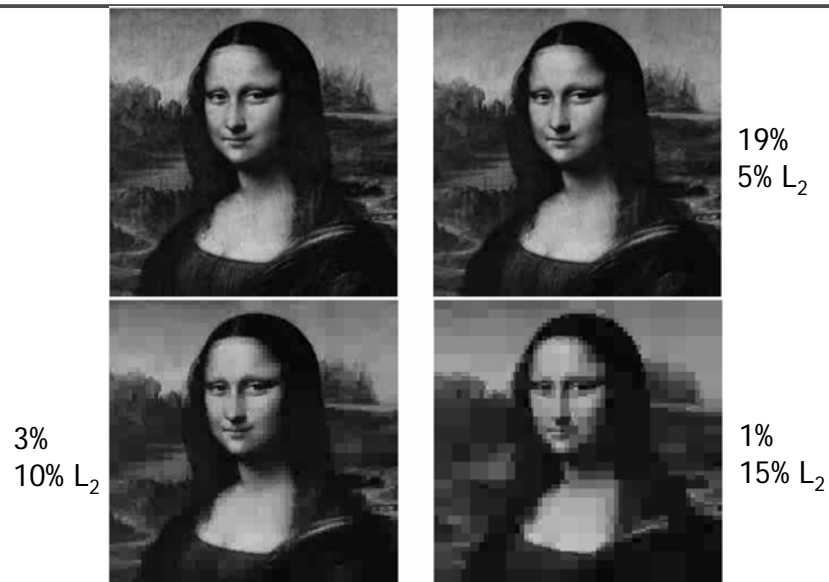
## 2D Harr wavelets



## Example for 2D Harr wavelets



## Applications



## Real spherical harmonics

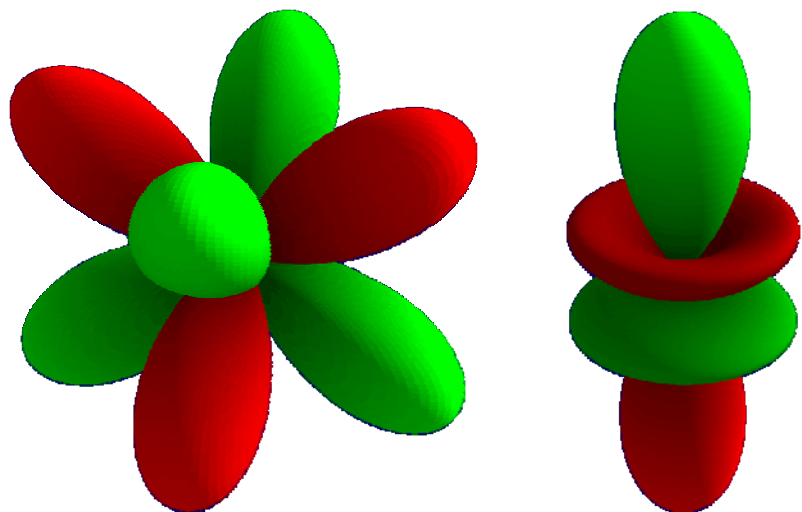


- A system of signed, orthogonal functions over the sphere
- Represented in spherical coordinates by the function

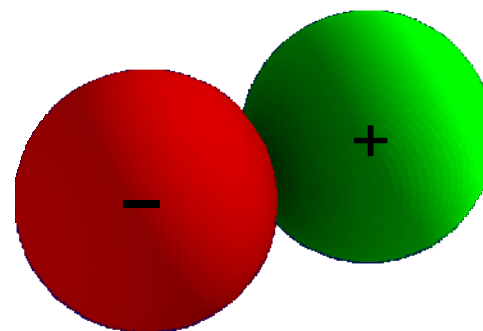
$$y_l^m(\theta, \varphi) = \begin{cases} \sqrt{2} K_l^m \cos(m\varphi) P_l^m(\cos\theta), & m > 0 \\ \sqrt{2} K_l^m \sin(-m\varphi) P_l^{-m}(\cos\theta), & m < 0 \\ K_l^0 P_l^0(\cos\theta), & m = 0 \end{cases}$$

where  $l$  is the band and  $m$  is the index within the band

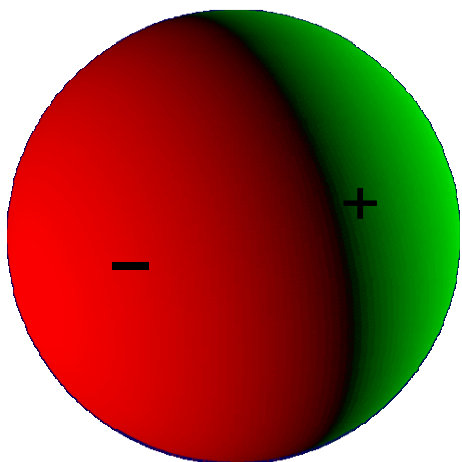
## Real spherical harmonics



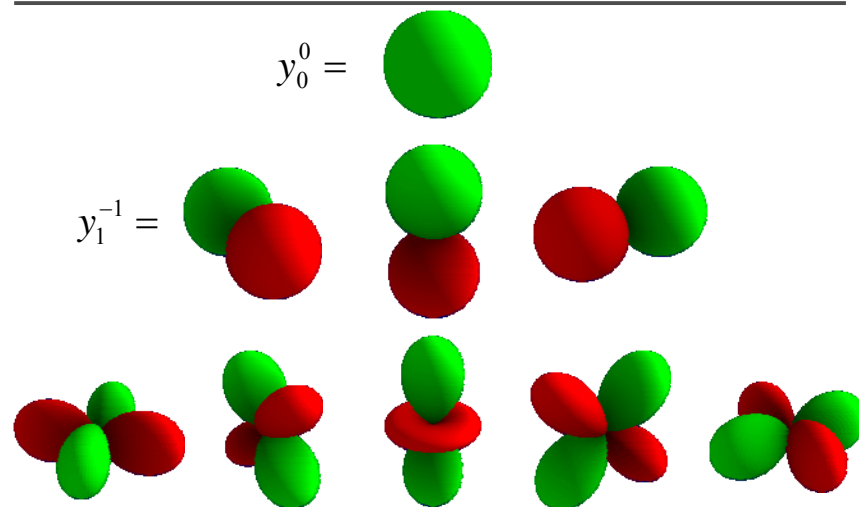
## Reading SH diagrams



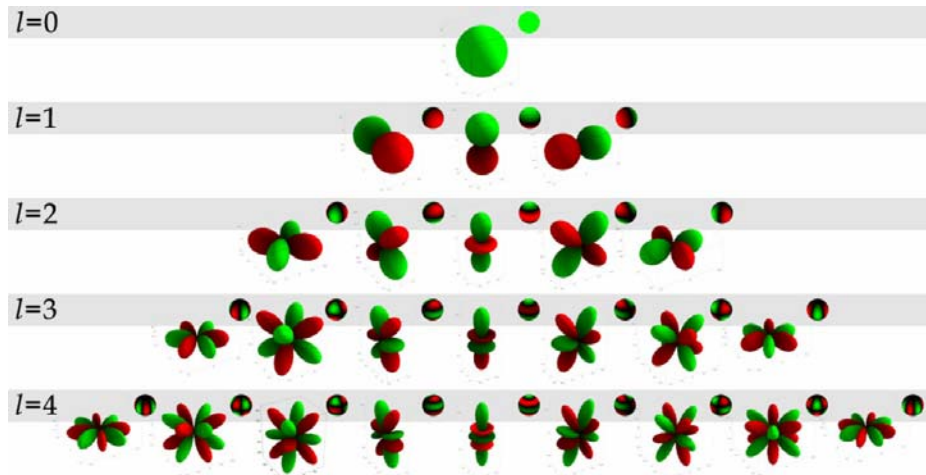
## Reading SH diagrams



## The SH functions



## The SH functions



## Spherical harmonics



$$(x, y, z) = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$$

$$Y_{00}(\theta, \phi) = 0.282095$$

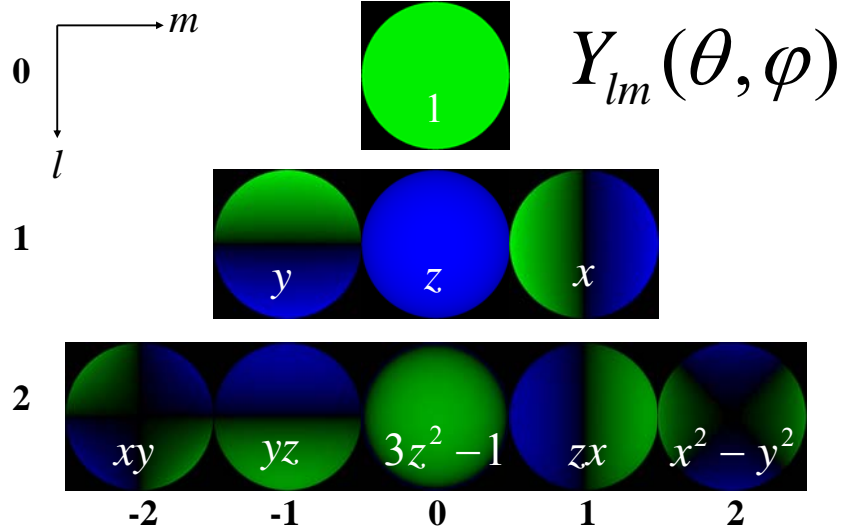
$$(Y_{11}; Y_{10}; Y_{1-1})(\theta, \phi) = 0.488603 (x; z; y)$$

$$(Y_{21}; Y_{2-1}; Y_{2-2})(\theta, \phi) = 1.092548 (xz; yz; xy)$$

$$Y_{20}(\theta, \phi) = 0.315392 (3z^2 - 1)$$

$$Y_{22}(\theta, \phi) = 0.546274 (x^2 - y^2)$$

## Spherical harmonics



## SH projection



- First we define a strict order for SH functions

$$i = l(l+1) + m$$

- Project a spherical function into a vector of SH coefficients

$$c_i = \int_S f(s) y_i(s) ds$$

## SH reconstruction

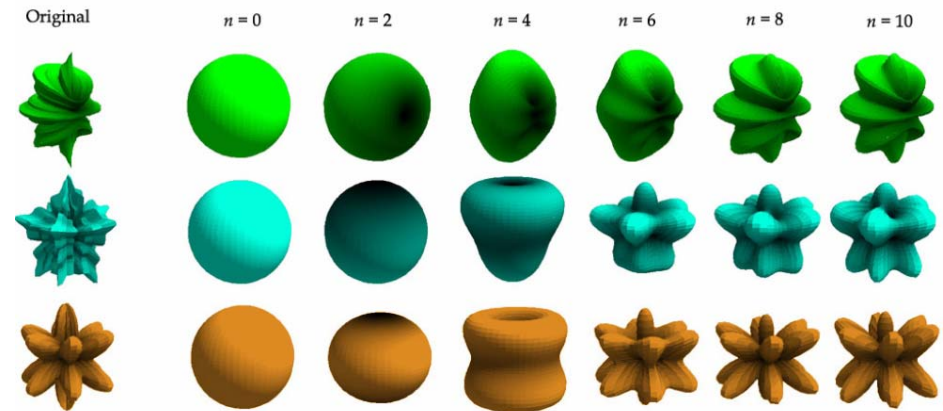


- To reconstruct the approximation to a function

$$\tilde{f}(s) = \sum_{i=0}^{N^2} c_i y_i(s)$$

- We truncate the infinite series of SH functions to give a low frequency approximation

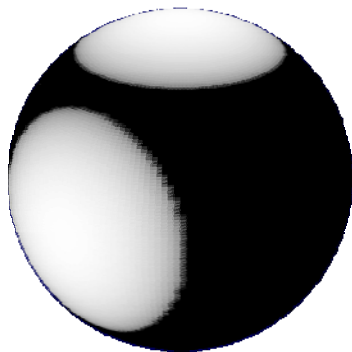
## Examples of reconstruction



## An example



- Take a function comprised of two area light sources
  - SH project them into 4 bands = 16 coefficients


$$\begin{bmatrix} 1.329, \\ -0.679, 0.930, 0.908, \\ -0.940, 0, 0.417, 0, 0.278, \\ -0.642, 0.001, 0.317, 0.837, \\ -0.425, 0, -0.238 \end{bmatrix}$$

## Low frequency light source



- We reconstruct the signal
  - Using only these coefficients to find a low frequency approximation to the original light source

