Color and Radiometry

Digital Image Synthesis

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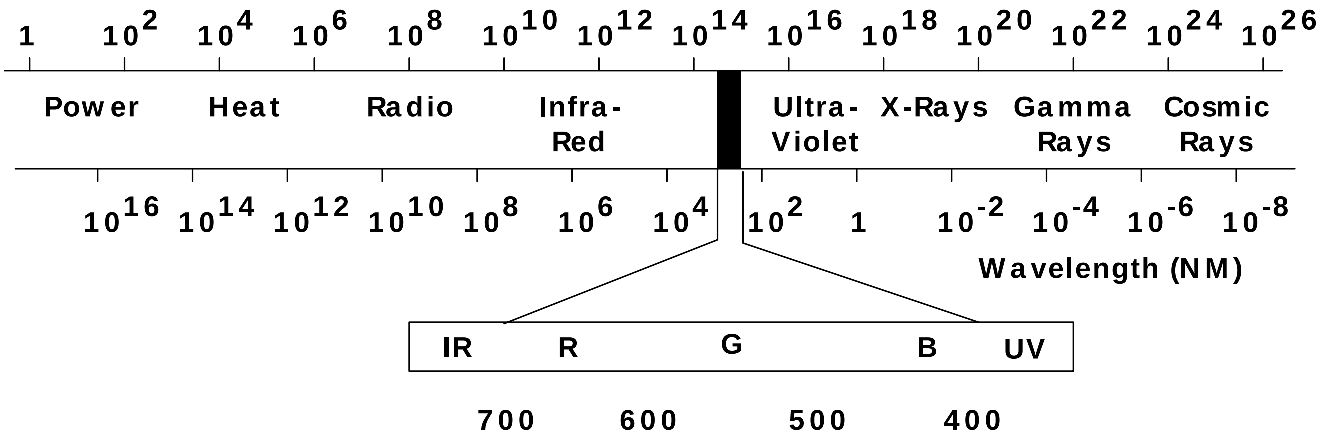
10/19/2006

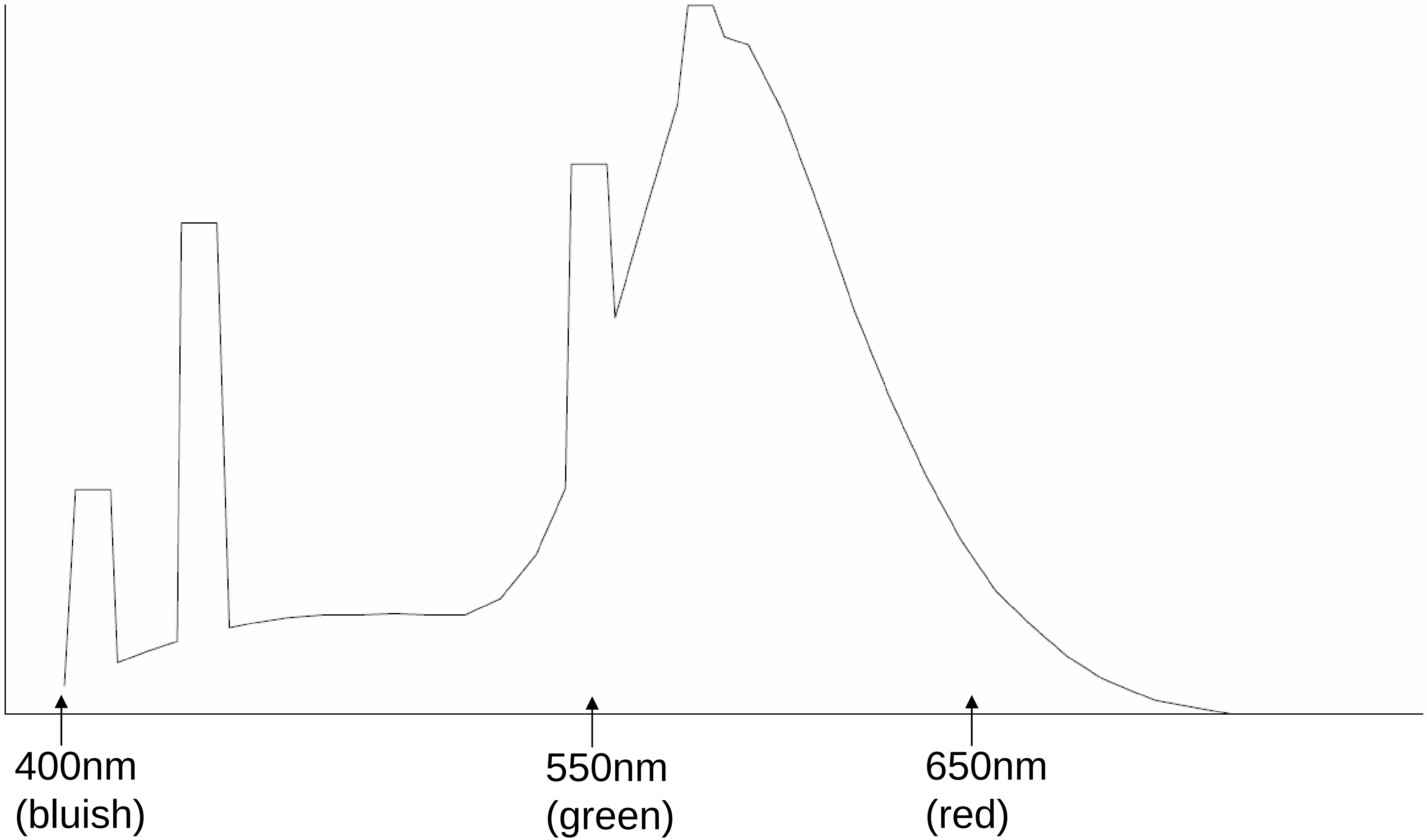
with slides by Pat Hanrahan and Matt Pharr

Radiometry

- Radiometry: study of the propagation of electromagnetic radiation in an environment
- Four key quantities: flux, intensity, irradiance and radiance
- These radiometric quantities are described by their spectral power distribution (SPD)
- Human visible light ranges from 370nm to 730nm

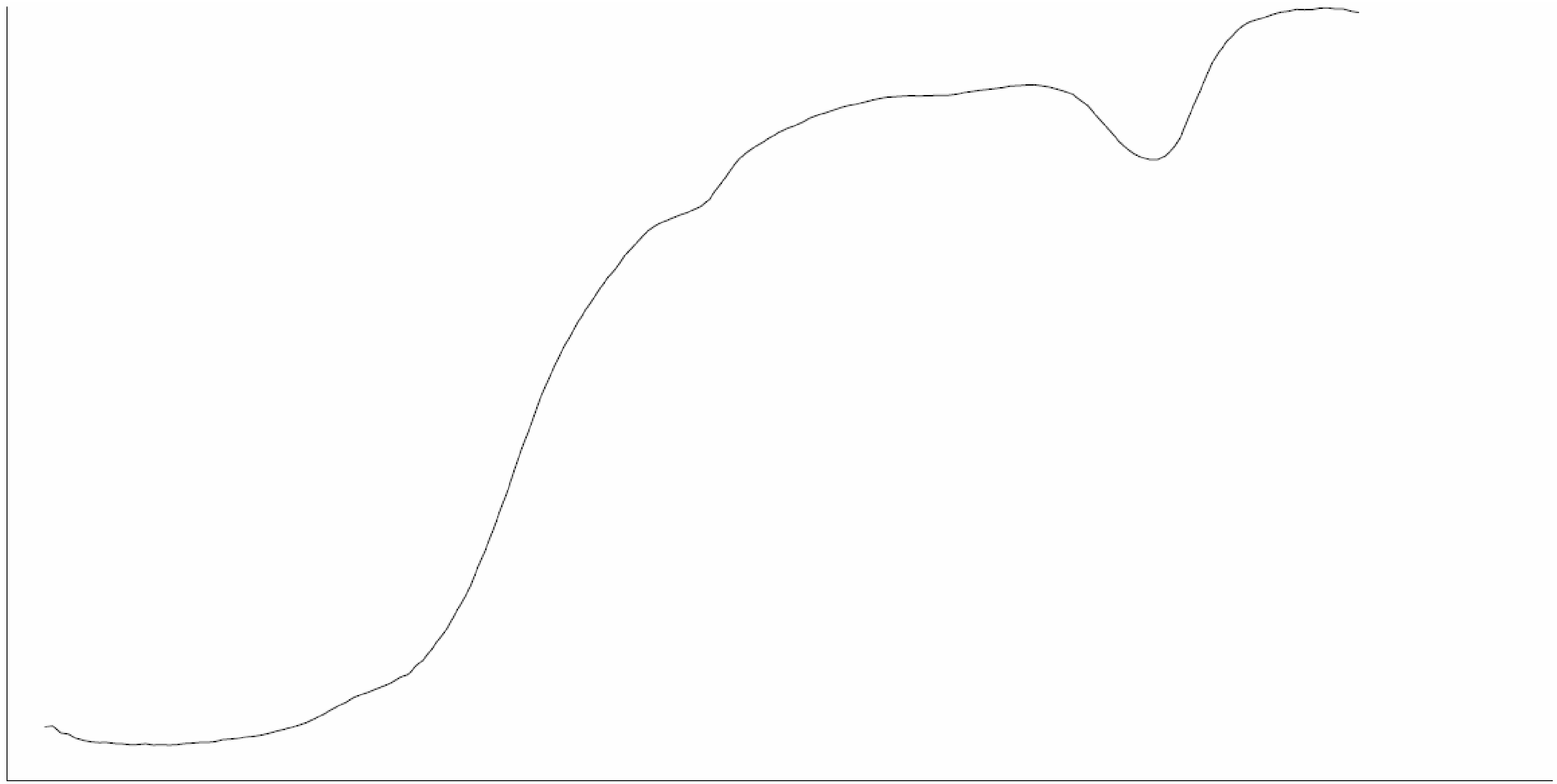


Spectral power distribution



fluorescent light (日光燈)

Spectral power distribution



lemmon skin

Color

- Need a compact, efficient and accurate way to represent functions like these
- Find proper basis functions to map the infinite-dimensional space of all possible SPD functions to a low-dimensional space of coefficients
- For example, $B(\lambda)=1$, a bad approximation

Spectrum

- In **`core/color.*`**
- Not a plug-in, to use inline for performance
- **`Spectrum`** stores a fixed number of samples at a fixed set of wavelengths. Better for smooth functions. Why is this possible? Human vision system

```
#define COLOR_SAMPLE 3
class COREDLL Spectrum {
public:
  <arithmetic operations>
private:
  float c[COLOR_SAMPLES];
  ...
}
```

We actually sample RGB

component-wise
+ - * / comparison...

Human visual system

- Tristimulus theory: all visible SPDs can be accurately represented for human observers with three values, x_λ, y_λ and z_λ.
- The basis are the *spectral matching curves*, X(λ), Y(λ) and Z(λ).

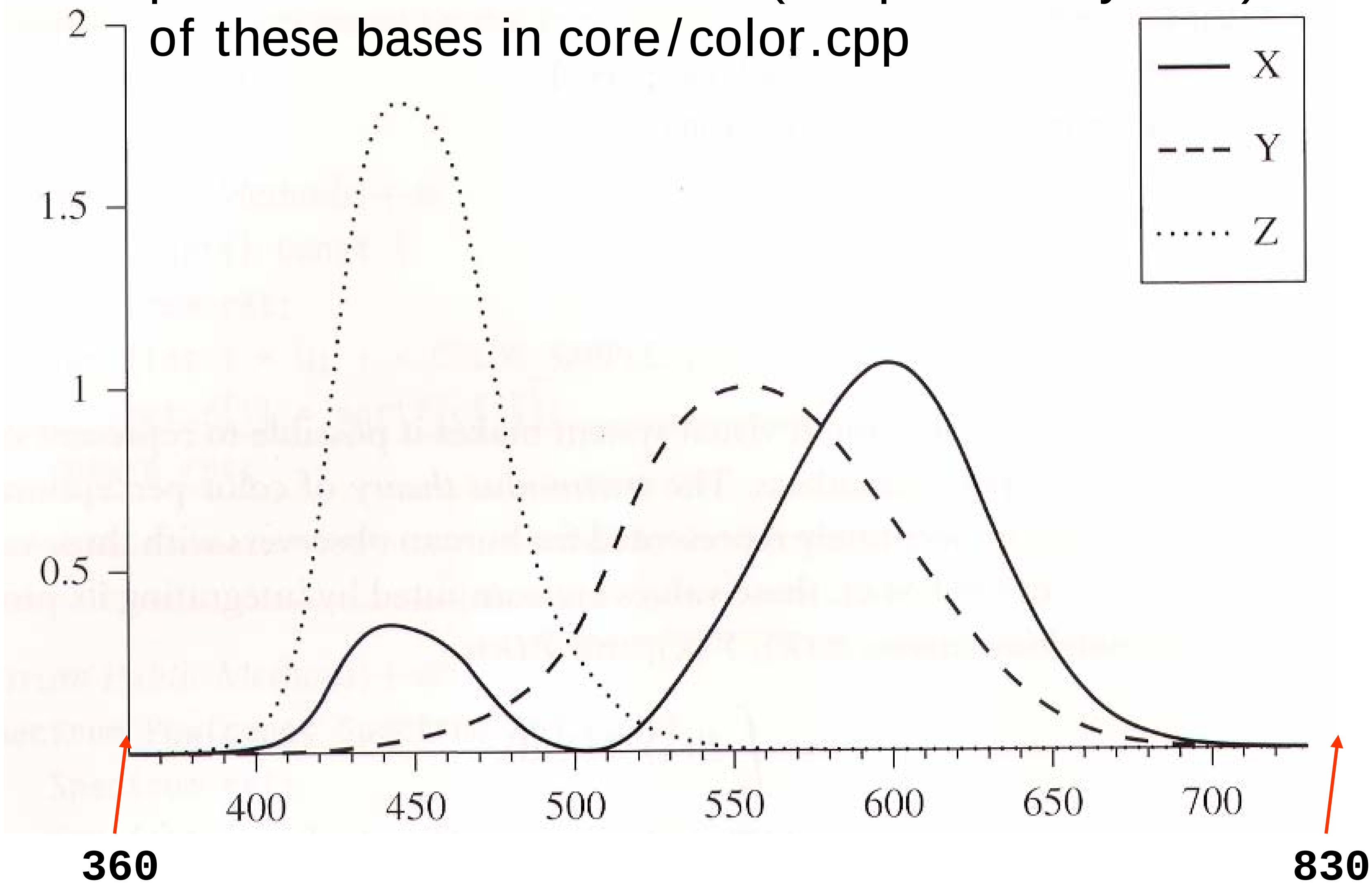
$$x_\lambda = \int_\lambda S(\lambda)X(\lambda)d\lambda$$

$$y_\lambda = \int_\lambda S(\lambda)Y(\lambda)d\lambda$$

$$z_\lambda = \int_\lambda S(\lambda)Z(\lambda)d\lambda$$

XYZ basis

pbrt has discrete versions (sampled every 1nm) of these bases in core/color.cpp



XYZ color

- It's, however, not good for spectral computation. A product of two SPD's XYZ values is likely different from the XYZ values of the SPD which is the product of the two original SPDs.
- Hence, we often have to convert our samples (RGB) into XYZ

```
void XYZ(float xyz[3]) const {
  xyz[0] = xyz[1] = xyz[2] = 0.;
  for (int i = 0; i < COLOR_SAMPLES; ++i) {
    xyz[0] += XWeight[i] * c[i];
    xyz[1] += YWeight[i] * c[i];
    xyz[2] += ZWeight[i] * c[i];
  }
}
```

Conversion between XYZ and RGB

```
float Spectrum::XWeight[COLOR_SAMPLES] = {
  0.412453f, 0.357580f, 0.180423f
};
float Spectrum::YWeight[COLOR_SAMPLES] = {
  0.212671f, 0.715160f, 0.072169f
};
float Spectrum::ZWeight[COLOR_SAMPLES] = {
  0.019334f, 0.119193f, 0.950227f
};
Spectrum FromXYZ(float x, float y, float z) {
  float c[3];
  c[0] =  3.240479f * x + -1.537150f * y + -
  0.498535f * z;
  c[1] = -0.969256f * x +  1.875991f * y +
  0.041556f * z;
  c[2] =  0.055648f * x + -0.204043f * y +
  1.057311f * z;
  return Spectrum(c);
}
```

Basic radiometry

- pbrt is based on radiative transfer: study of the transfer of radiant energy based on radiometric principles and operates at the geometric optics level (light interacts with objects much larger than the light's wavelength)
- It is based on the particle model. Hence, **diffraction** and **interference** can't be easily accounted for.

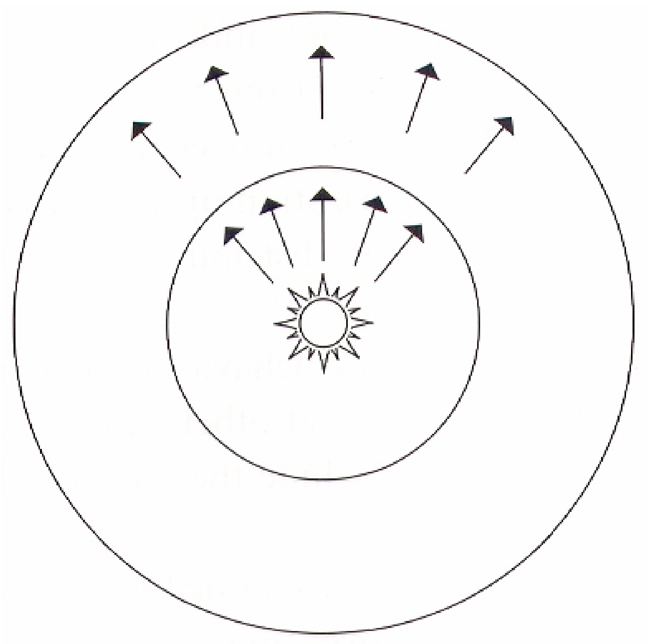
Basic assumptions about light behavior

- **Linearity**: the combined effect of two inputs is equal to the sum of effects
- **Energy conservation**: scattering event can't produce more energy than they started with
- **No polarization**: that is, we only care the frequency of light but not other properties
- **No fluorescence or phosphorescence**: behavior of light at a wavelength doesn't affect the behavior of light at other wavelengths
- **Steady state**: light is assumed to have reached equilibrium, so its radiance distribution isn't changing over time.

Fluorescent materials

Flux (Φ)

- Radiant flux, power
- Total amount of energy passing through a surface per unit of time (J/s,W)

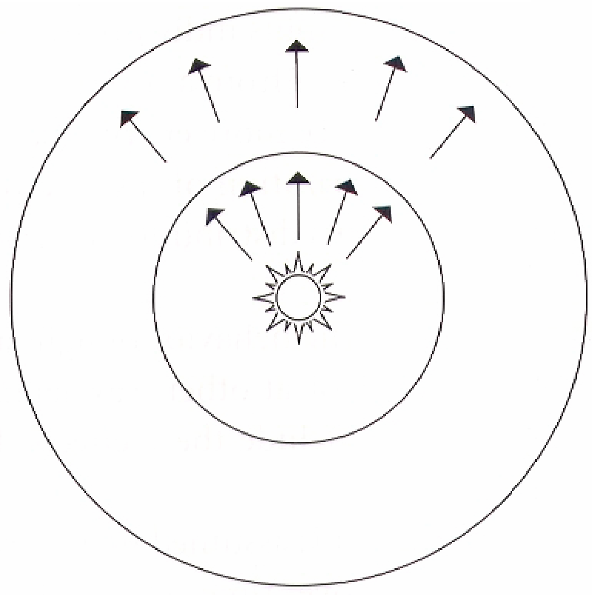


Irradiance (E)

- Area density of flux (W/m²) $E = \frac{d\Phi}{dA}$

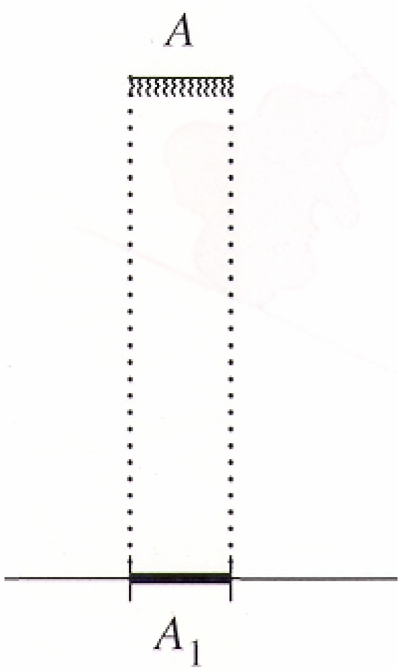
Inverse square law

$$E = \frac{\Phi}{4\pi r^2}$$

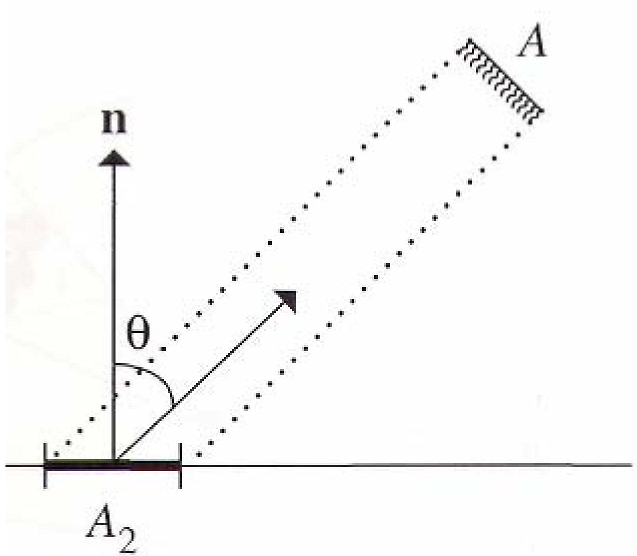


Lambert's law

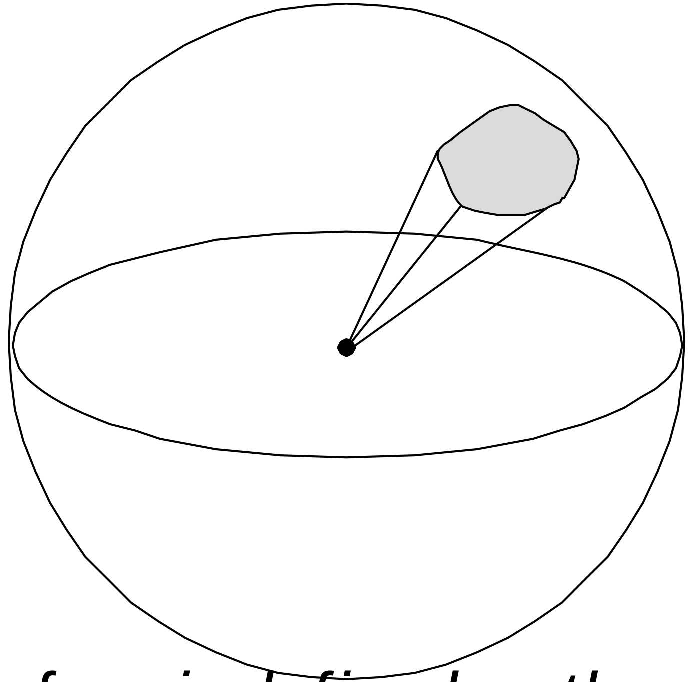
$$E = \frac{\Phi}{A}$$



$$E = \frac{\Phi \cos\theta}{A}$$



Angles and Solid Angles



- Angle $\theta = \frac{l}{r}$

  $\Rightarrow$ **circle has 2π radians**

- Solid angle $\Omega = \frac{A}{R^2}$

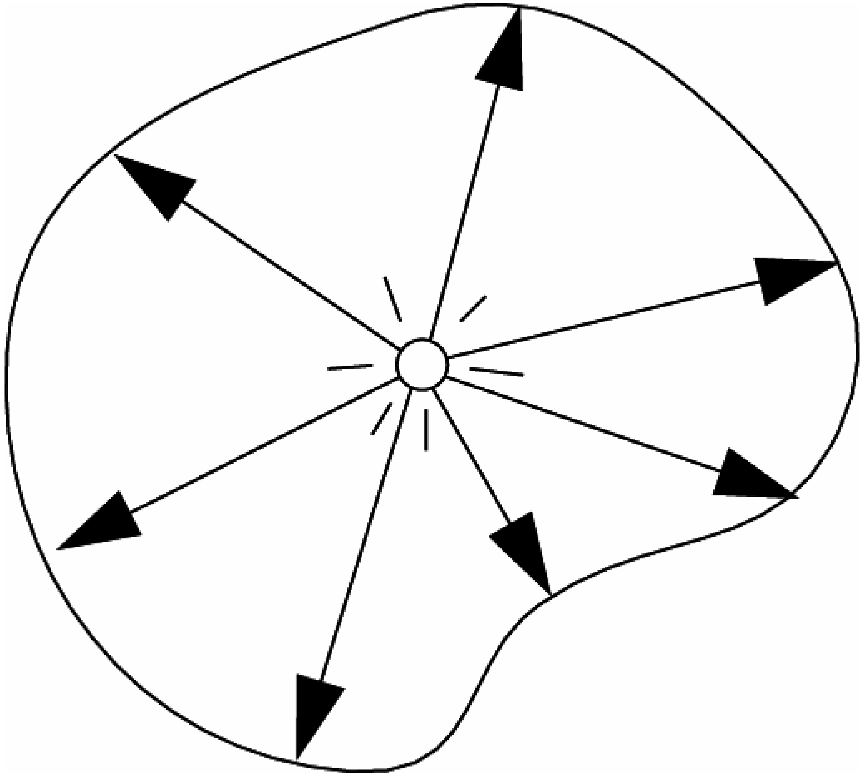
The solid angle subtended by a surface is defined as the surface area of a unit sphere covered by the surface's projection onto the sphere.

$\Rightarrow$ **sphere has 4π steradians**

Intensity (I)

- Flux density per solid angle $I = \frac{d\Phi}{d\omega}$
- Intensity describes the directional distribution of light

$$I(\omega) \equiv \frac{d\Phi}{d\omega}$$

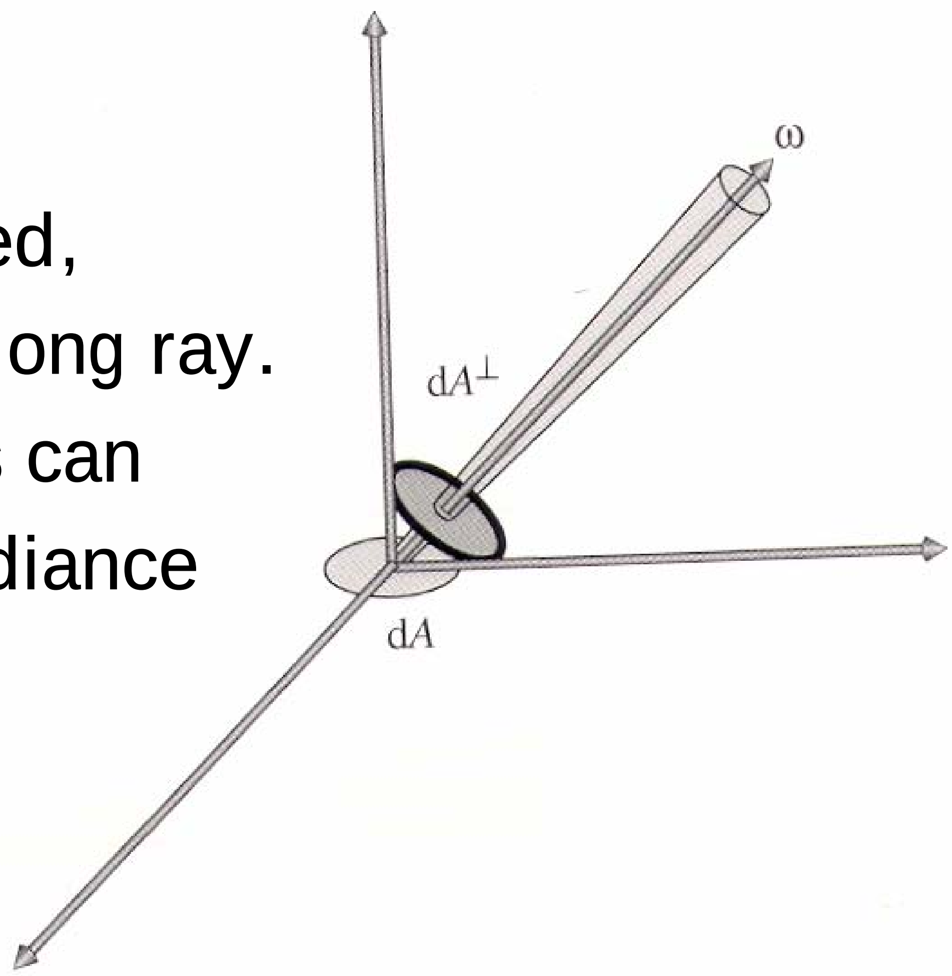


Radiance (L)

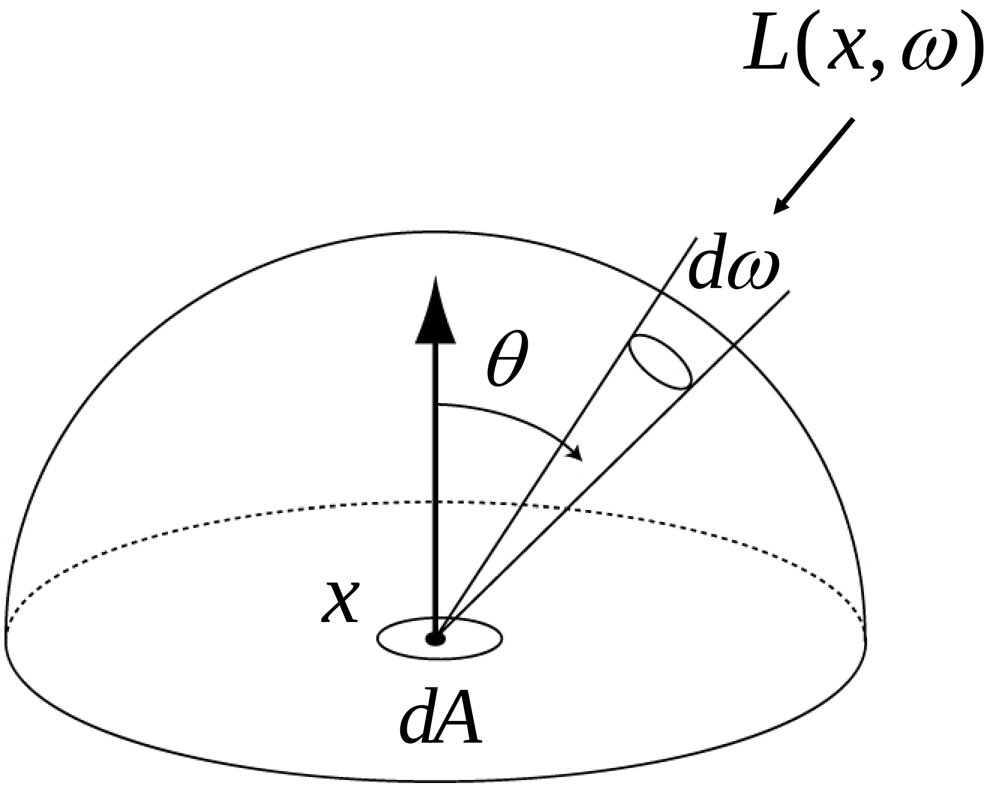
- Flux density per unit area per solid angle

$$L = \frac{d\Phi}{d\omega dA^{\perp}}$$

- Most frequently used, remains constant along ray.
- All other quantities can be derived from radiance



Calculate irradiance from radiance

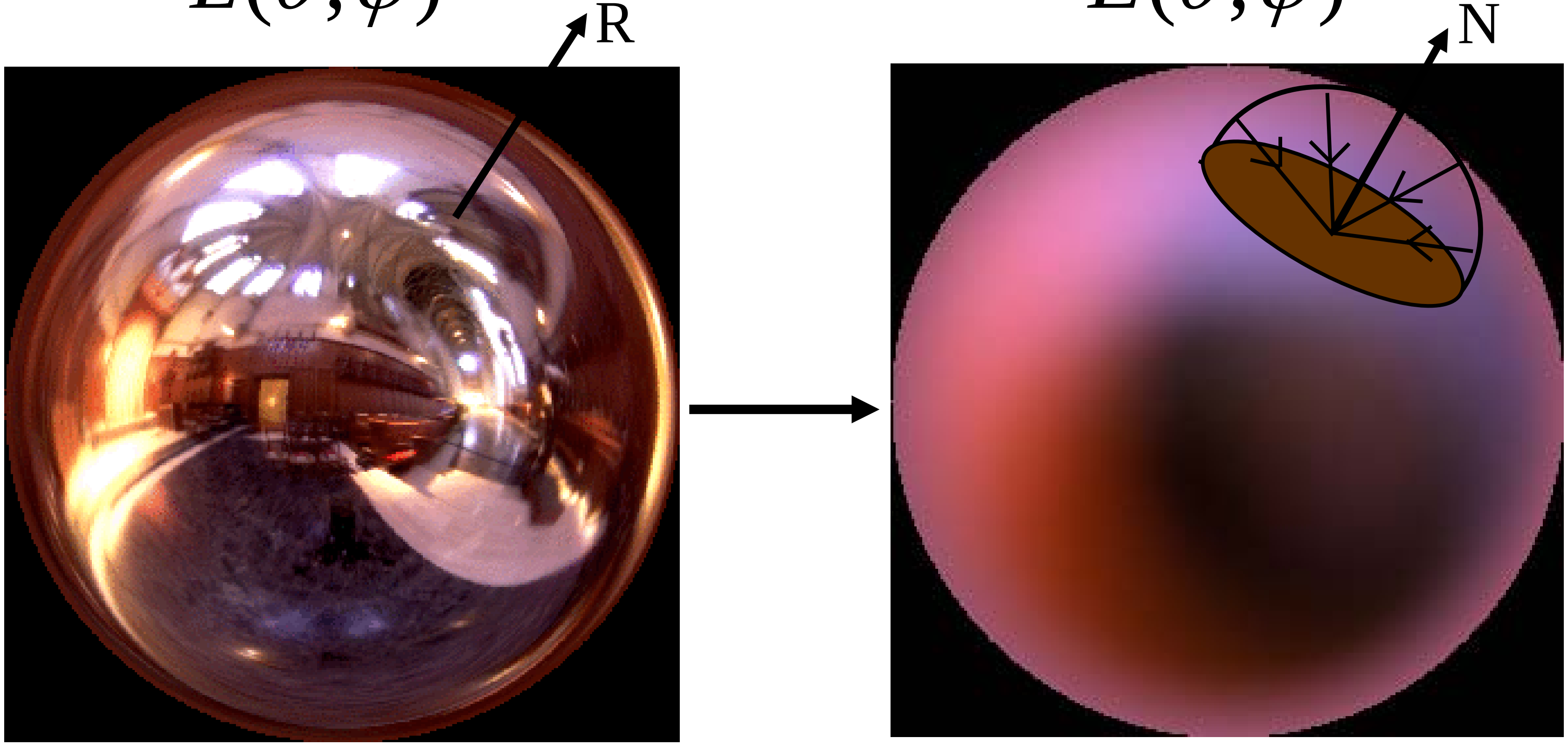


Light meter

$$E(x) = \frac{d\Phi}{dA} = \int_{\Omega} L(x,\omega)\cos\theta d\omega$$

Irradiance Environment Maps

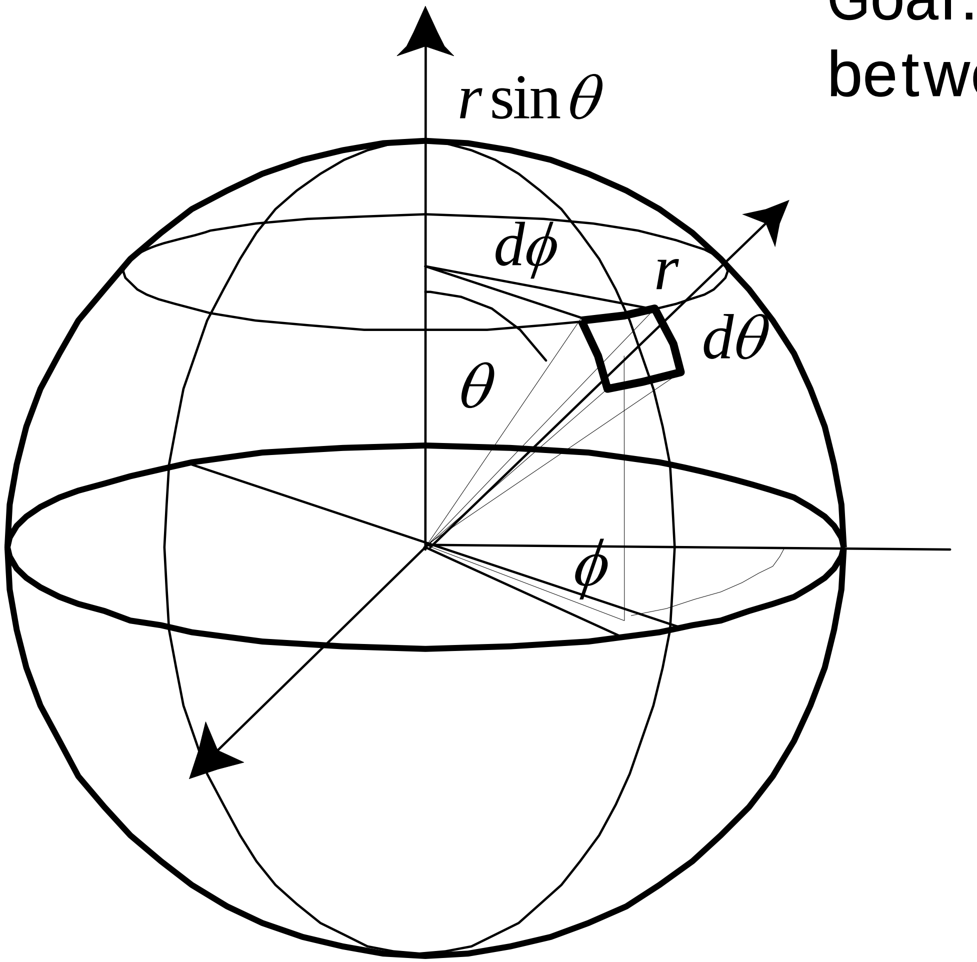
$L(\theta,\varphi)$ $E(\theta,\varphi)$



Radiance Environment Map

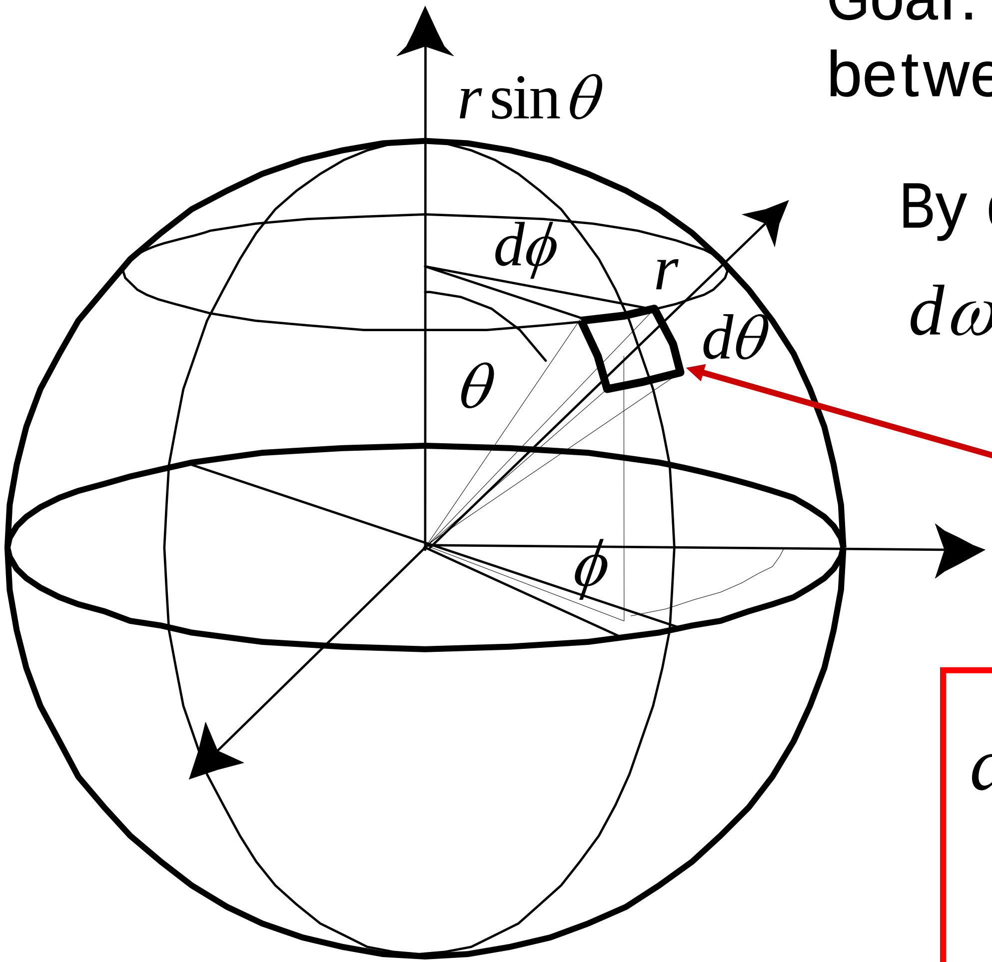
Irradiance Environment Map

Differential solid angles



Goal: find out the relationship between $d\omega$ and $d\theta$, $d\phi$

Differential solid angles



Goal: find out the relationship between $d\omega$ and $d\theta$, $d\phi$

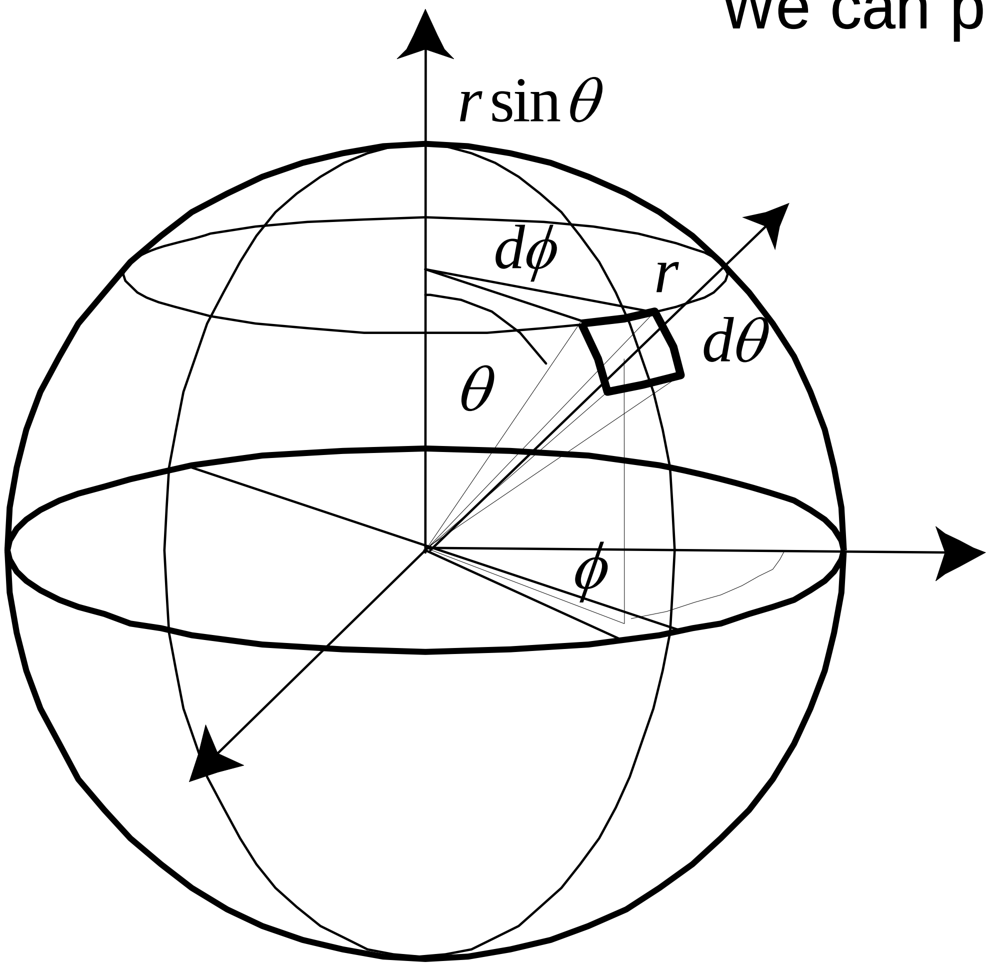
By definition, we know that

$$d\omega = dA \quad \text{when} \quad r = 1$$

$$dA = (r\,d\theta)(r\sin\theta\,d\phi)$$
$$= r^2\sin\theta\,d\theta\,d\phi$$

$$d\omega = \frac{dA}{r^2} = \sin\theta\,d\theta\,d\phi$$
$$= -d\cos\theta d\phi$$

Differential solid angles



We can prove that $\Omega = \int_{S^2} d\omega = 4\pi$

Differential solid angles

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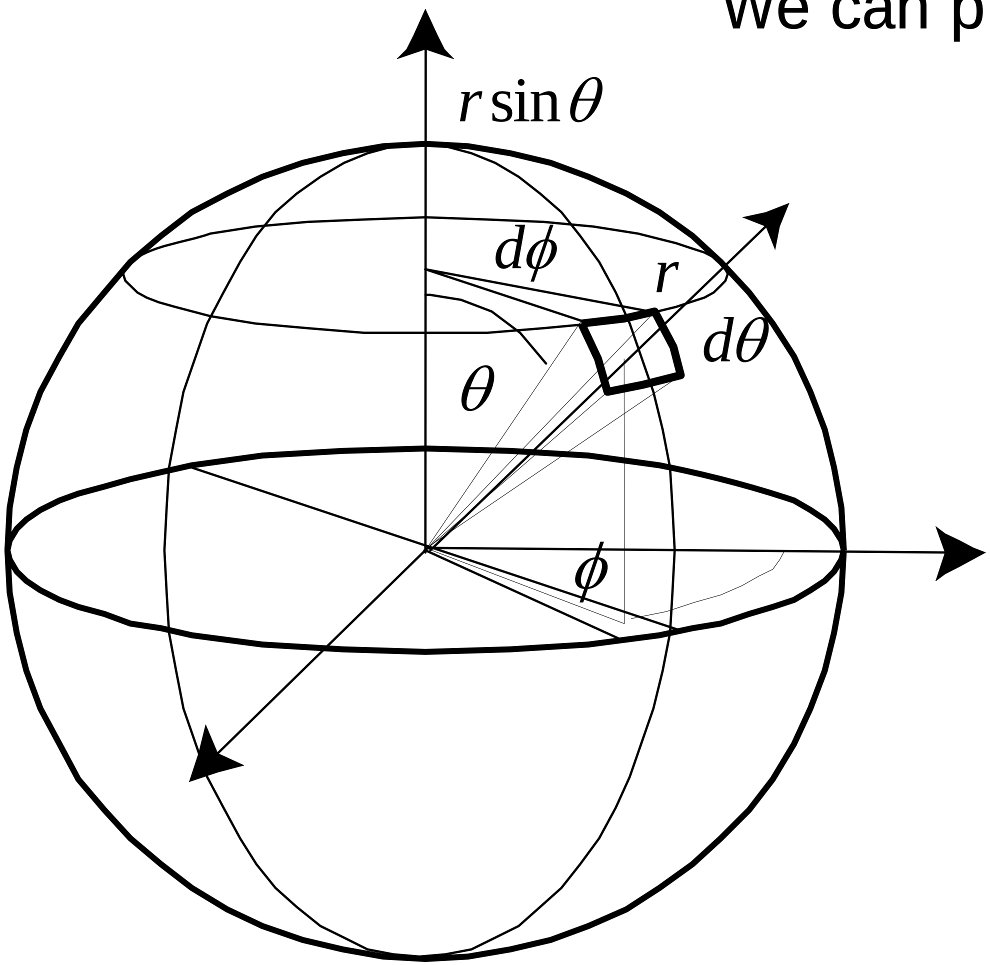
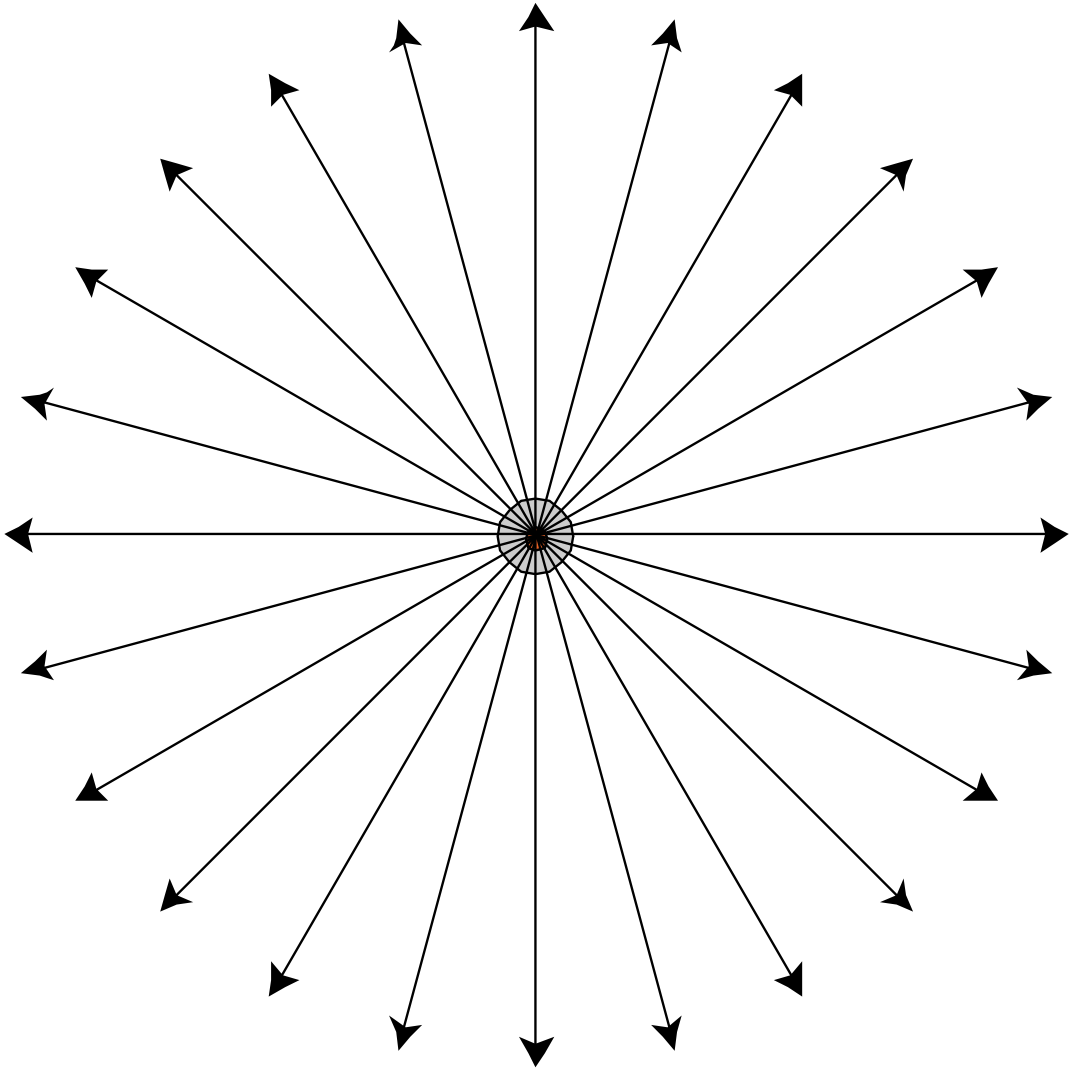


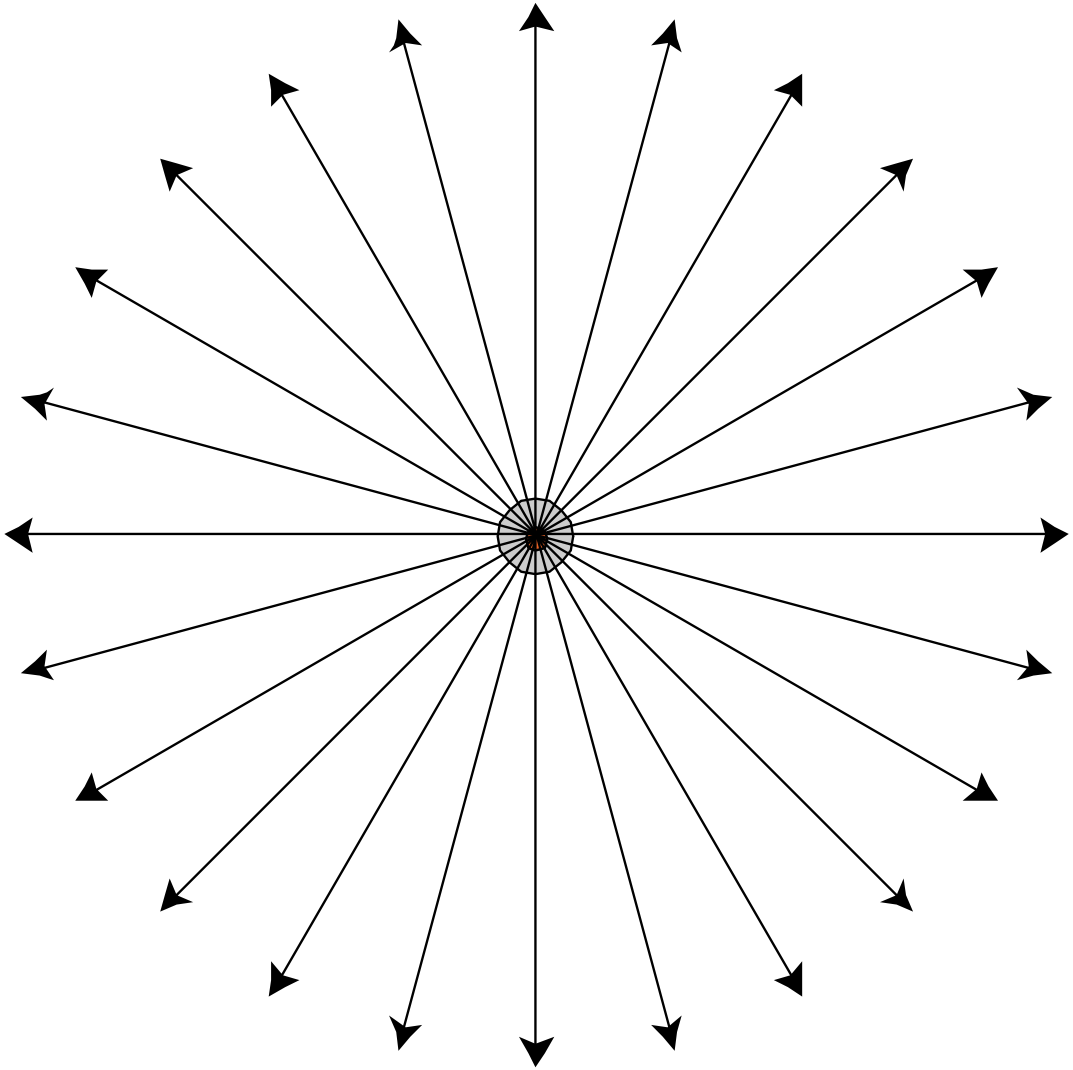
$$\begin{aligned}
\Omega &= \int_{S^2} d\omega \\
&= \int_0^{2\pi} \int_0^{\pi} \sin\theta \, d\theta \, d\phi \\
&= \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta \, d\theta \\
&= 2\pi \int_1^{-1} -d\cos\theta \\
&= 4\pi
\end{aligned}$$

Isotropic point source



If the total flux of the light source is Φ, what is the intensity?

Isotropic point source



If the total flux of the light source is Φ, what is the intensity?

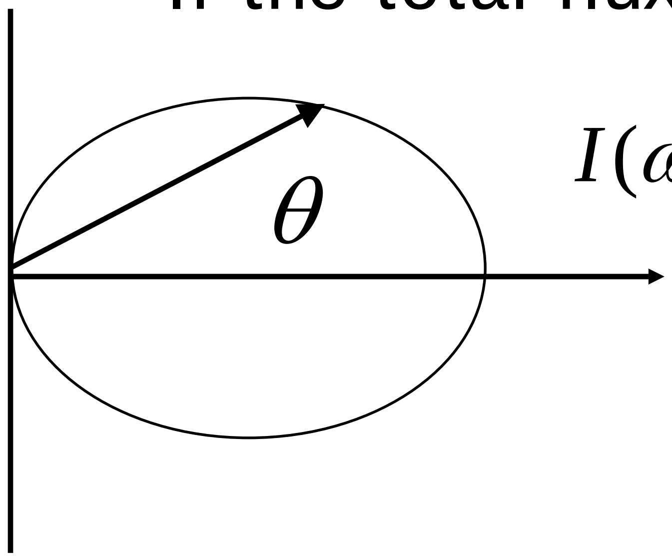
$$\Phi = \int_{S^2} I \, d\omega$$

$$= 4\pi I$$

$$I = \frac{\Phi}{4\pi}$$

Warn's spotlight

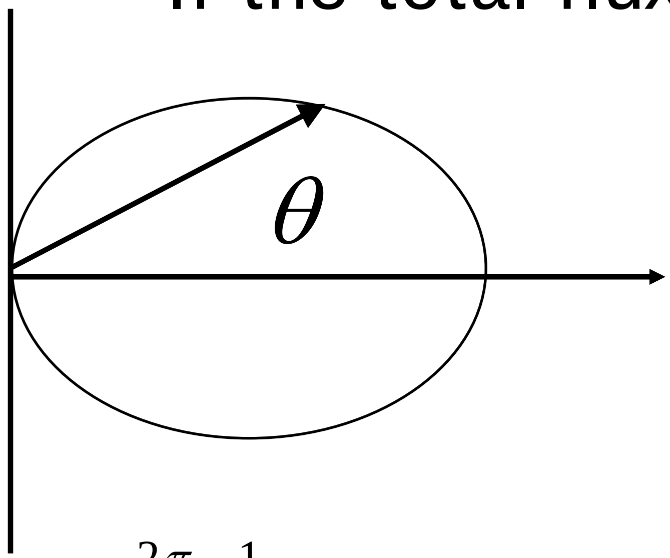
If the total flux is Φ, what is the intensity?



$$I(\omega) \propto \cos^{S}\theta$$

Warn's spotlight

If the total flux is Φ, what is the intensity?

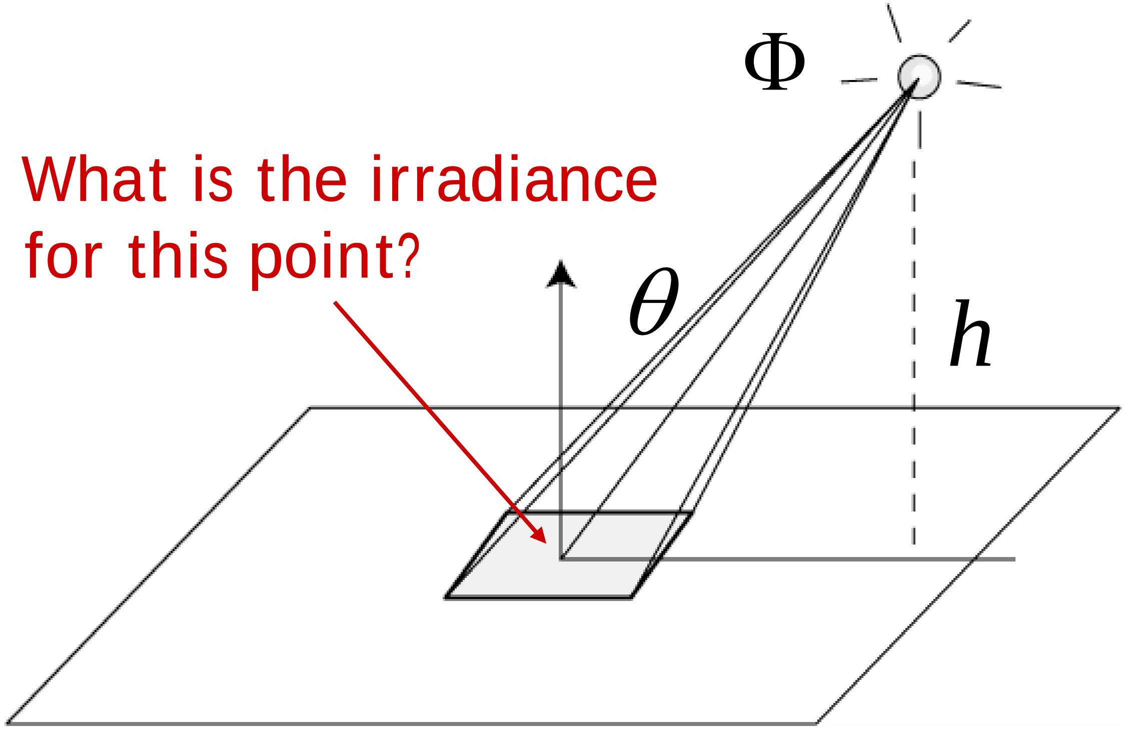


$$I(\omega)=\begin{cases} c\cos^S\theta & \theta \geq \pi/2 \\ 0 & otherwise \end{cases}$$

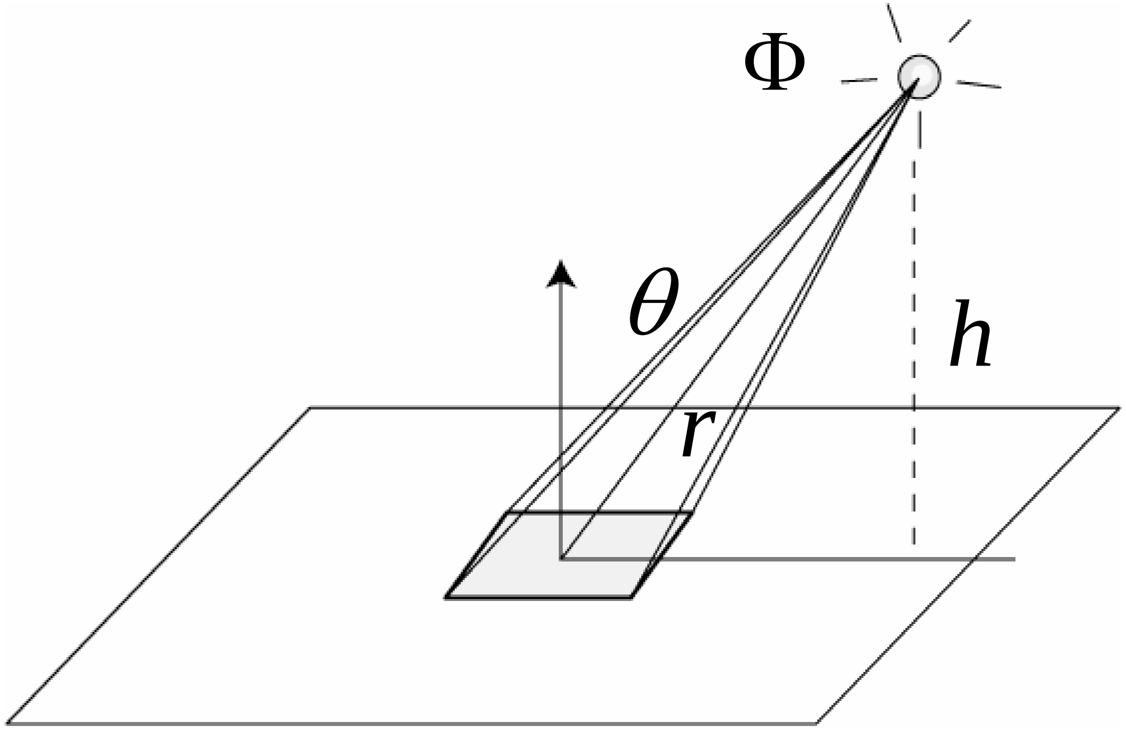
$$\Phi = c\int_0^{2\pi}\int_0^1 \cos^S\theta d\cos\theta d\phi = 2\pi c\int_0^1 \cos^S\theta d\cos\theta$$

$$= 2\pi c\left.\frac{y^{S+1}}{S+1}\right|_{y=0}^{y=1} = \frac{2\pi c}{S+1} \longrightarrow c = \frac{S+1}{2\pi}\Phi$$

Irradiance: isotropic point source



Irradiance: isotropic point source



$$r = \frac{h}{\cos\theta}$$

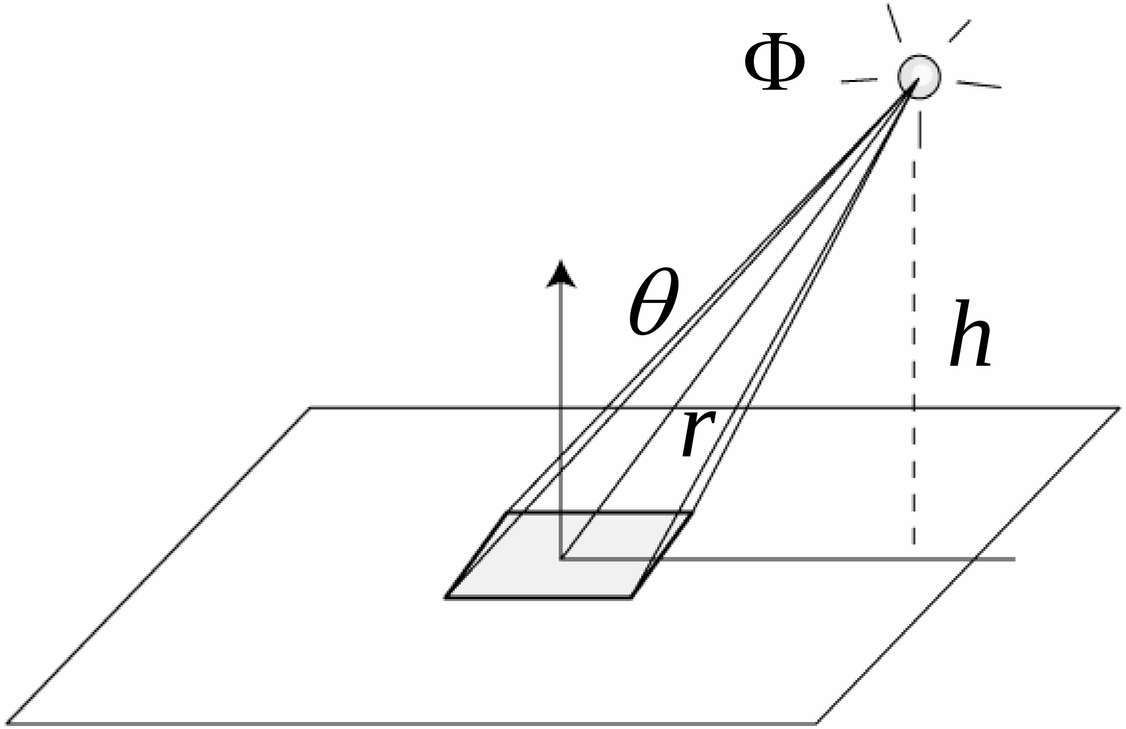
$$I = \frac{d\Phi}{d\omega} = \frac{\Phi}{4\pi}$$

Lambert law

$$E = \frac{d\Phi}{dA} = \frac{I d\omega}{dA} = \frac{\Phi}{4\pi}\frac{d\omega}{dA} = \frac{\Phi}{4\pi}\frac{\cos\theta}{r^2}$$

Inverse square law

Irradiance: isotropic point source



$$r = \frac{h}{\cos\theta}$$

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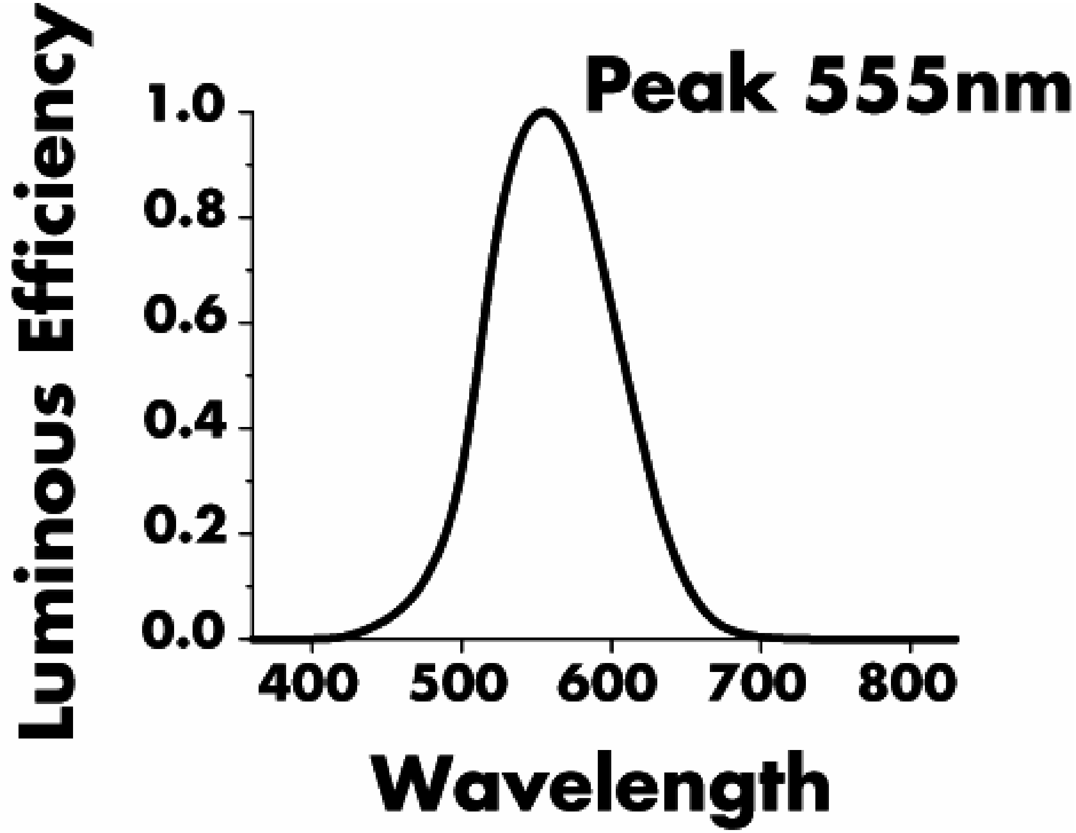
$$E = \frac{d\Phi}{dA} = \frac{I d\omega}{dA} = \frac{\Phi}{4\pi}\frac{d\omega}{dA} = \frac{\Phi}{4\pi}\frac{\cos\theta}{r^2} = \frac{\Phi}{4\pi}\frac{\cos^3\theta}{h^2}$$

Photometry



Luminance

$$Y = \int V(\lambda) L(\lambda) d\lambda$$