Reflection models

Digital Image Synthesis

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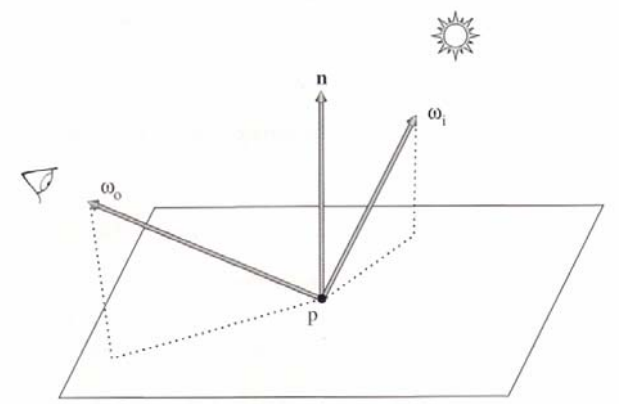
with slides by Pat Hanrahan and Matt Pharr

Rendering equation



$$L(\omega_o) = \int_\Omega f(\omega_i \rightarrow \omega_o) L(\omega_i) \cos\theta_i d\omega_i$$

Taxonomy 1

$$(x, y, t, \theta, \phi, \lambda)_{in} \rightarrow (x, y, t, \theta, \phi, \lambda)_{out}$$

General function = 12D

↓ assume time doesn't matter (no phosphorescence)
↓ assume wavelengths are equal (no fluorescence)

Scattering function = 9D

↓ assume wavelength is discretized or integrated into RGB (This is a common assumption for computer graphics)

Single-wavelength Scattering function = 8D

$$(x, y, \theta, \phi)_{in} \rightarrow (x, y, \theta, \phi)_{out}$$

Taxonomy 2

$$(x, y, \theta, \phi)_{in} \rightarrow (x, y, \theta, \phi)_{out}$$

Single-wavelength Scattering function = 8D

ignore subsurface scattering (x,y) in = (x,y) out

ignore dependence on position

Bidirectional Texture Function (BTF)
Spatially-varying BRDF (SVBRDF) = 6D

Bidirectional Subsurface Scattering Distribution Function (BSSRDF) = 6D

ignore direction of incident light

ignore dependence on position

ignore subsurface scattering

Light Fields, Surface LFs = 4D

$$(x, y, \theta, \phi)_{out}$$

assume Lambertian

Texture Maps = 2D

$$(x, y)_{out}$$

BRDF = 4D

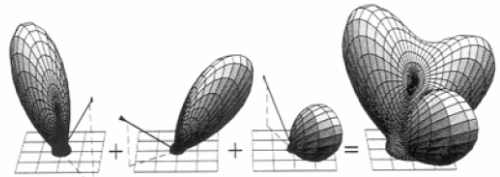
$$(\theta, \phi)_{in} \rightarrow (\theta, \phi)_{out}$$

assume isotropy

3D

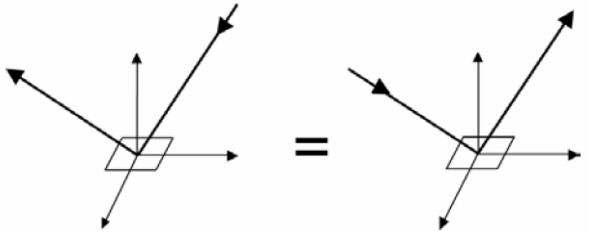
Properties of BRDFs

1. Linear



From Sillion, Arvo, Westin, Greenberg

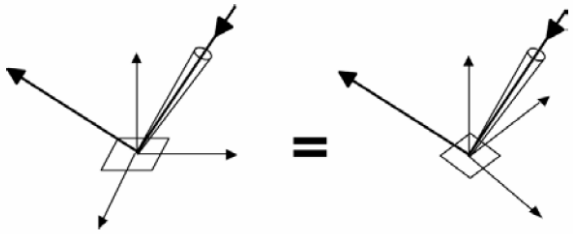
2. Reciprocity principle $f_r(\omega_r \to \omega_i) = f_r(\omega_i \to \omega_r)$



Properties of BRDFs

3. Isotropic vs. anisotropic

$$f_r(\theta_i, \varphi_i; \theta_r, \varphi_r) = f_r(\theta_i, \theta_r, \varphi_r - \varphi_i)$$

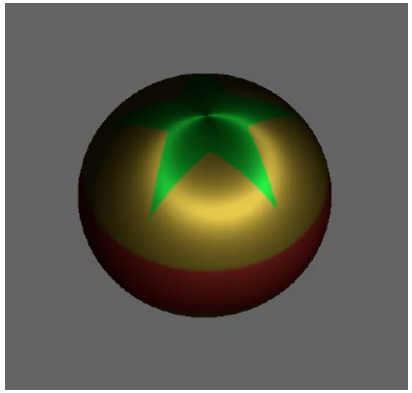


Reciprocity *and* isotropy

$$f_r(\theta_i, \theta_r, \varphi_r - \varphi_i) = f_r(\theta_r, \theta_i, \varphi_i - \varphi_r) = f_r(\theta_i, \theta_r, |\varphi_r - \varphi_i|)$$

4. Energy conservation $\int_\Omega f_r(\omega_o, \omega_i) \cos\theta_i d\omega_i \le 1$

Isotropic and anisotropic



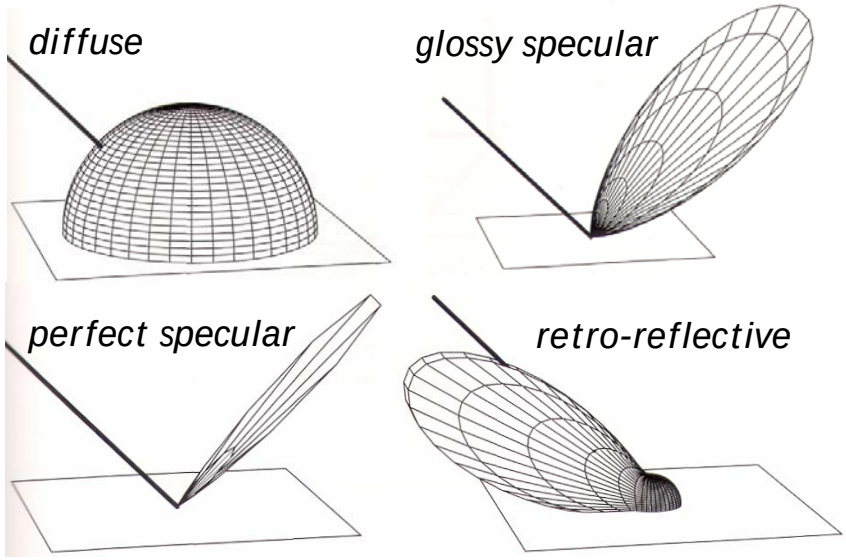
Reflection models

- BRDF/BTDF/BSDF
- Scattering from realistic surfaces is best described as a mixture of multiple BRDFs and BSDFs.
- `core/reflection.*`
- Material = BSDF that combines multiple BRDFs and BSDFs. (chap. 10)
- Textures = reflection and transmission properties that vary over the surface. (chap. 11)

Surface reflection models

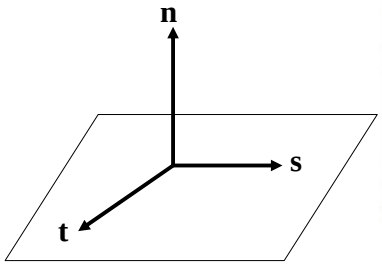
- **Measured data:** usually described in tabular form or coefficients of a set of basis functions
- **Phenomenological models**: *qualitative* approach; models with intuitive parameters
- Simulation: simulates light scattering from microgeometry and known reflectance properties
- Physical optics: solve Maxwell's equation
- **Geometric optics**: microfacet models

Reflection categories

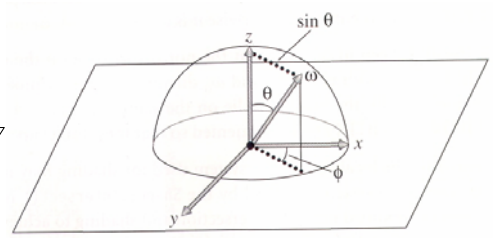


Geometric setting

incident and outgoing directions are normalized and outward facing after being transformed into the local frame



local frame



$$\cos\theta = \omega_z, \quad \sin\theta = \sqrt{1-\omega_z^2}$$

$$\cos\phi = \frac{\omega_x}{\sin\theta}, \quad \sin\phi = \frac{\omega_y}{\sin\theta}$$

BxDF

- **`BxDFType`**
  - **`BSDF_REFLECTION`**, **`BSDF_TRANSMISSION`**
  - **`BSDF_DIFFUSE`**, **`BSDF_GLOSSY`** (retro-reflective), **`BSDF_SPECULAR`**
- **`Spectrum f(Vector &wo, Vector &wi)=0;`**
- **`Spectrum Sample_f(Vector &wo, Vector *wi, float u1, float u2, float *pdf);`**
  used to find an incident direction for an outgoing direction; especially useful for reflection with a delta distribution
- **`Spectrum rho(Vector &wo, int nSamples=16, float *samples=NULL);`**

hemispherical-directional reflectance; computed analytically or by sampling

$$\rho_{hd}(\omega_o) = \int_\Omega f_r(p,\omega_o,\omega_i)\,|\cos\theta_i|\,d\omega_i$$

- **`Spectrum rho(int nSamples, float *samples);`**

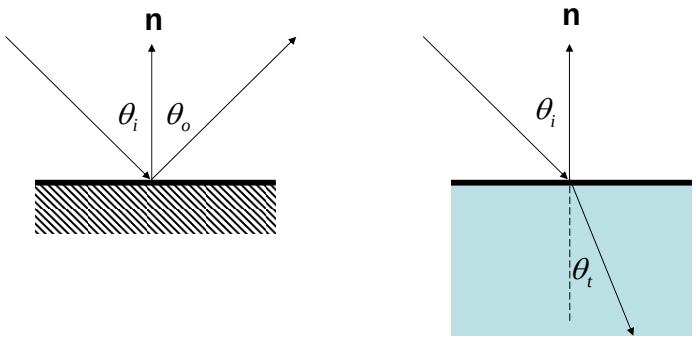
hemispherical-hemispherical reflectance

$$\rho_{hh} = \frac{1}{\pi}\int_\Omega\int_\Omega f_r(p,\omega_o,\omega_i)\,|\cos\theta_i\cos\theta_o|\,d\omega_i d\omega_o$$

Specular reflection and transmission

- Reflection: $\theta_i = \theta_o$
- Transmission: $\eta_i \sin\theta_i = \eta_t \sin\theta_t$ (Snell's law)

index of refraction *dispersion*



Fresnel reflectance

- Reflectivity and transmissiveness: fraction of incoming light that is reflected or transmitted; they are usually uniform over the surface but view dependent. Hence, the reflectivity should be corrected by *Fresnel equation*
- *Fresnel reflectance* for dielectrics

$$r_{\parallel} = \frac{\eta_t \cos\theta_i - \eta_i \cos\theta_t}{\eta_t \cos\theta_i + \eta_i \cos\theta_t} \qquad F_r(\omega_i) = \frac{1}{2}\left(r_{\parallel}^2 + r_{\perp}^2\right)$$

$$r_{\perp} = \frac{\eta_i \cos\theta_i - \eta_t \cos\theta_t}{\eta_i \cos\theta_i + \eta_t \cos\theta_t} \qquad F_t(\omega_i) = \left(1 - F_r(\omega_i)\right)$$

Indices of refraction

medium	Index of refraction
Vaccum	1.0
Air at sea level	1.00029
Ice	1.31
Water (20˚C)	1.333
Fused quartz	1.46
Glass	1.5~1.6
Sapphire	1.77
Diamond	2.42

Fresnel reflectance

- *Fresnel reflectance* for conductors (no transmission)

index of refraction, absorption coefficient

$$r_{\parallel}^2 = \frac{\left(\eta^2 + k^2\right)\cos^2\theta_i - 2\eta\cos\theta_i + 1}{\left(\eta^2 + k^2\right)\cos^2\theta_i + 2\eta\cos\theta_i + 1}$$

$$r_{\perp}^2 = \frac{\left(\eta^2 + k^2\right) - 2\eta\cos\theta_i + \cos^2\theta_i}{\left(\eta^2 + k^2\right) + 2\eta\cos\theta_i + \cos^2\theta_i}$$

$$F_r(\omega_i) = \frac{1}{2}\left(r_{\parallel}^2 + r_{\perp}^2\right)$$

η and k for a few conductors

Object	η	k
Gold	0.370	2.820
Silver	0.177	3.638
Copper	0.617	2.630
Steel	2.485	3.433

- However, for most conductors, these coefficients are unknown. Approximations are used to find plausible values for these quantities if reflectance at the normal incidence is known.

Approximation

- Measure F_r for θ_i=0

1. Assume $k = 0$

$$r_\perp^2 = r_\parallel^2 = \frac{(\eta-1)^2}{(\eta+1)^2} \qquad \eta = \frac{1+\sqrt{F_r(0)}}{1-\sqrt{F_r(0)}}$$

2. Assume $\eta = 1$

$$r_\perp^2 = r_\parallel^2 = \frac{k^2}{k^2+4} \qquad k = 2\sqrt{\frac{F_r(0)}{1-F_r(0)}}$$

Fresnel class

```
class Fresnel {
public:
  virtual Spectrum Evaluate(float cosi) const = 0;
};
class FresnelConductor : public Fresnel {
public:
  FresnelConductor(Spectrum &e, Spectrum &kk)
      : eta(e), k(kk) {}
private:
  Spectrum eta, k;
};
class FresnelDielectric : public Fresnel {
public:
  FresnelDielectric(float ei, float et) {
      eta_i = ei; eta_t = et; }
private:
  float eta_i, eta_t;
};
```

Evaluate directly implements Fresnel formula for conductor

Evaluate directly implements Fresnel formula for dielectric

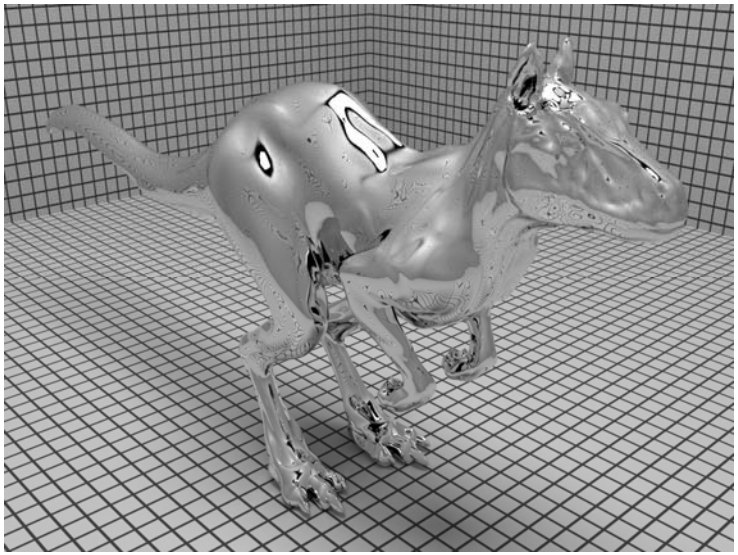
Specular reflection

```
class SpecularReflection : public BxDF {
public:
  SpecularReflection(const Spectrum &r, Fresnel *f)
  : BxDF(BxDFType(BSDF_REFLECTION | BSDF_SPECULAR)),
    R(r), fresnel(f) { }
  Spectrum f(const Vector &, const Vector &) const {
    return Spectrum(0.);
  }
  Spectrum Sample_f(const Vector &wo, Vector *wi,
                    float u1, float u2, float *pdf) const;
  float Pdf(const Vector &wo, const Vector &wi) const{
    return 0.;
   }
private:
  Spectrum R;
  Fresnel *fresnel;
};
```

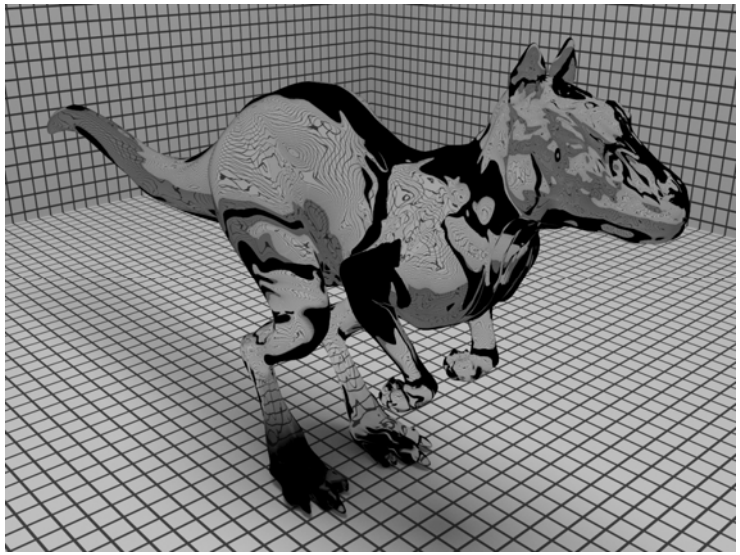
Specular reflection

```
Spectrum SpecularReflection::Sample_f(Vector &wo,
  Vector *wi, float u1, float u2, float *pdf) const{
  // Compute perfect specular reflection direction
  *wi = Vector(-wo.x, -wo.y, wo.z);
  *pdf = 1.f;
  return fresnel->Evaluate(CosTheta(wo)) * R /
         fabsf(CosTheta(*wi));
}
```

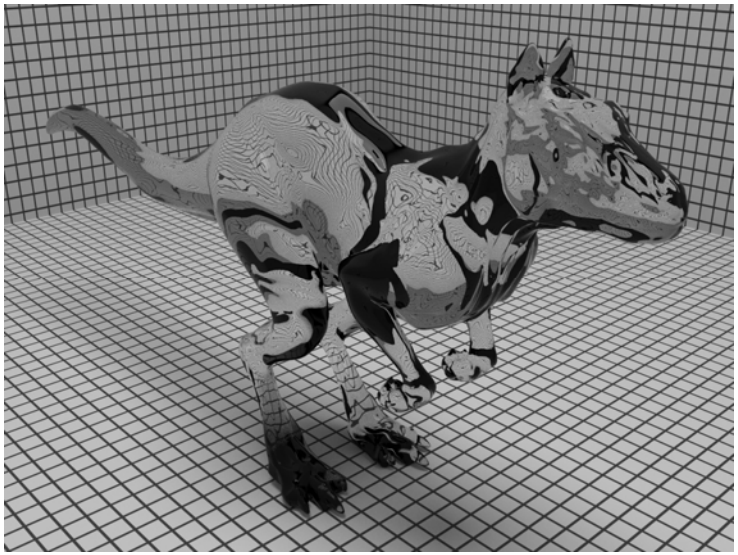
Perfect specular reflection



Perfect specular transmission



Fresnel modulation



Lambertian reflection

- It is not physically feasible, but provides a good approximation to many real-world surfaces.

```
class COREDLL Lambertian : public BxDF {
public:
  Lambertian(Spectrum &reflectance)
  : BxDF(BxDFType(BSDF_REFLECTION | BSDF_DIFFUSE)),
    R(reflectance), RoverPI(reflectance * INV_PI) {}
  Spectrum f(Vector &wo, Vector &wi) {return RoverPI}
  Spectrum rho(Vector &, int, float *) { return R; }
  Spectrum rho(int, float *) { return R; }
private:
  Spectrum R, RoverPI;
};
```

Derivations

$$\rho_{hh} = \frac{1}{\pi}\int_{\Omega}\int_{\Omega} f_r(p,\omega_o,\omega_i)\,|\cos\theta_i \cos\theta_o|\,d\omega_i d\omega_o$$

$$R = \frac{1}{\pi}\int_{\Omega}\int_{\Omega} c\,|\cos\theta_i \cos\theta_o|\,d\omega_i d\omega_o$$

$$R = \frac{c}{\pi}\cdot\int_{\Omega}\cos\theta_i d\omega_i \cdot \int_{\Omega}\cos\theta_o d\omega_o = c\pi$$

$$c = \frac{R}{\pi}$$

$$\int_{\Omega}\cos\theta_i d\omega_i = \int_0^{2\pi}\int_0^{\pi/2}\cos\theta_i \sin\theta_i d\theta_i d\phi_i$$

$$= \int_0^{2\pi} d\phi_i \int_0^{\pi/2}\cos\theta_i \sin\theta_i d\theta_i$$

$$= 2\pi\int_0^{\pi/2}\tfrac{1}{2}\sin(2\theta_i)\,\tfrac{1}{2}d(2\theta_i)$$

$$= \frac{\pi}{2}\cdot -\cos(2\theta_i)\Big|_0^{\pi/2} = \pi$$

Derivations

$$\rho_{hd}(\omega_o) = \int_{\Omega} f_r(p,\omega_o,\omega_i)\,|\cos\theta_i|\,d\omega_i$$

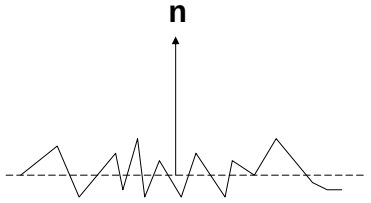
$$= \int_{\Omega}\tfrac{R}{\pi}\cos\theta_i d\omega_i$$

$$= \tfrac{R}{\pi}\int_{\Omega}\cos\theta_i d\omega_i$$

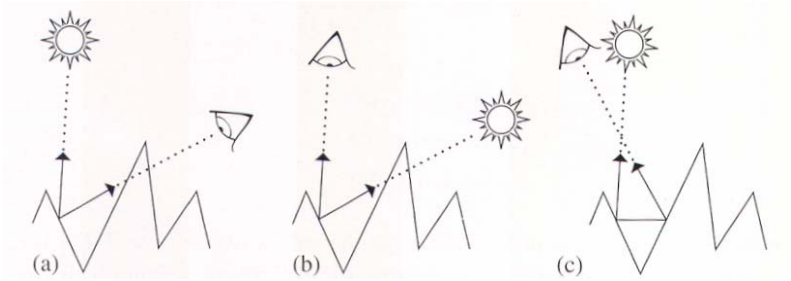
$$= \tfrac{R}{\pi}\cdot\pi = R$$

Microfacet models

- Rough surfaces can be modeled as a collection of small microfacets. Their aggregate behavior determines the scattering.
- Two components: distribution of microfacets and how light scatters from individual microfacet → closed-form BRDF expression



Important geometric effects to consider



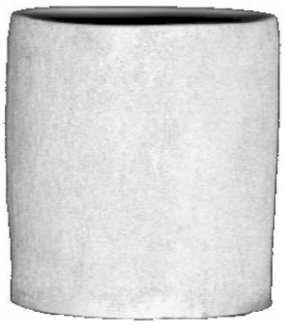
masking shadowing interreflection

Most microfacet models assume that all microfacets make up symmetric V-shaped grooves so that only neighboring microfacet needs to be considered. Particular models consider these effects with varying degrees of accuracy.

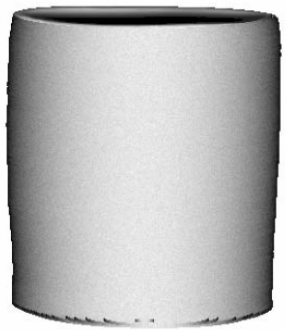
Oren-Nayar model

- Many real-world materials such as concrete, sand and cloth are not real Lambertian. Specifically, rough surfaces generally appear brighter as the illumination direction approaches the viewing direction.
- A collection of symmetric V-shaped perfect Lambertian grooves whose orientation angles follow a Gaussian distribution.
- Don't have a closed-form solution, instead they used an approximation

Oren-Nayar model



(a) Real image



(b) Lambertian model

Oren-Nayar model

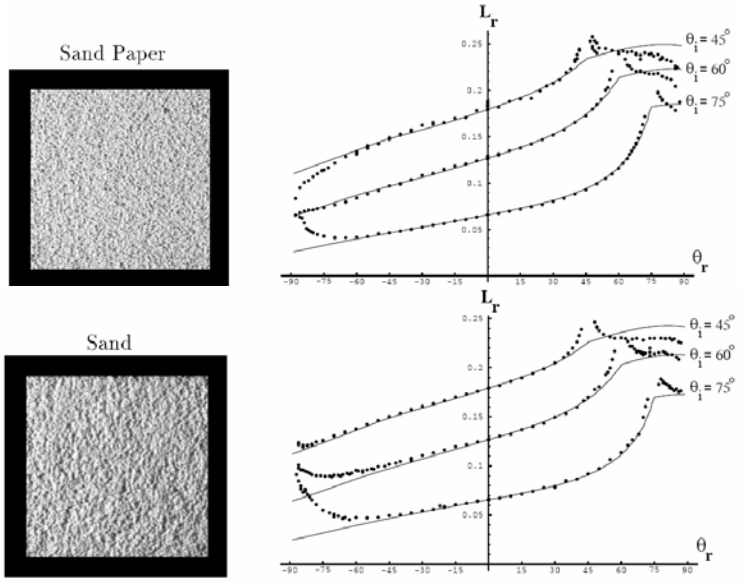
standard deviation for Gaussian

$$A = 1 - \frac{\sigma^2}{2(\sigma^2 + 0.33)}, \quad B = \frac{0.45\sigma^2}{\sigma^2 + 0.09}$$

$$\alpha = \max(\theta_i, \theta_o), \qquad \beta = \min(\theta_i, \theta_o)$$

$$f_r(\omega_i, \omega_o) = \frac{\rho}{\pi}(A + B\max(0, \cos(\phi_i - \phi_o))\sin\alpha\tan\beta)$$

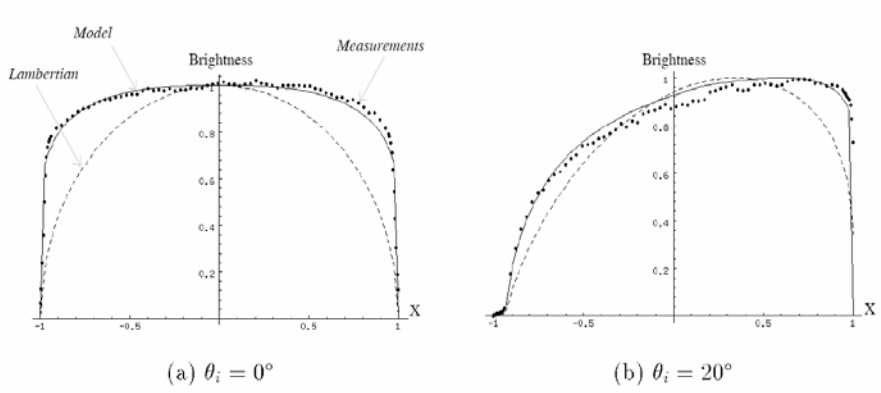
Oren-Nayar model



Oren-Nayar model

(a) Real image (b) Lambertian model (c) Proposed model

Oren-Nayar model



(a) $\theta_i = 0^\circ$ (b) $\theta_i = 20^\circ$

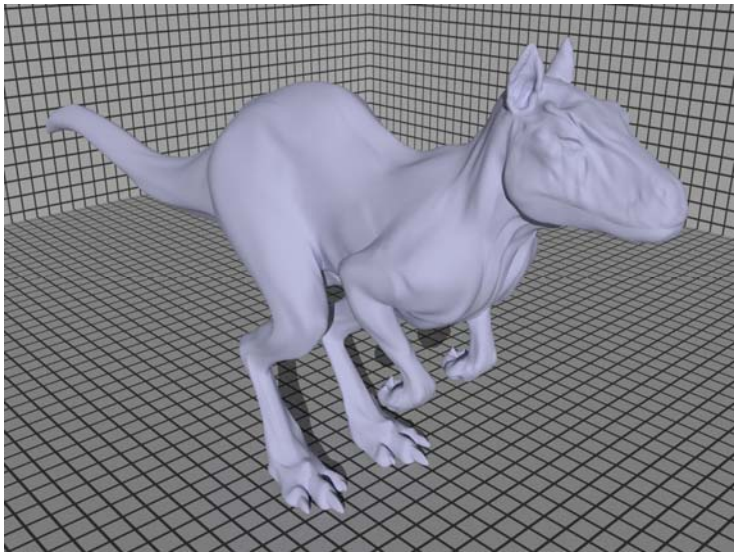
Oren-Nayar model

```
class OrenNayar : public BxDF {
public:
  Spectrum f(const Vector &wo, const Vector &wi)const;
  OrenNayar(const Spectrum &reflectance, float sig)
  : BxDF(BxDFType(BSDF_REFLECTION | BSDF_DIFFUSE)),
    R(reflectance) {
    float sigma = Radians(sig);
    float sigma2 = sigma*sigma;
    A = 1.f - (sigma2 / (2.f * (sigma2 + 0.33f)));
    B = 0.45f * sigma2 / (sigma2 + 0.09f);
  }
private:
  Spectrum R;
  float A, B;
};
```

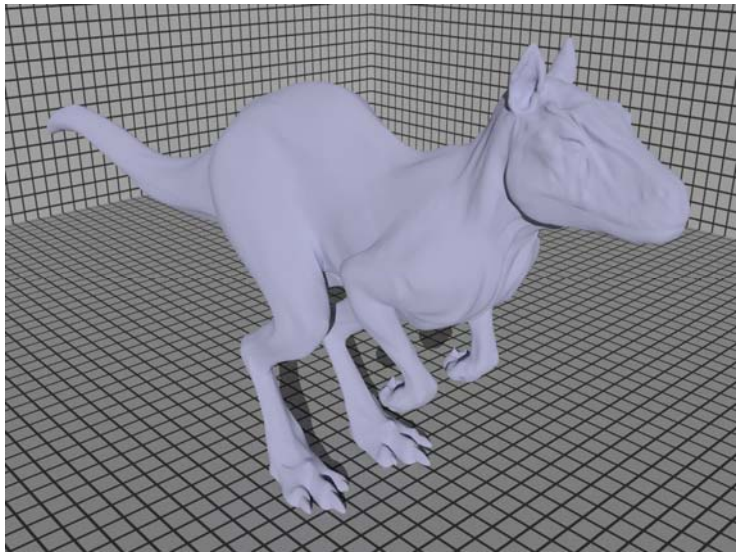
Oren-Nayar model

```
Spectrum OrenNayar::f(Vector &wo, Vector &wi)vconst{
  float sinthetai = SinTheta(wi);
  float sinthetao = SinTheta(wo);
  float sinphii = SinPhi(wi), cosphii = CosPhi(wi);
  float sinphio = SinPhi(wo), cosphio = CosPhi(wo);
  float dcos = cosphii * cosphio + sinphii * sinphio;
  float maxcos = max(0.f, dcos);
  float sinalpha, tanbeta;
  if (fabsf(CosTheta(wi)) > fabsf(CosTheta(wo))) {
    sinalpha = sinthetao;
    tanbeta = sinthetai / fabsf(CosTheta(wi));
  } else {
    sinalpha = sinthetai;
    tanbeta = sinthetao / fabsf(CosTheta(wo));
  }
  return R * INV_PI *
         (A + B * maxcos * sinalpha * tanbeta);
}
```

Lambertian

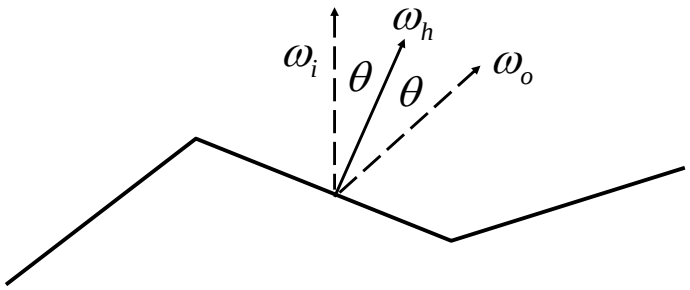


Oren-Nayer model



Torrance-Sparrow model

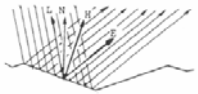
- One of the first microfacet models, designed to model metallic surfaces
- A collection of perfectly smooth mirrored microfacets with distribution $D(\omega_h)$



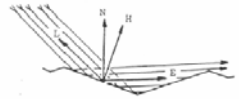
Torrance-Sparrow model

- Microfacet distribution D
- Fresnel reflection F
- Geometric attenuation G

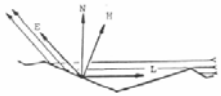
$$f_r(\omega_i\omega_o) = \frac{D(\omega_h)\,G(\omega_i,\omega_o)\,F(\omega_i,\omega_h)}{4\cos\theta_i\cos\theta_o}$$



$G = 1$



$G = \frac{2(N\cdot H)(N\cdot\omega_i)}{(H\cdot\omega_i)}$

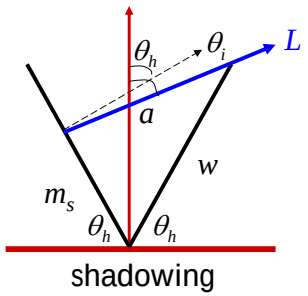


$G = \frac{2(N\cdot H)(N\cdot\omega_o)}{(H\cdot\omega_o)}$

Geometry attenuation factor

$$G = \frac{\text{facet area that is both visible and illuminated}}{\text{total facet area}}$$

$$= \frac{1\cdot\min(w-m_s, w-m_v)}{1\cdot w} = \min\left(1-\frac{m_s}{w}, 1-\frac{m_v}{w}\right)$$



shadowing

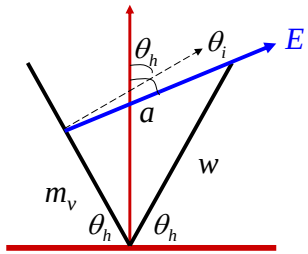
$$a\sin\theta_i = w\cos\theta_h + m_s\cos\theta_h \quad \times\cos\theta_i$$

$$a\cos\theta_i = w\sin\theta_h - m_s\sin\theta_h \quad \times -\sin\theta_i$$

$$\frac{m_s}{w} = -\frac{\cos(\theta_h+\theta_i)}{\cos(\theta_h-\theta_i)}$$

$$1-\frac{m_s}{w} = \frac{2\cos\theta_h\cos\theta_i}{\cos(\theta_h-\theta_i)}$$

Geometry attenuation factor



masking

$$1-\frac{m_v}{w} = \frac{2\cos\theta_h\cos\theta_o}{\cos(\theta_h-\theta_o)}$$

$$G = \min\left(1-\frac{m_s}{w}, 1-\frac{m_v}{w}\right) = \min\left(\frac{2\cos\theta_h\cos\theta_i}{\cos(\theta_h-\theta_i)}, \frac{2\cos\theta_h\cos\theta_o}{\cos(\theta_h-\theta_o)}\right)$$

$$G(\omega_o,\omega_i) = \min\left(1, \min\left(\frac{2(n\cdot\omega_h)(n\cdot\omega_o)}{\omega_o\cdot\omega_h}, \frac{2(n\cdot\omega_h)(n\cdot\omega_i)}{\omega_o\cdot\omega_h}\right)\right)$$

Microfacet model


```
class COREDLL MicrofacetDistribution {
public:
  virtual ~MicrofacetDistribution() { }
  virtual float D(const Vector &wh) const=0;
  virtual void Sample_f(const Vector &wo,
    Vector *wi, float u1, float u2,
    float *pdf) const = 0;
  virtual float Pdf(const Vector &wo,
                    const Vector &wi) const = 0;
};
```

Microfacet model

```
class Microfacet : public BxDF {
public:
  Microfacet(const Spectrum &reflectance, Fresnel *f,
             MicrofacetDistribution *d);
  Spectrum f(const Vector &wo,const Vector &wi) const;
  float G(Vector &wo, Vector &wi, Vector &wh) const {
    float NdotWh = fabsf(CosTheta(wh));
    float NdotWo = fabsf(CosTheta(wo));
    float NdotWi = fabsf(CosTheta(wi));
    float WOdotWh = AbsDot(wo, wh);
    return min(1.f, min((2.f*NdotWh*NdotWo/WOdotWh),
                        (2.f*NdotWh*NdotWi/WOdotWh)));
  }
  ...
private:
  Spectrum R;   Fresnel *fresnel;
  MicrofacetDistribution *distribution;
};
```

Microfacet model

```
Spectrum Microfacet::f(const Vector &wo,
                       const Vector &wi)
{
  float cosThetaO = fabsf(CosTheta(wo));
  float cosThetaI = fabsf(CosTheta(wi));
  Vector wh = Normalize(wi + wo);
  float cosThetaH = Dot(wi, wh);
  Spectrum F = fresnel->Evaluate(cosThetaH);
  return R * distribution->D(wh)
           * G(wo, wi, wh) * F
         / (4.f * cosThetaI * cosThetaO);
}
```

Blinn microfacet distribution

- Distribution of microfacet normals is modeled by an exponential falloff

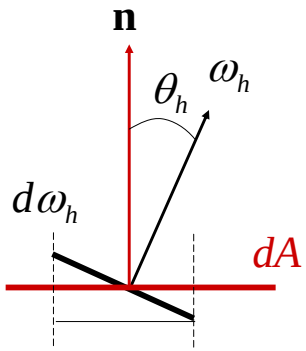
$$D(\omega_h) \propto (\omega_h \cdot n)^e = (\cos\theta_h)^e$$

- For smooth surfaces, this falloff happens very quickly; for rough surfaces, it is more gradual.
- Microfacet distribution must be normalized to ensure that they are physically plausible. The projected area of all microfacet faces over some area dA, the sum should be dA.

$$\int_\Omega D(\omega_h)\cos\theta_h d\omega_h = 1$$

Blinn microfacet distribution

$$\int_\Omega D(\omega_h)\cos\theta_h d\omega_h = 1 \qquad \int_\Omega c(\omega_h \cdot \mathbf{n})^e \cos\theta_h d\omega_h = 1$$



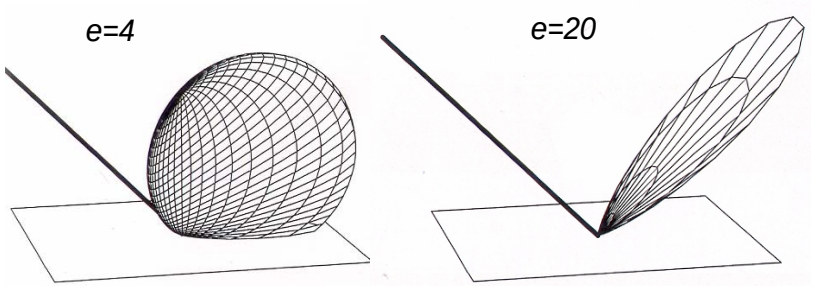
$$\int_0^{2\pi}\int_0^{\frac{\pi}{2}} c(\cos\theta_h)^{e+1}\sin\theta_h d\theta_h d\phi_h = 1$$

$$2\pi c\int_0^{\frac{\pi}{2}} (\cos\theta_h)^{e+1}(-d\cos\theta_h) = 1$$

$$-2\pi c\frac{(\cos\theta_h)^{e+2}}{e+2}\Bigg|_{\cos\theta_h=1}^{\cos\theta_h=0} = 1$$

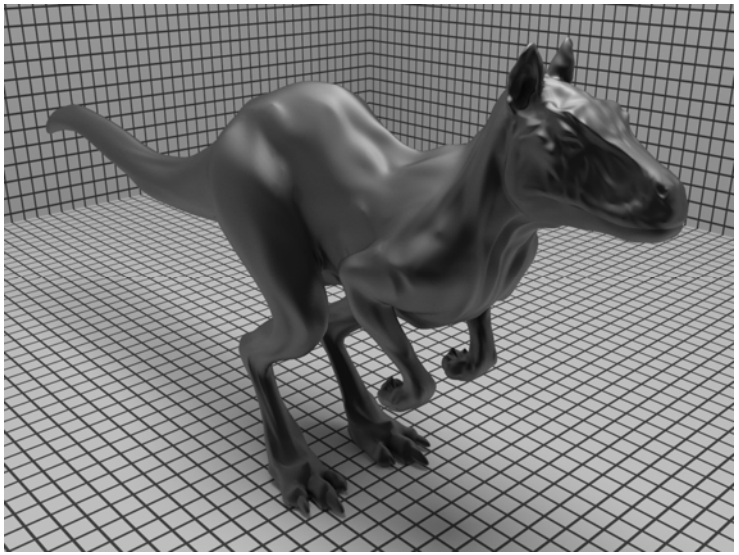
$$d\omega_h \cos\theta_h \qquad c = \frac{e+2}{2\pi} \qquad D(\omega_h) = \frac{e+2}{2\pi}(\omega_h \cdot n)^e$$

Blinn microfacet distribution



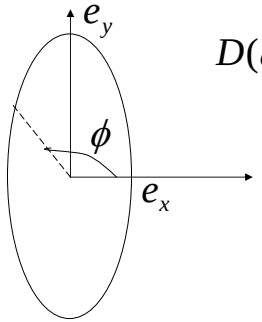

```
class Blinn : public MicrofacetDistribution
{
  ...
  float Blinn::D(const Vector &wh) const {
    float costhetah = fabsf(CosTheta(wh));
    return (exponent+2) * INV_TWOPI *
      powf(max(0.f, costhetah), exponent);
  }
}
```

Torrance-Sparrow with Blinn distribution



Anisotropic microfacet model

- Blinn microfacet model is radially symmetric (only depending on θ_h); hence, it is isotropic.
- Ashikmin and Shirley have developed a microfacet model for anisotropic surfaces



$$D(\omega_h) \propto (\omega_h \cdot n)^{e_x \cos^2\phi + e_y \sin^2\phi}$$

Ashikmin-Shirley model

$$\int_\Omega c(\omega_h \cdot \mathbf{n})^{e_x \cos^2\phi + e_y \sin^2\phi} \cos\theta_h \, d\omega_h = 1$$

$$\int_0^{2\pi} \int_0^{\pi/2} c(\cos\theta_h)^{e_x \cos^2\phi_h + e_y \sin^2\phi_h + 1} \sin\theta_h \, d\theta_h \, d\phi_h = 1$$

$$c \int_0^{2\pi} \int_0^{\pi/2} (\cos\theta_h)^{e_x \cos^2\phi_h + e_y \sin^2\phi_h + 1} d\cos\theta_h \, d\phi_h = -1$$

$$c \int_0^{2\pi} \frac{(\cos\theta_h)^{e_x \cos^2\phi_h + e_y \sin^2\phi_h + 2}}{e_x \cos^2\phi_h + e_y \sin^2\phi_h + 2} \Bigg|_1^0 d\phi_h = -1$$

$$c \int_0^{2\pi} \frac{1}{e_x \cos^2\phi_h + e_y \sin^2\phi_h + 2} d\phi_h = 1$$

Ashikmin-Shirley model

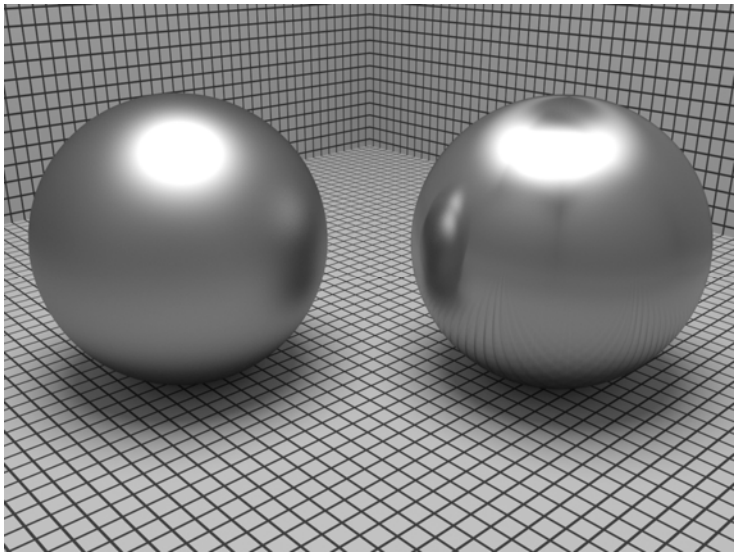
$$c\int_0^{2\pi}\frac{1}{e_x\cos^2\phi_h+e_y\sin^2\phi_h+2}d\phi_h=1$$

$$\int\frac{1}{a\cos^2(x)+b\sin^2(x)+2}dx=\frac{\tan^{-1}\left(\frac{\sqrt{b+2}\tan(x)}{\sqrt{a+2}}\right)}{\sqrt{a+2}\sqrt{b+2}}$$

$$c\frac{2\pi}{\sqrt{e_x+2}\sqrt{e_y+2}}=1$$

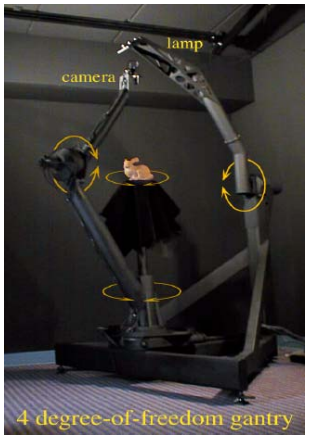
$$D(\omega_h)=\frac{\sqrt{(e_x+2)(e_y+2)}}{2\pi}(\omega_h\cdot n)^{e_x\cos^2\phi+e_y\sin^2\phi}$$

Anisotropic microfacet model



Lafortune model

- An efficient BRDF model to fit measured data to a parameterized model with a relatively small number of parameters



modifed Phong model

$$f_r(p,\omega_o,\omega_i)=\left(\omega_o\cdot R(\omega_i,\mathbf{n})\right)^e$$

Lafortune model

$$f_r(p,\omega_o,\omega_i)$$

$$=\frac{\rho_d}{\pi}+\sum_{i=1}^{n}(\omega_o\cdot(\omega_{ix}o_{i,x},\omega_{iy}o_{i,y},\omega_{iz}o_{i,z}))^{e_i}$$

Lafortune model (for a measured clay)