Real-time environment map lighting

Digital Image Synthesis

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with slides by Ravi Ramamoorthi and Robin Green

Realistic rendering

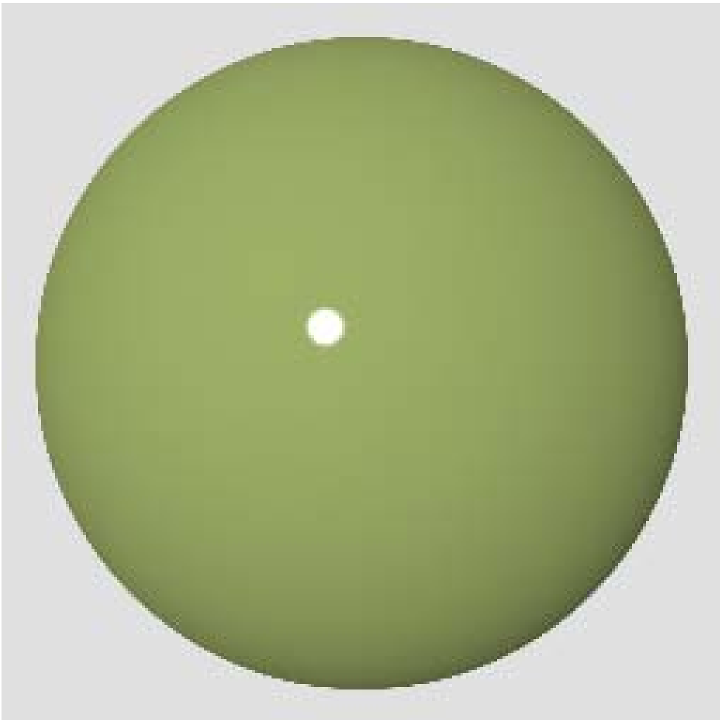
- So far, we have talked photorealistic rendering including complex materials, complex geometry and complex lighting. They are slow.

Real-time rendering

- Goal is to achieve interactive rendering with reasonable quality. It's important in many applications such as games, visualization, computer-aided design, …

Natural illumination

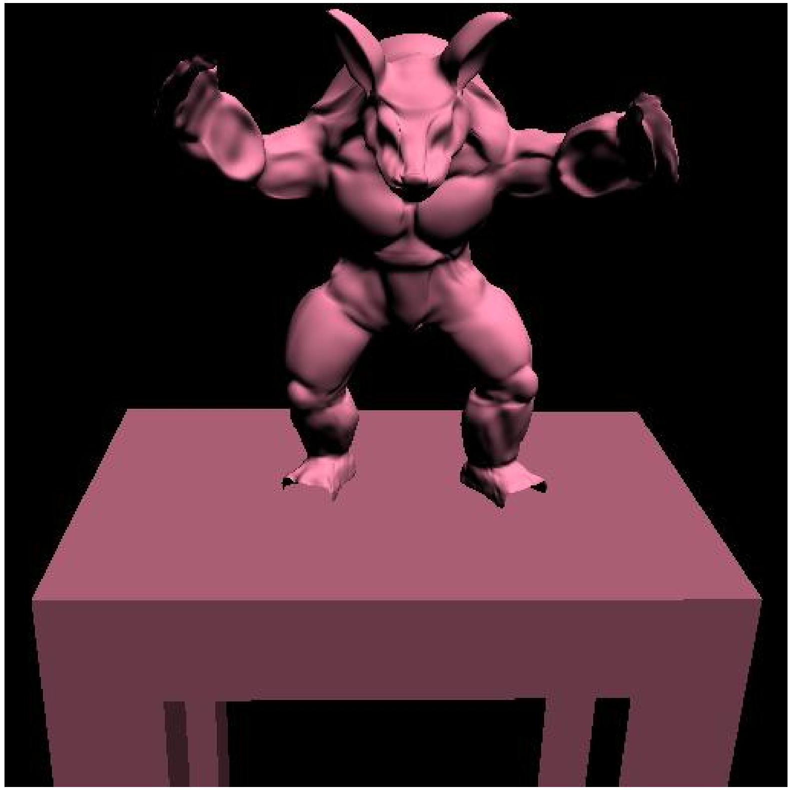
People perceive materials more easily under natural illumination than simplified illumination.



Images courtesy Ron Dror and Ted Adelson

Natural illumination

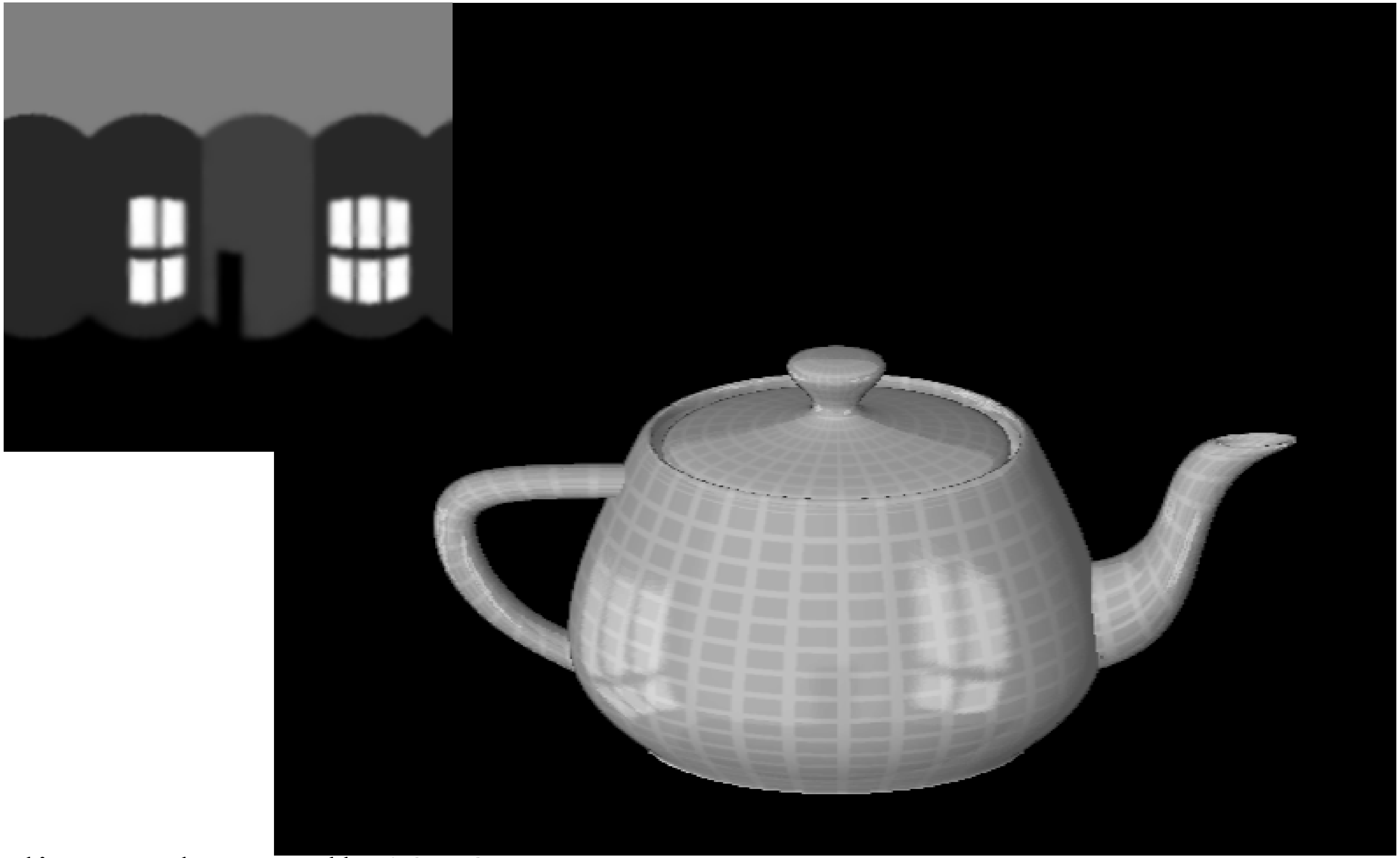
Classically, rendering with natural illumination is very expensive compared to using simplified illumination



directional source

natural illumination

Reflection maps

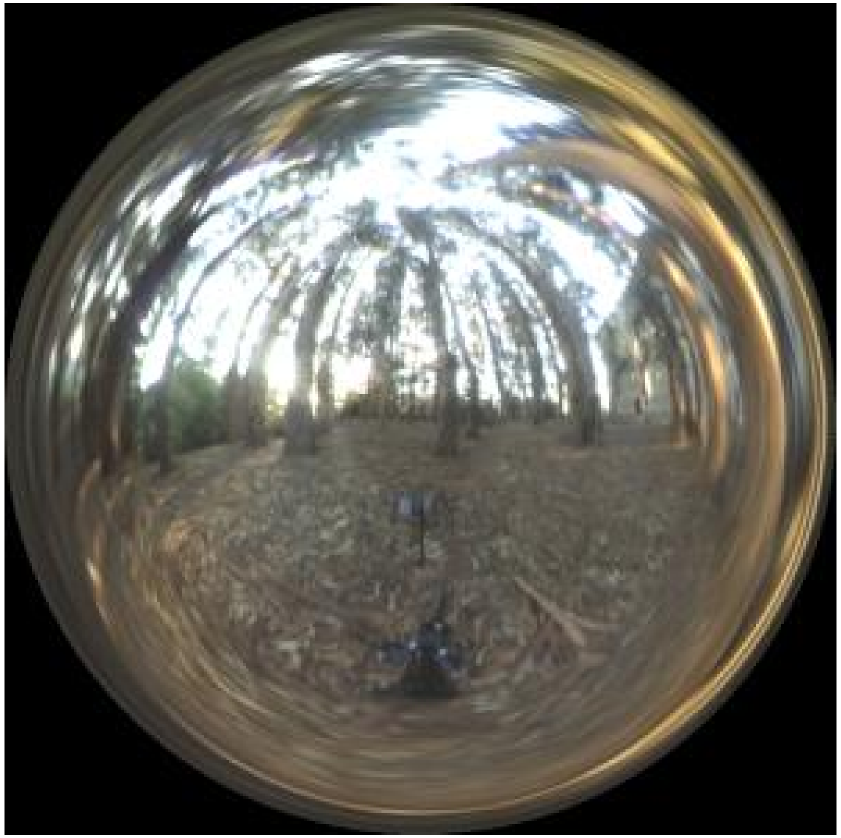


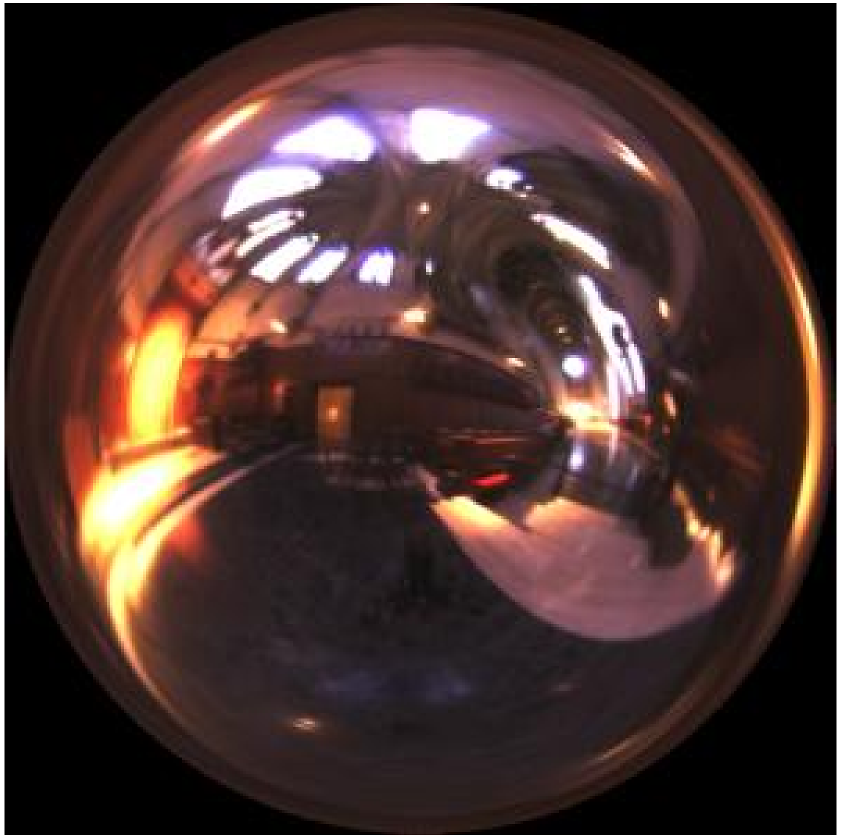
Blinn and Newell, 1976

Environment maps

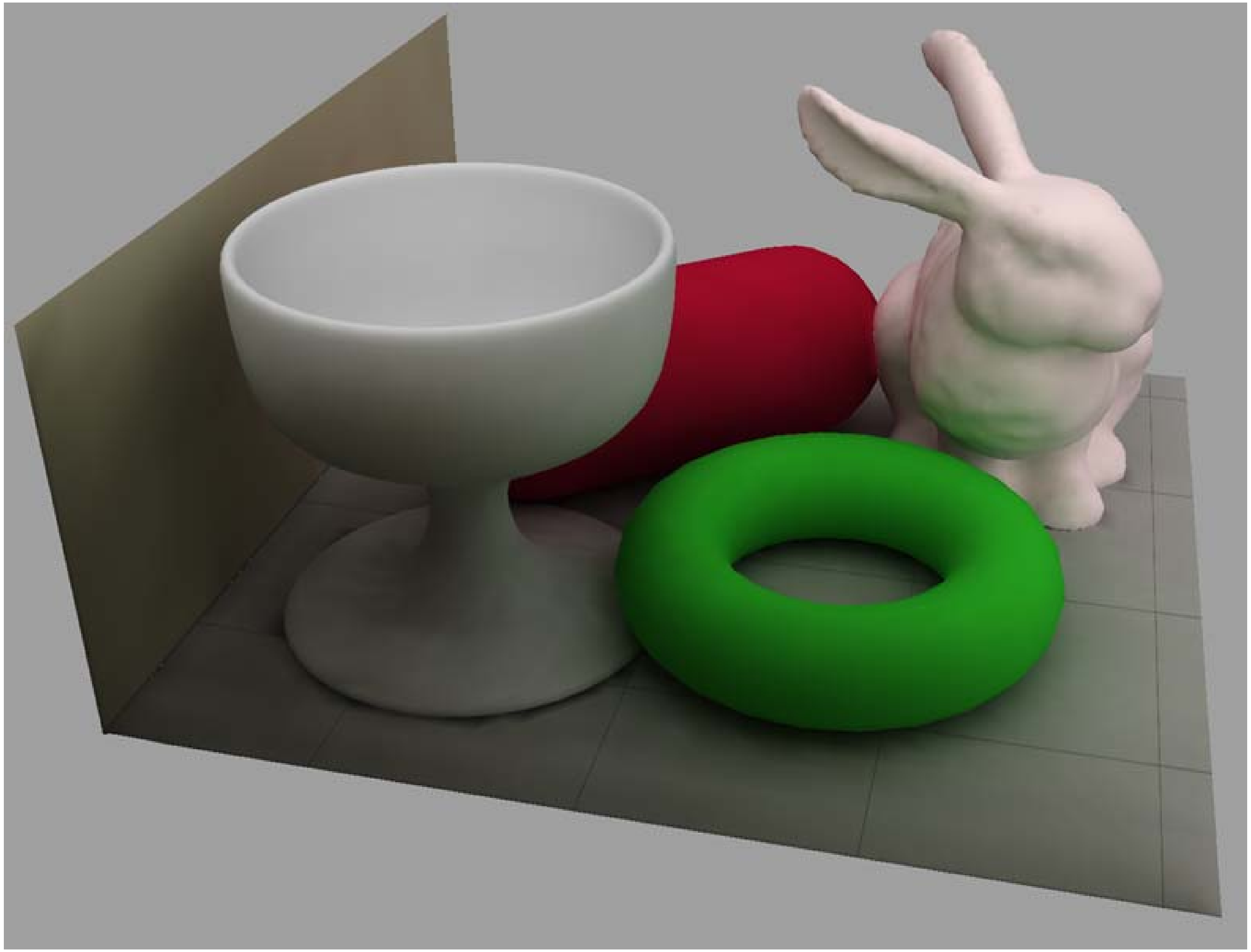
Miller and Hoffman, 1984

HDR lighting





Examples



Examples

Examples

Complex illumination

$$L_o(\mathrm{p},\omega_\mathrm{o}) = L_e(\mathrm{p},\omega_\mathrm{o}) + \int_{s^2} f(\mathrm{p},\omega_\mathrm{o},\omega_\mathrm{i}) L_i(\mathrm{p},\omega_\mathrm{i}) \left|\cos\theta_\mathrm{i}\right| d\omega_\mathrm{i}$$

$$B(\mathrm{p},\omega_\mathrm{o}) = \int_{s^2} f(\mathrm{p},\omega_\mathrm{o},\omega_\mathrm{i}) L_d(\mathrm{p},\omega_\mathrm{i}) \left|\cos\theta_\mathrm{i}\right| d\omega_\mathrm{i}$$

Function approximation

- G(x) … function to approximate
- $B_1(x)$, $B_2(x)$, ... $B_n(x)$ ... basis functions
- We want

$$G(x) = \sum_{i=1}^{n} c_i B_i(x)$$

- Storing a finite number of coefficients c_i gives an approximation of G(x)

Function approximation

- How to find coefficients c_i?
  - Minimize an error measure
- What error measure?
  - L_2 error

$$E_{L_2} = \int_I [G(x) - \sum_i c_i B_i(x)]^2$$

Function approximation

- Orthonormal basis

$$\langle F|G\rangle = \int_I F(x)G(x)dx \qquad \langle B_i|B_j\rangle = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

- If basis is orthonormal then

$$\begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} = \begin{bmatrix} \langle G|B_1\rangle \\ \langle G|B_2\rangle \\ \vdots \\ \langle G|B_n\rangle \end{bmatrix}$$

- ➔ we want our bases to be orthonormal

Orthogonal basis functions

- Orthogonal Basis Functions
  - These are families of functions with special properties

$$\int B_i(x) B_j(x)\, dx = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

  - Intuitively, it’s like functions don’t overlap each other’s footprint
    - A bit like the way a Fourier transform breaks a functions into component sine waves

Integral of product

$$I = \int F(x) G(x)\, dx$$

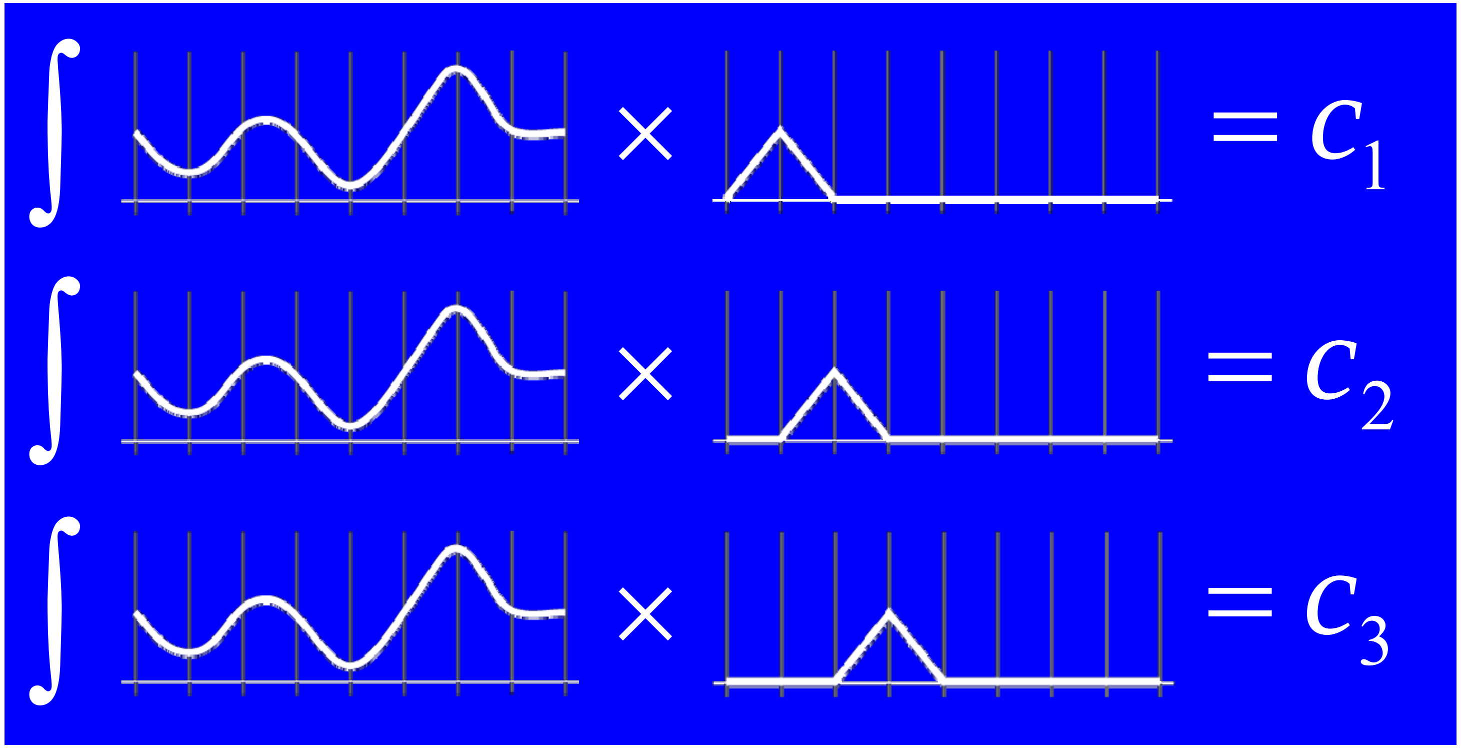
$$F(x) = \sum_i f_i B_i(x) \qquad G(x) = \sum_j g_j B_j(x)$$

$$\int F(x) G(x)\, dx = \int \left(\sum_i f_i B_i(x) \sum_j g_j B_j(x) \right) dx$$

$$= \int \sum_i \sum_j f_i g_j B_i(x) B_j(x) dx = \int \sum_i f_i g_i dx = \hat{F} \cdot \hat{G}$$

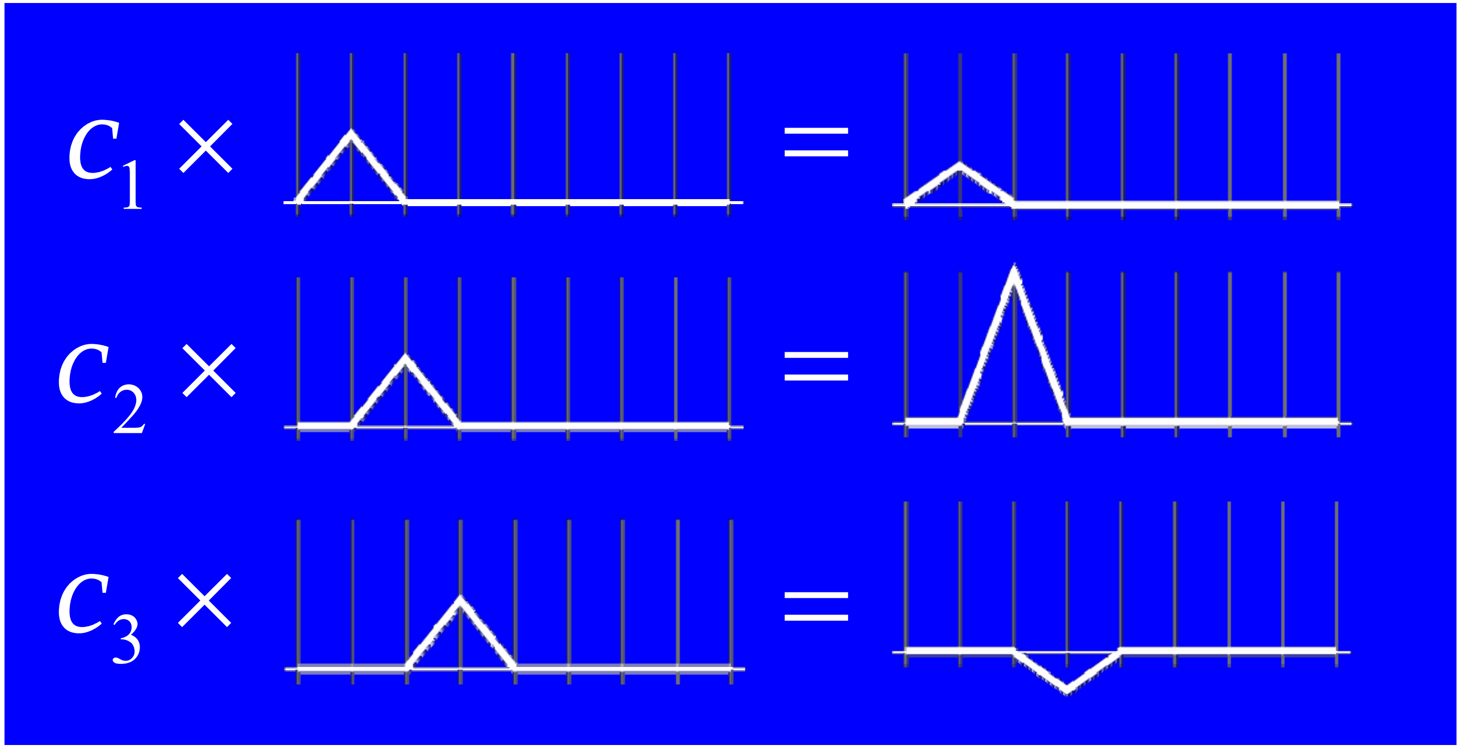
Function approximation

- Basis Functions are pieces of signal that can be used to produce approximations to a function



Function approximation

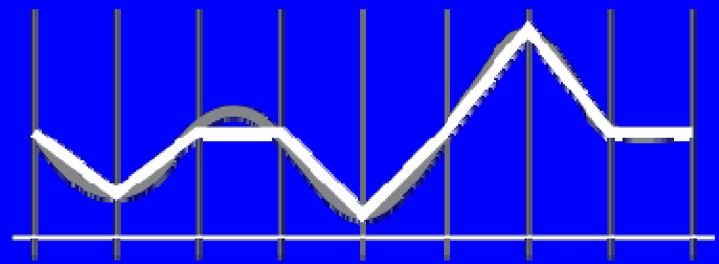
- We can then use these coefficients to reconstruct an approximation to the original signal



Function approximation

- We can then use these coefficients to reconstruct an approximation to the original signal

$$\sum_{i=1}^{N} c_i B_i(x) =$$



Basis functions

- Transform data to a space in which we can capture the essence of the data better
- Here, we use spherical harmonics, similar to Fourier transform in spherical domain

Harr wavelets

- Scaling functions ($\mathbf{V}^j$)

$$\phi(x) := \begin{cases} 1 & \text{for } 0 \le x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

$$\phi_i^j(x) := \phi(2^j x - i), \qquad i = 0, \ldots, 2^j - 1$$

- Wavelet functions ($\mathbf{W}^j$)

$$\psi(x) := \begin{cases} 1 & \text{for } 0 \le x < 1/2 \\ -1 & \text{for } 1/2 \le x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

$$\psi_i^j(x) := \psi(2^j x - i), \qquad i = 0, \ldots, 2^j - 1$$

- The set of scaling functions and wavelet functions forms an orthogonal basis

Harr wavelets

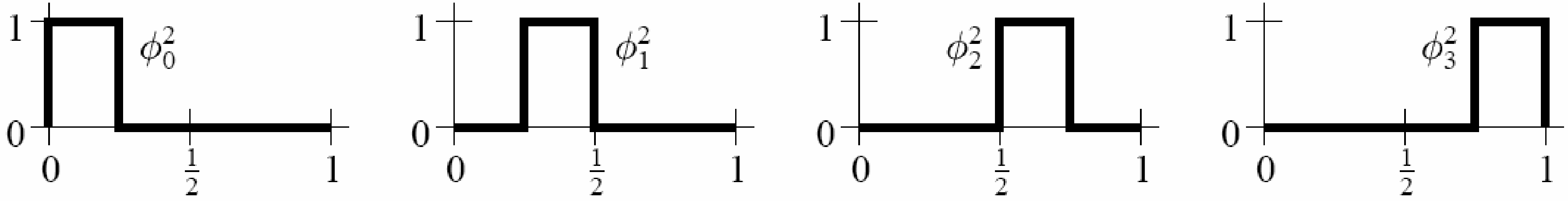


Figure 2 The box basis for V^2.

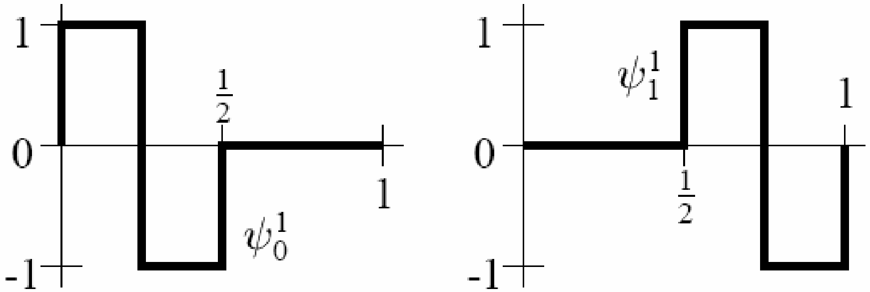
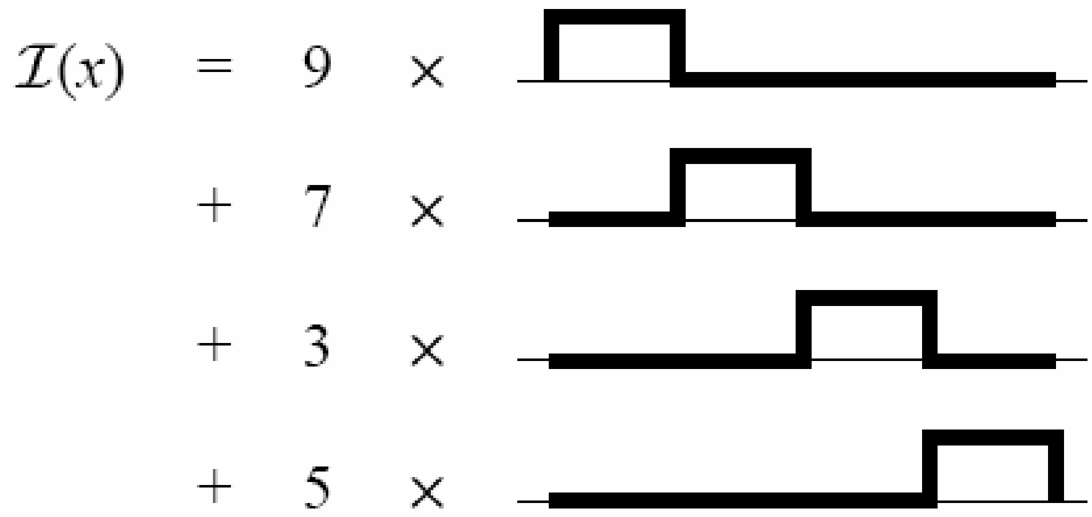


Figure 3 The Haar wavelets for W^1.

Example for wavelet transform

- Delta functions, f=(9,7,3,5) in $\mathbf{V}^2$

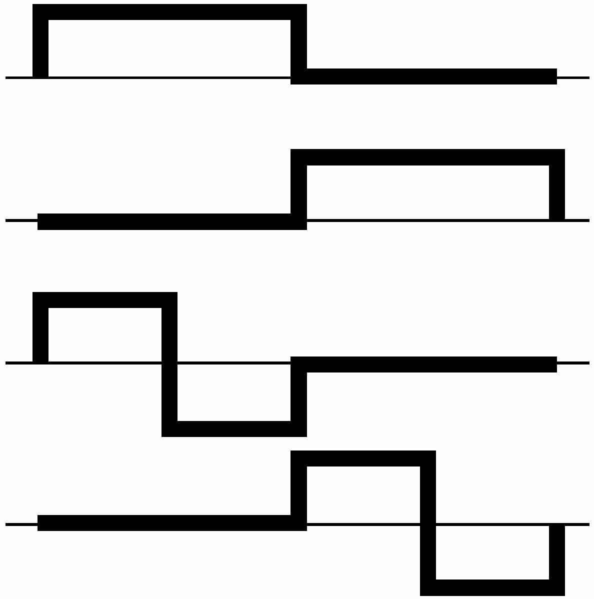


Wavelet transform

- $\mathbf{V^1, W^1}$

$$\mathcal{I}(x) = c_0^1\,\phi_0^1(x) + c_1^1\,\phi_1^1(x) + d_0^1\,\psi_0^1(x) + d_1^1\,\psi_1^1(x)$$

$$= 8 \times$$
$$+ 4 \times$$
$$+ 1 \times$$
$$+ -1 \times$$



Example for wavelet transform

- $\mathbf{V^0, W^0, W^1}$

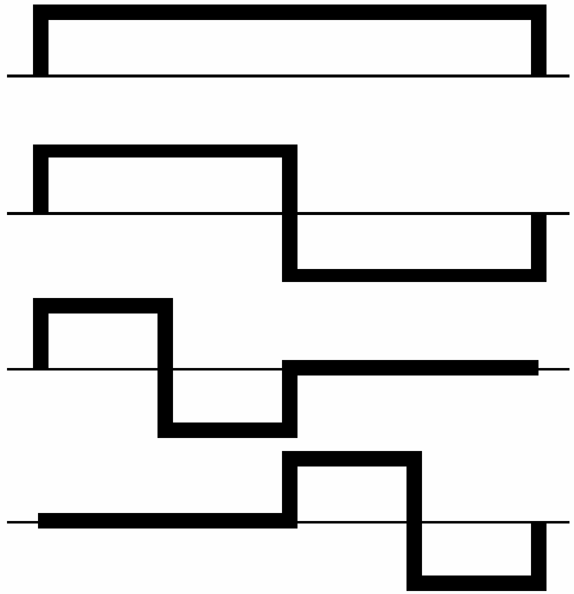
$$\mathcal{I}(x) \quad = \quad c_0^0\,\phi_0^0(x) + d_0^0\,\psi_0^0(x) + d_0^1\,\psi_0^1(x) + d_1^1\,\psi_1^1(x)$$

$$= \quad 6 \quad \times$$

$$+ \quad 2 \quad \times$$

$$+ \quad 1 \quad \times$$

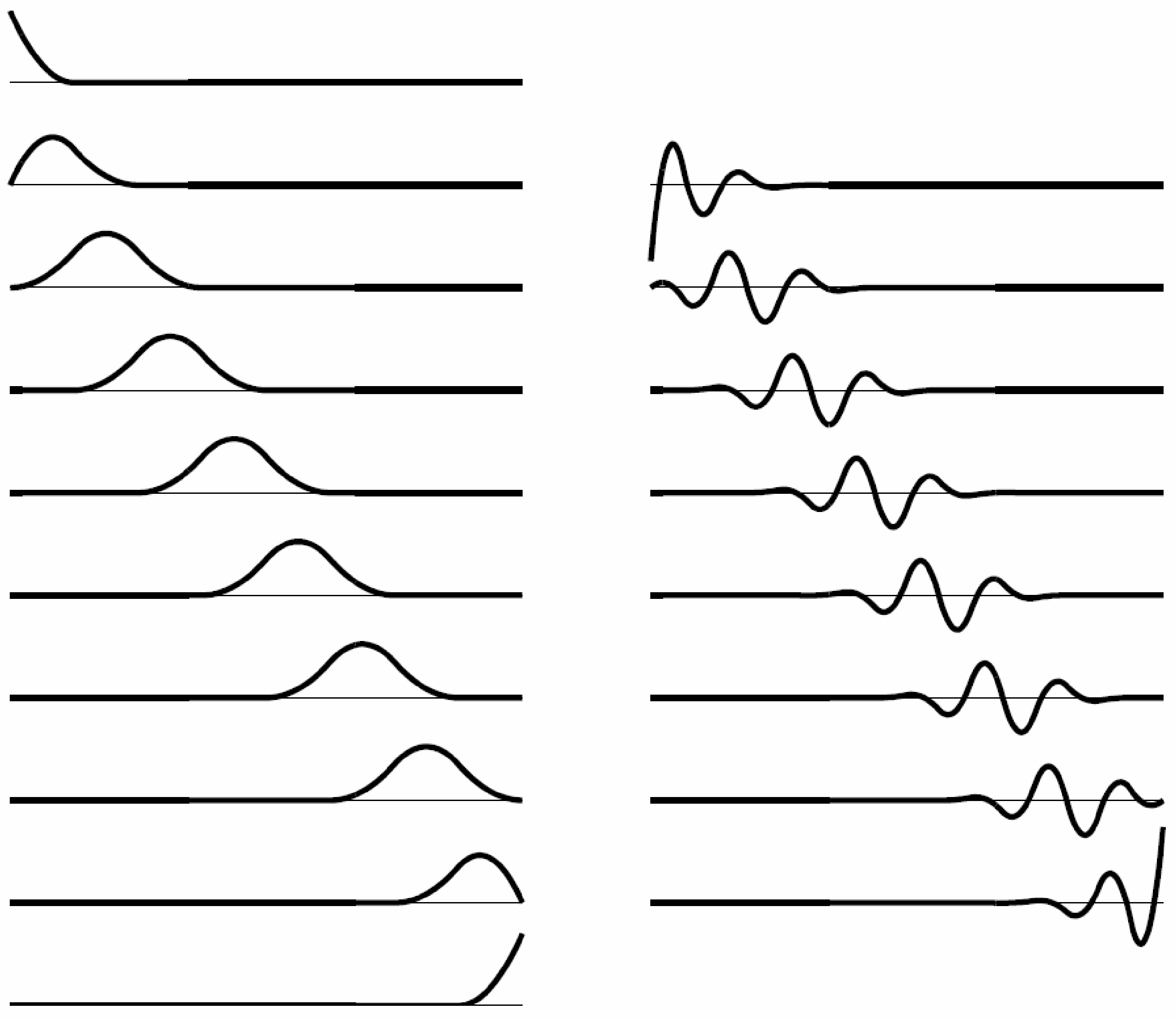
$$+ \quad -1 \quad \times$$



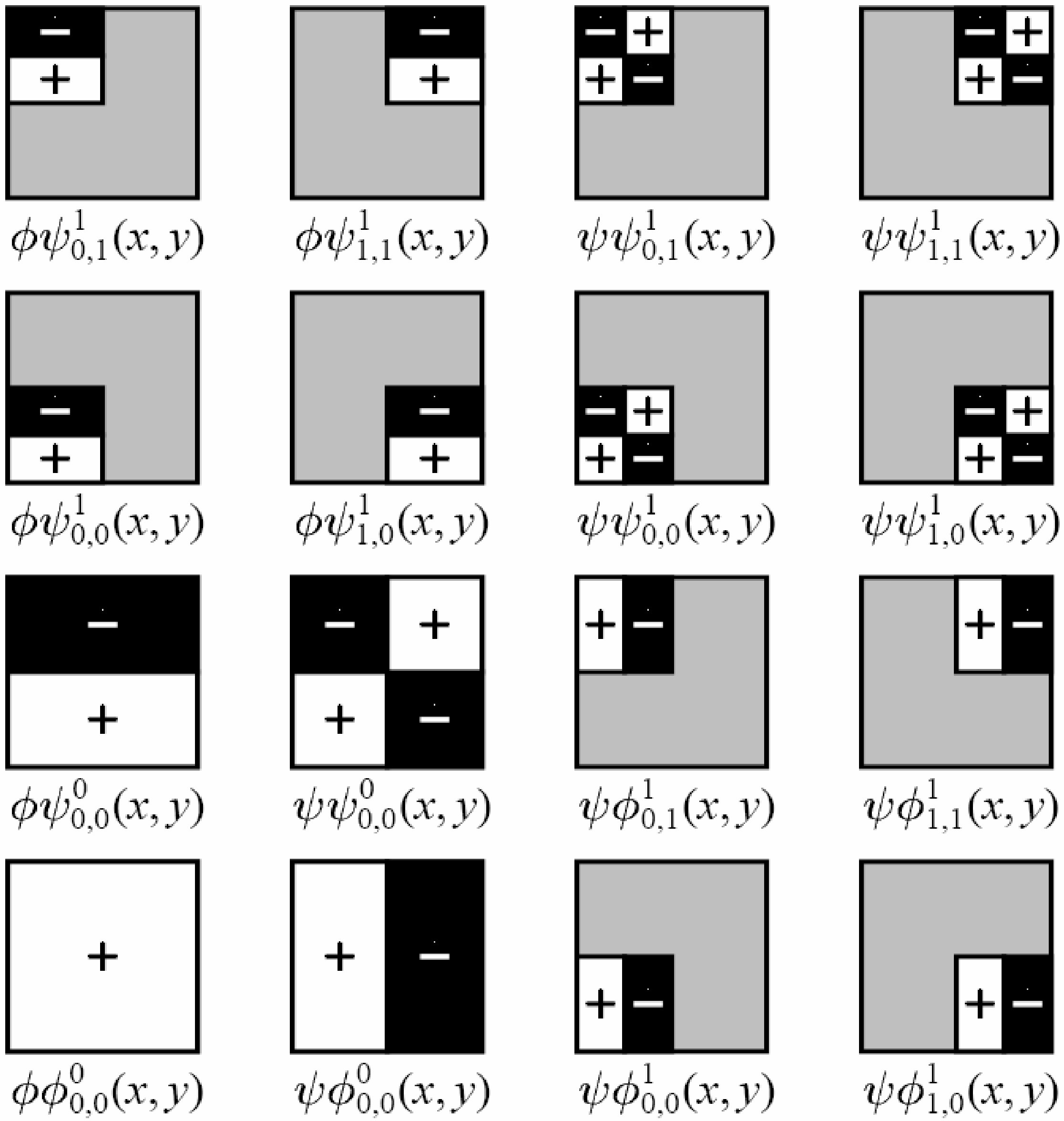
Example for wavelet transform

Resolution	Averages	Detail coefficients
4	[9 7 3 5]	
2	[8 4]	[1 −1]
1	[6]	[2]

Quadratic B-spline scaling and wavelets



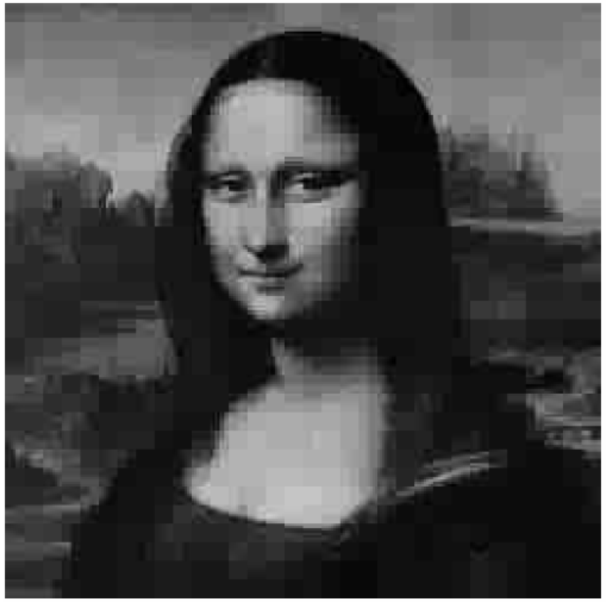
2D Harr wavelets



Example for 2D Harr wavelets

Applications

19%
5% L_2



3%
10% L_2

1%
15% L_2

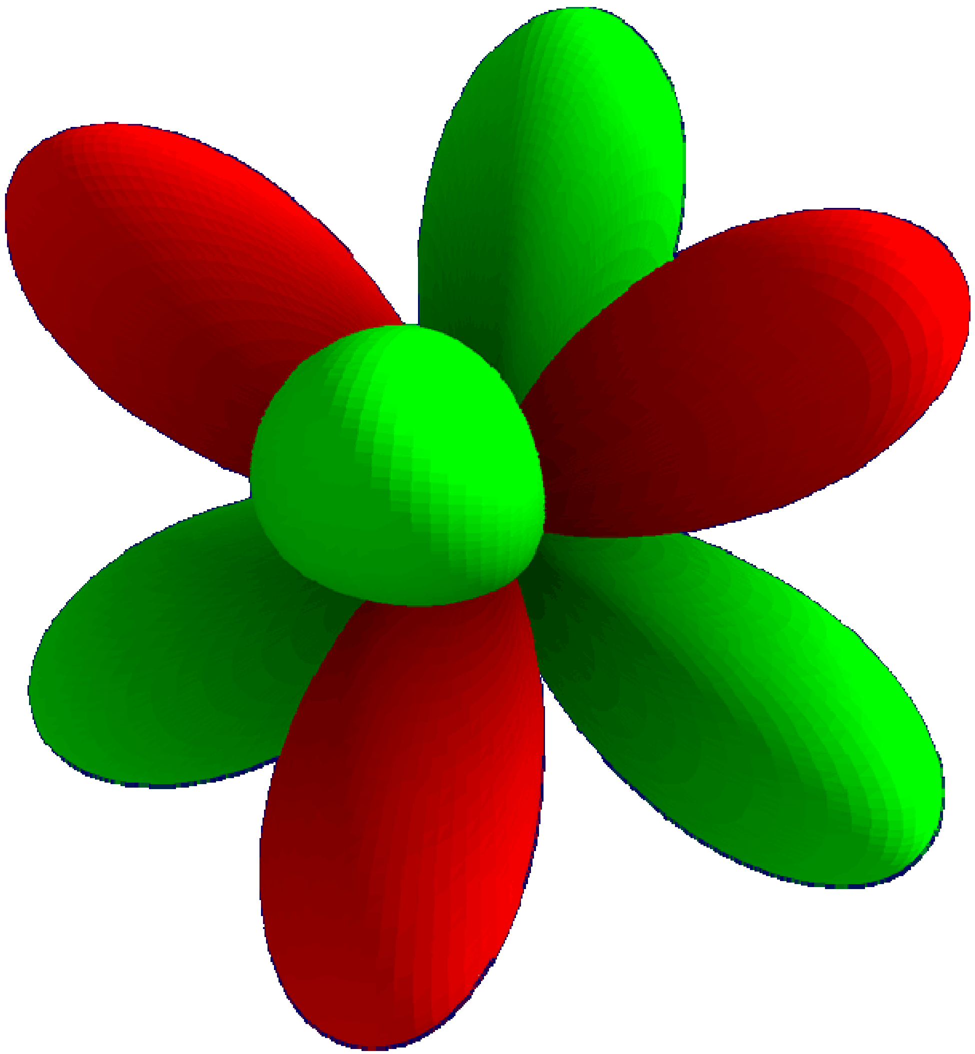
Real spherical harmonics

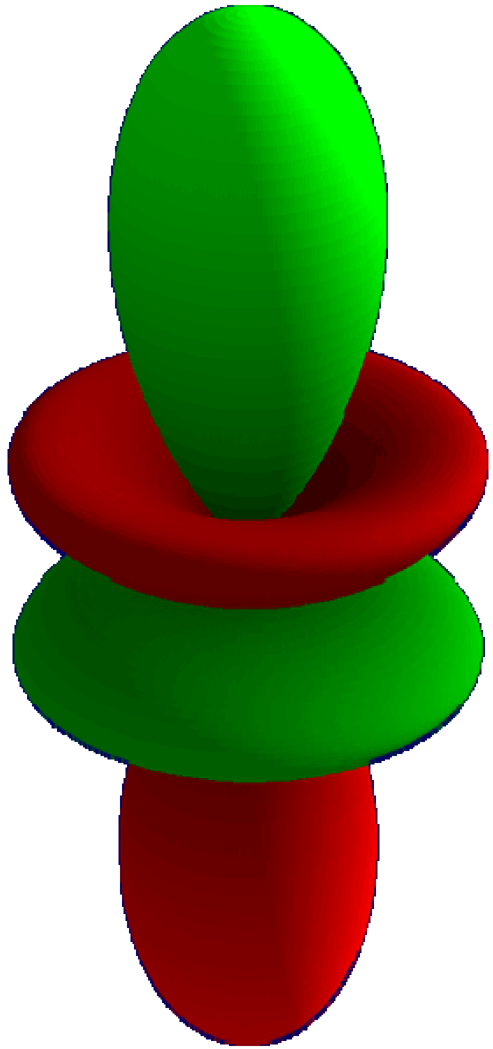
- A system of signed, orthogonal functions over the sphere
- Represented in spherical coordinates by the function

$$y_l^m(\theta,\varphi)=\begin{cases}\sqrt{2}K_l^m\cos(m\varphi)P_l^m(\cos\theta), & m>0\\ \sqrt{2}K_l^m\sin(-m\varphi)P_l^{-m}(\cos\theta), & m<0\\ K_l^0P_l^0(\cos\theta), & m=0\end{cases}$$

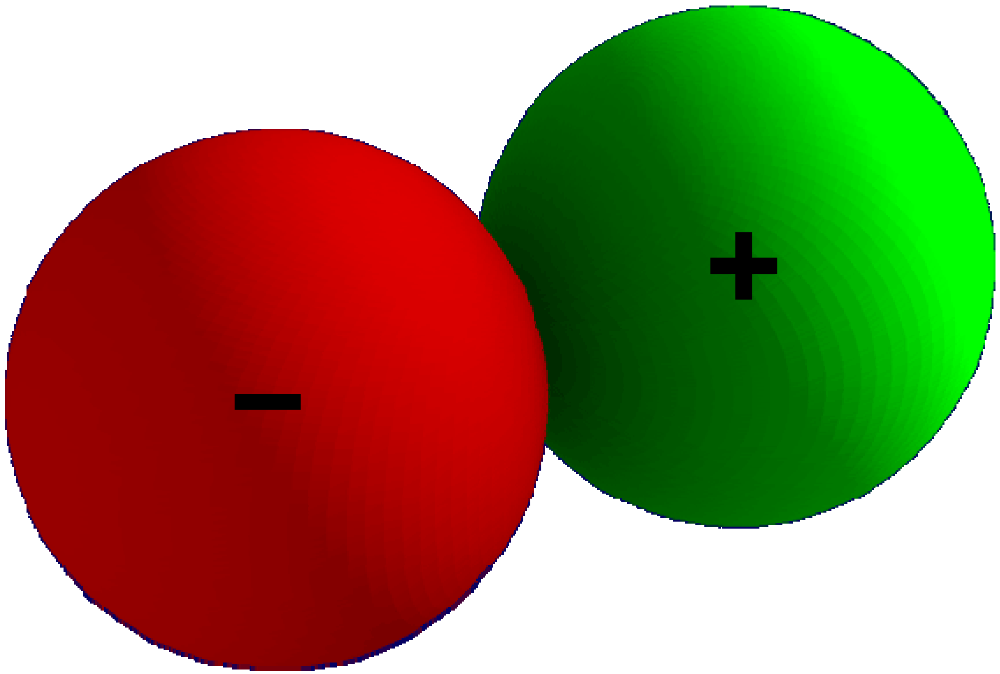
where l is the band and m is the index within the band

Real spherical harmonics

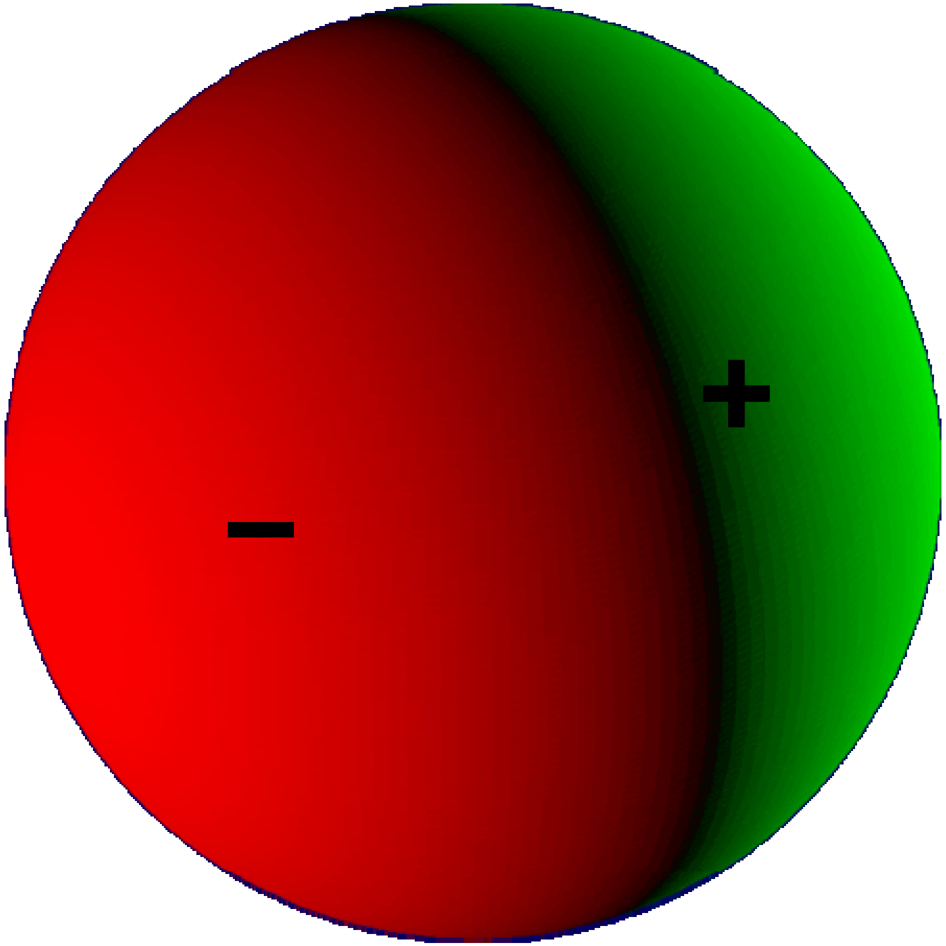




Reading SH diagrams

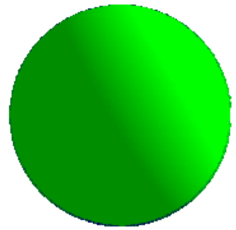


Reading SH diagrams

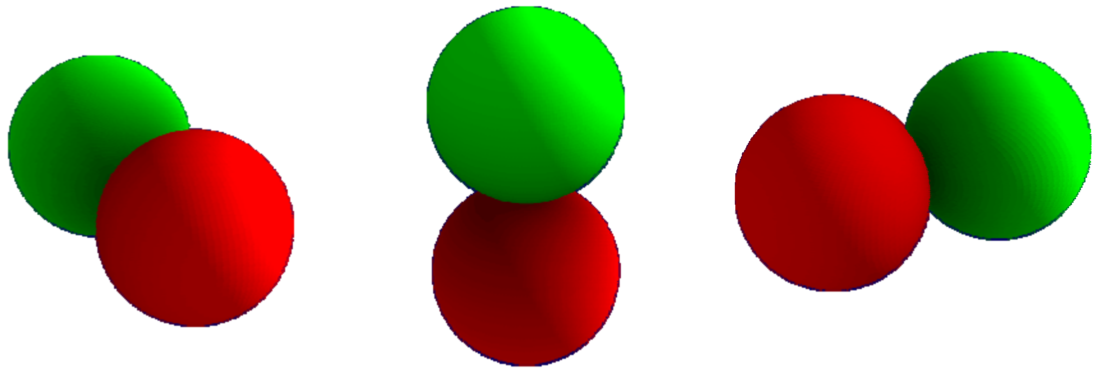


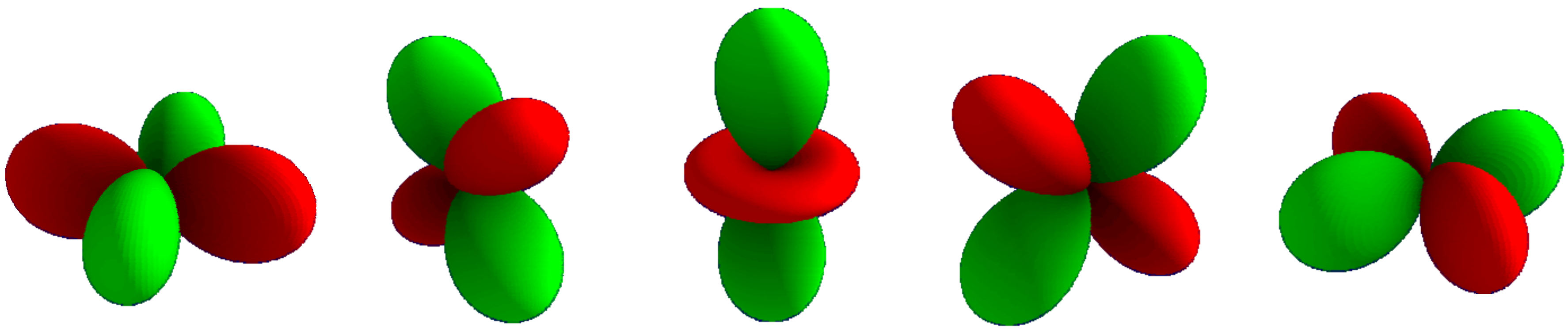
The SH functions

$y_0^0 =$

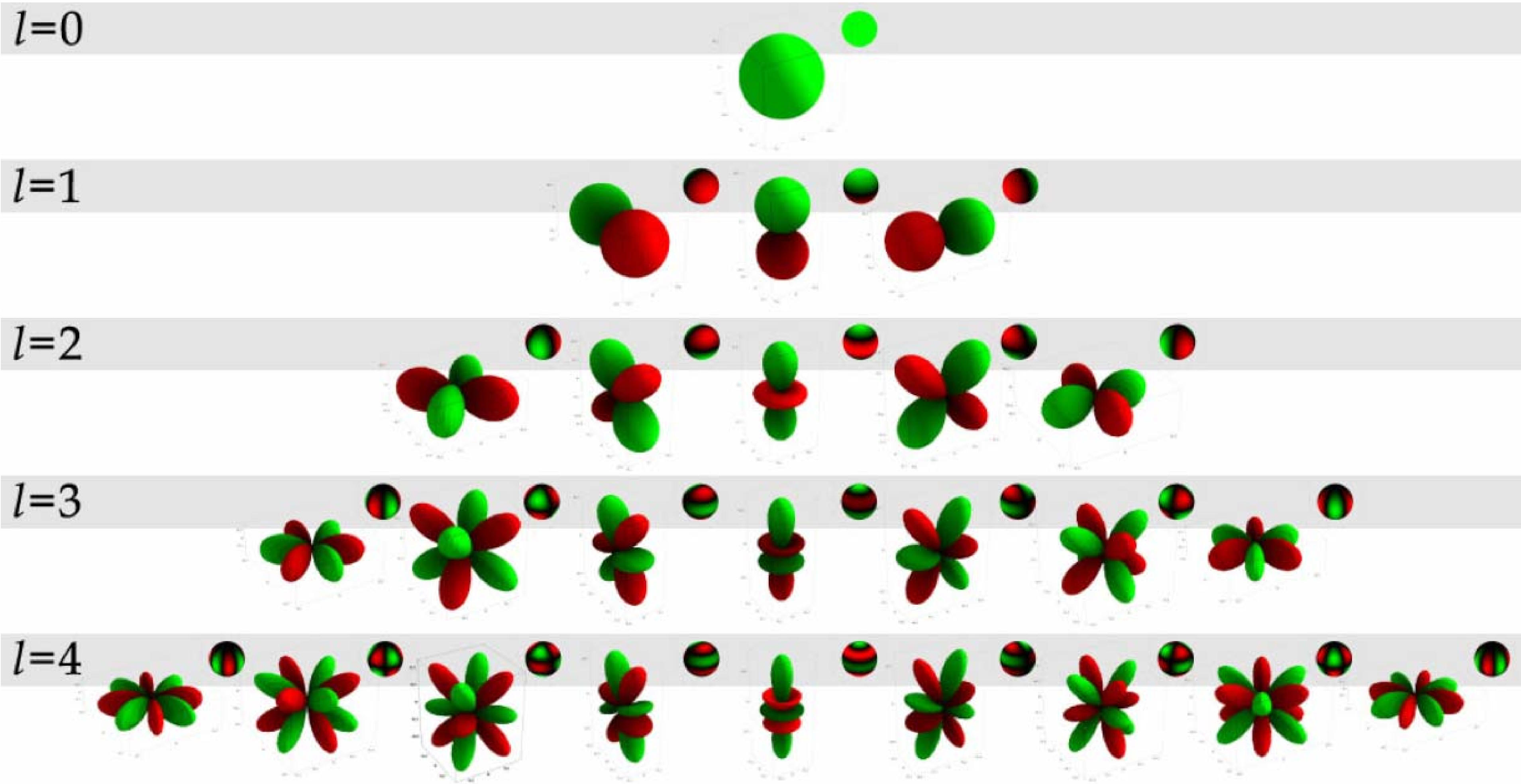


$y_1^{-1} =$





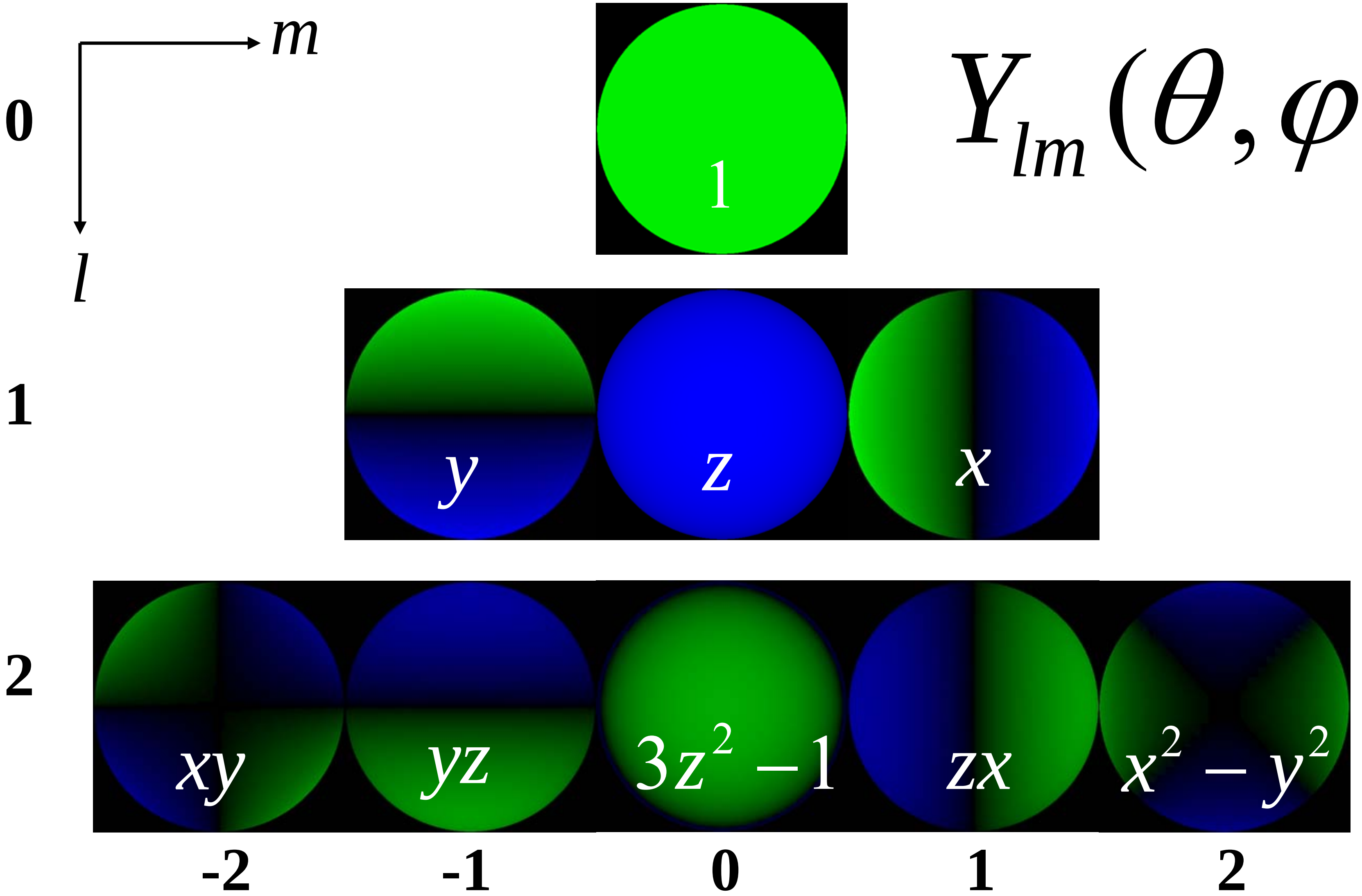
The SH functions



Spherical harmonics

$$
\begin{aligned}
(x, y, z) &= (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta) \\
Y_{00}(\theta,\phi) &= 0.282095 \\
(Y_{11}; Y_{10}; Y_{1-1})\,(\theta,\phi) &= 0.488603\,(x; z; y) \\
(Y_{21}; Y_{2-1}; Y_{2-2})\,(\theta,\phi) &= 1.092548\,(xz; yz; xy) \\
Y_{20}(\theta,\phi) &= 0.315392\left(3z^2 - 1\right) \\
Y_{22}(\theta,\phi) &= 0.546274\left(x^2 - y^2\right)
\end{aligned}
$$

Spherical harmonics



$$Y_{lm}(\theta, \varphi)$$

SH projection

- **First we define a strict order for SH functions**

$$i = l(l+1) + m$$

- **Project a spherical function into a vector of SH coefficients**

$$c_i = \int_S f(s) y_i(s) ds$$

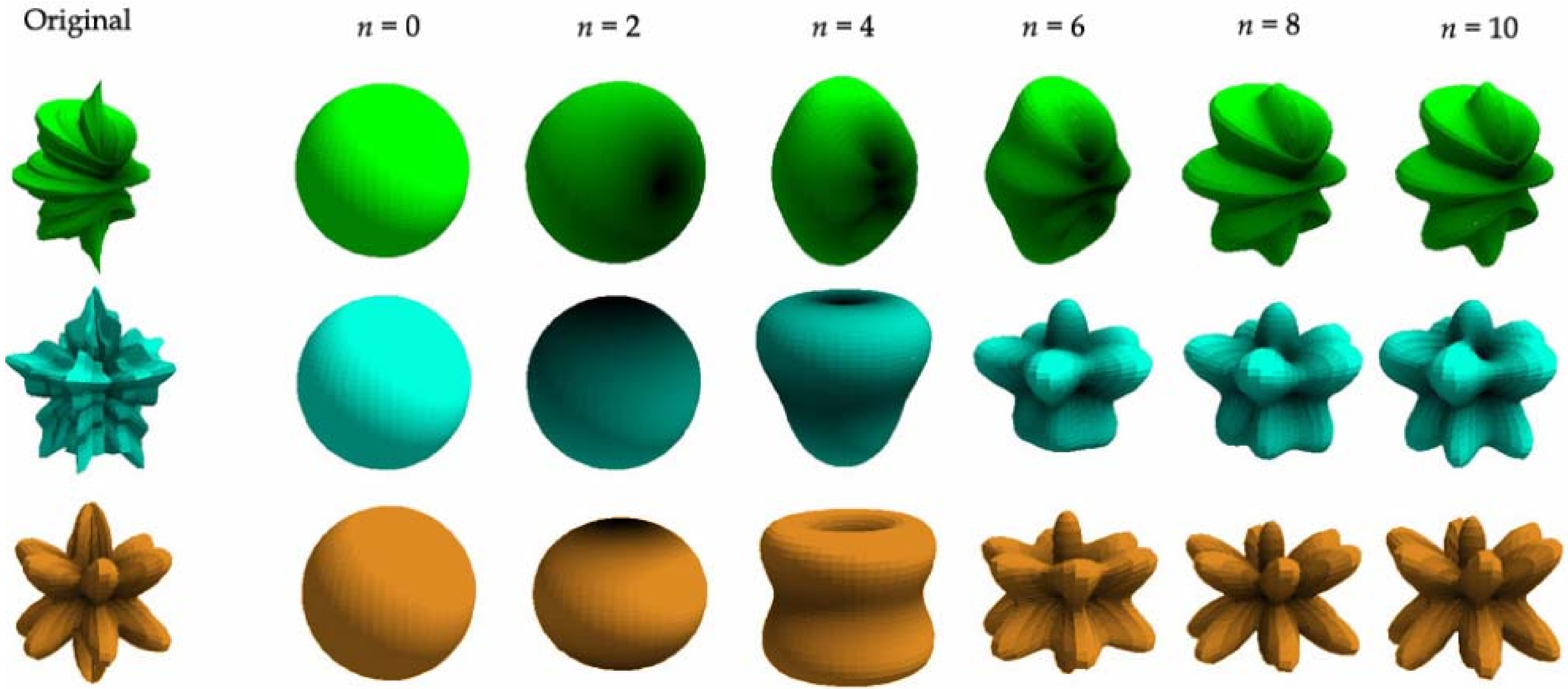
SH reconstruction

- To reconstruct the approximation to a function

$$\tilde{f}(s) = \sum_{i=0}^{N^2} c_i y_i(s)$$

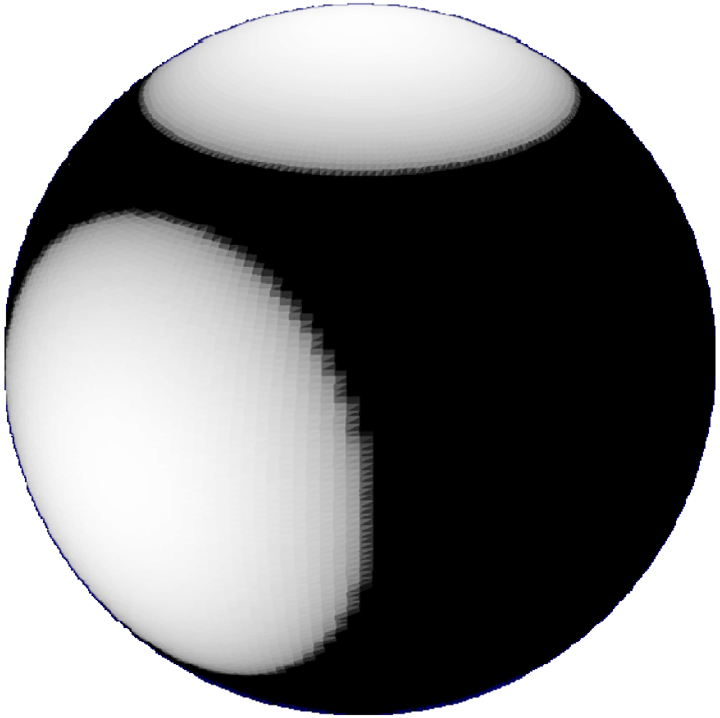
- We truncate the infinite series of SH functions to give a **low frequency** approximation

Examples of reconstruction



An example

- Take a function comprised of two area light sources
  - SH project them into 4 bands = 16 coefficients



$$\begin{bmatrix} 1.329, \\ -0.679, 0.930, 0.908, \\ -0.940, 0, 0.417, 0, 0.278, \\ -0.642, 0.001, 0.317, 0.837, \\ -0.425, 0, -0.238 \end{bmatrix}$$

Low frequency light source

- We reconstruct the signal
  - Using only these coefficients to find a low frequency approximation to the original light source