Shapes

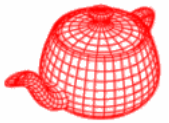
Digital Image Synthesis

Yung-Yu Chuang

10/05/2006

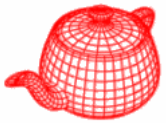
with slides by Pat Hanrahan

Announcements



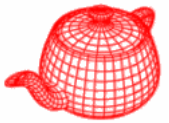
- 10/12's class is cancelled.
- 10/9 talks by Leif Kobbelt and Paul Debevec. 10:00am-12:00pm, Room 102.
- Assignment #1 will be handed out today. Due three weeks later.

Shapes



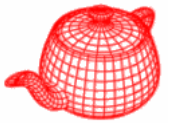
- One advantages of ray tracing is it can support various kinds of shapes as long as we can find ray-shape intersection.
- Careful abstraction of geometric shapes is a key component for a ray tracer. Ideal candidate for object-oriented design.
- All shape classes implement the same interface and the other parts of the ray tracer just use this interface without knowing the details about this shape.

Shapes



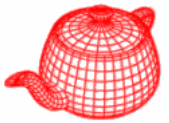
- **`Primitive=Shape+Material`**
- **`Shape`**: raw geometry properties of the primitive, implements interface such as surface area and bounding box.
- Source code in core/shape.* and shapes/*

Shapes



- `pbrt` provides the following shape plug-ins:
  - quadrics: sphere, cone, cylinder, disk, hyperboloid (雙曲面), paraboloid(拋物面) (surface described by quadratic polynomials in x, y, z)
  - triangle mesh
  - height field
  - NURBS
  - Loop subdivision surface
- Some possible extensions: other subdivision schemes, fractals, CSG, point-sampled geometry

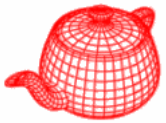
Shapes



- All shapes are defined in object coordinate space

```
class Shape : public ReferenceCounted {
public:
  <Shape Interface> all are virtual functions
  const Transform ObjectToWorld, WorldToObject;
  const bool reverseOrientation,
             transformSwapsHandedness;
}
```

Shape interface: bounding

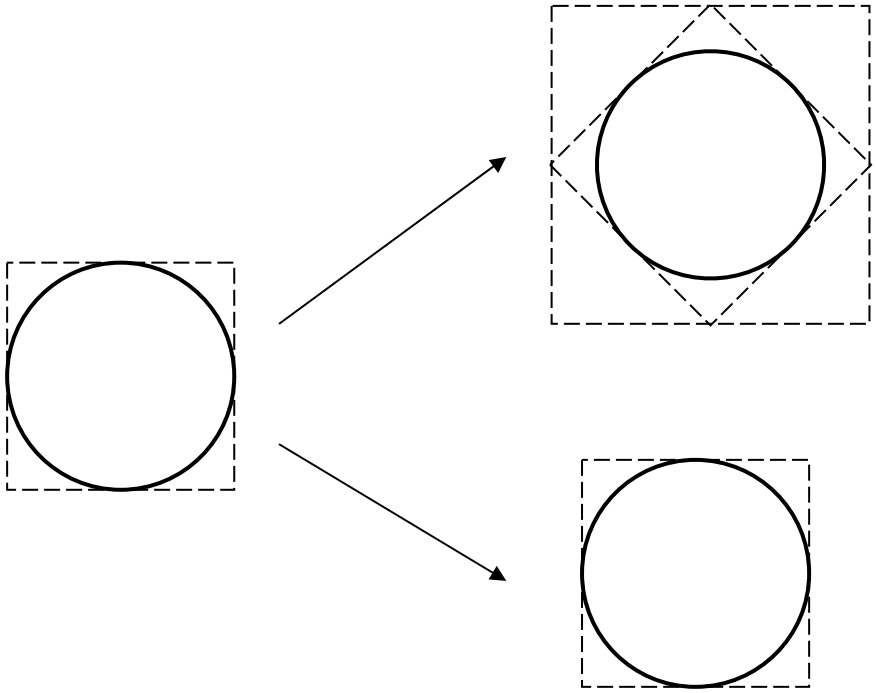


- `BBox ObjectBound() const=0;` *left to individual shape*
- `BBox WorldBound() {`

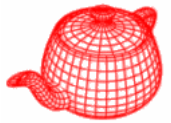
  default implementation; can be overridden

  `return ObjectToWorld(ObjectBound());`

  `}`



Shape interface: intersecting



- **`bool CanIntersect()`** *returns whether this shape can do intersection test; if not, the shape must provide*

  `void Refine(vector<Reference<Shape>>&refined)`

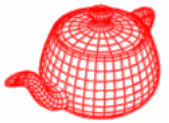
  examples include complex surfaces (which need to be tessellated first) or placeholders (which store geometry information in the disk)

in world space

- **`bool Intersect(const Ray &ray,`**
  `float *tHit, DifferentialGeometry *dg)`
- **`bool IntersectP(const Ray &ray)`**

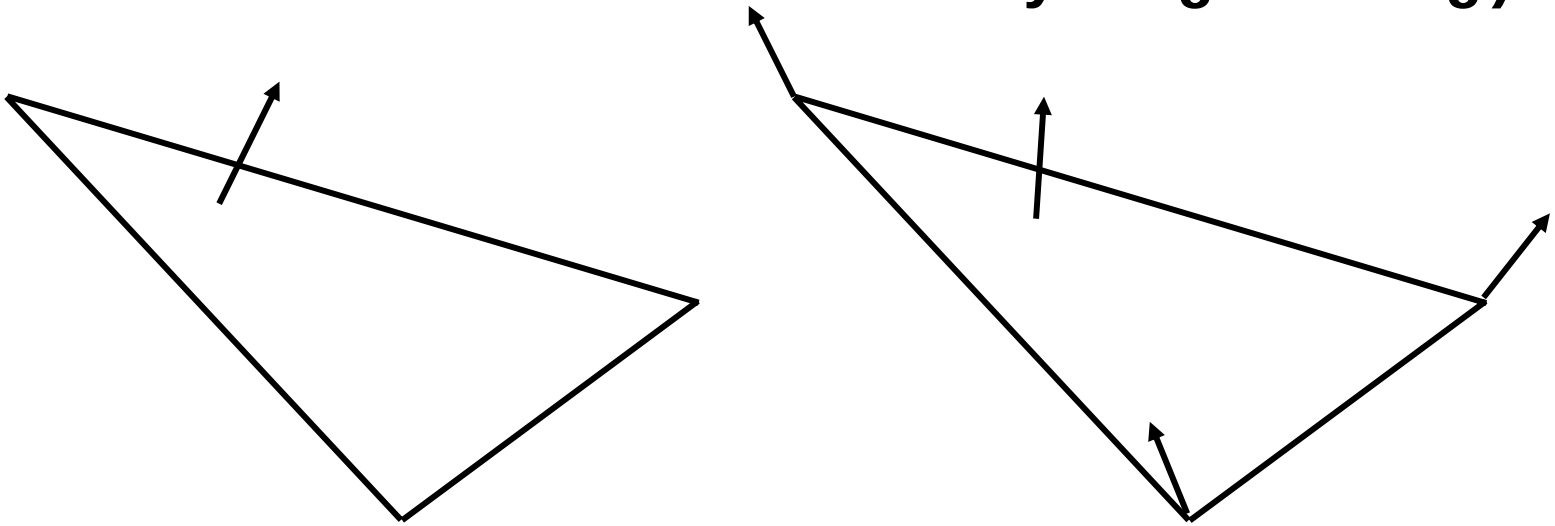
  not pure virtual functions so that non-intersectable shapes don't need to implement them; instead, a default implementation which prints error is provided.

Shape interface



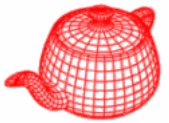
- `float Area()` *useful when using as a area light*
- `void GetShadingGeometry(` *for object instancing*

```
        const Transform &obj2world,
        const DifferentialGeometry &dg,
        DifferentialGeometry *dgShading)
```



- No back culling for that it doesn't save much for ray tracing and it is not physically correct

Surfaces



- Implicit: F(x,y,z)=0

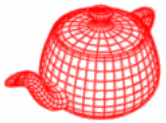
  you can check
- Explicit: (x(u,v),y(u,v),z(u,v))

  you can enumerate

  also called parametric
- Quadrics

$$\begin{bmatrix} x & y & z & 1 \end{bmatrix} \begin{bmatrix} A & B & C & D \\ B & E & F & G \\ C & F & H & I \\ D & G & I & J \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = 0$$

Sphere



- A sphere of radius r at the origin
- Implicit: $x^2+y^2+z^2-r^2=0$
- Parametric: $f(\theta, \phi)$

  $x=r\sin\theta\cos\phi$

  $y=r\sin\theta\sin\phi$

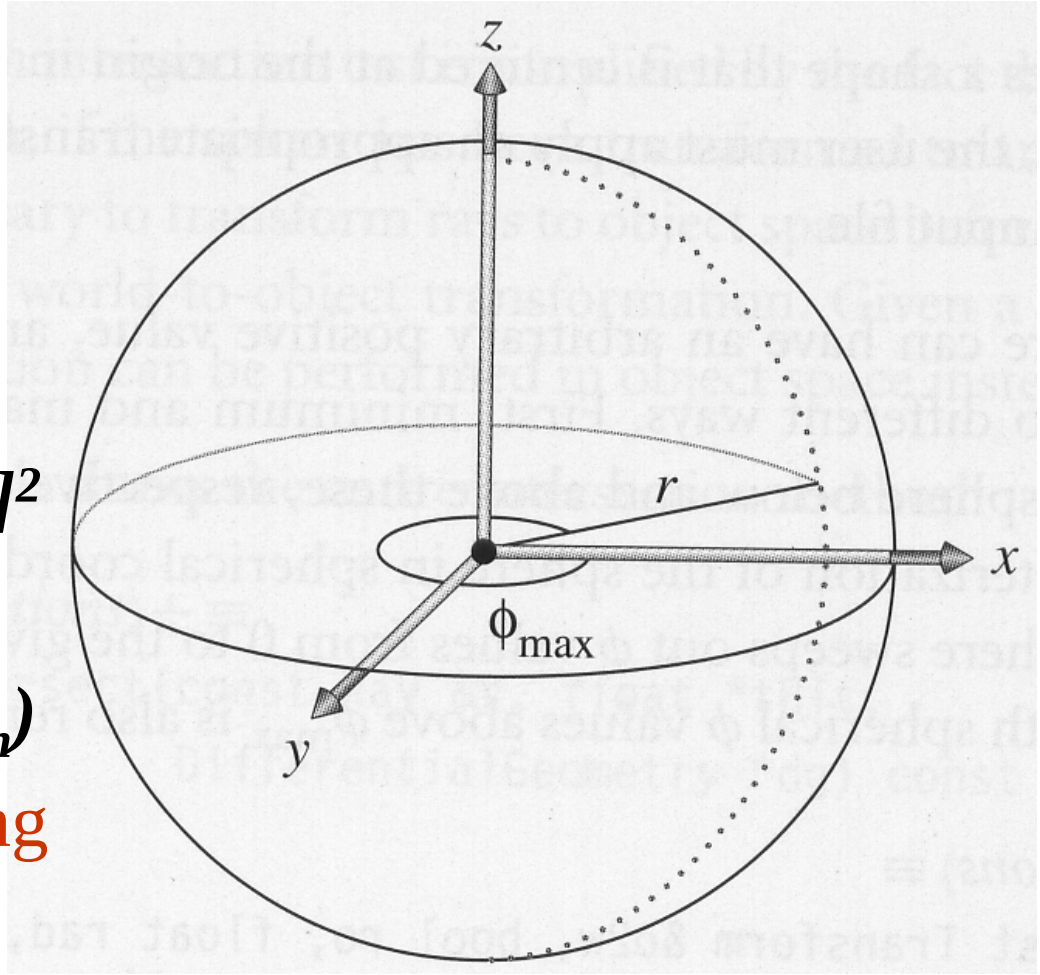
  $z=r\cos\theta$

mapping $f(u,v)$ over $[0,1]^2$

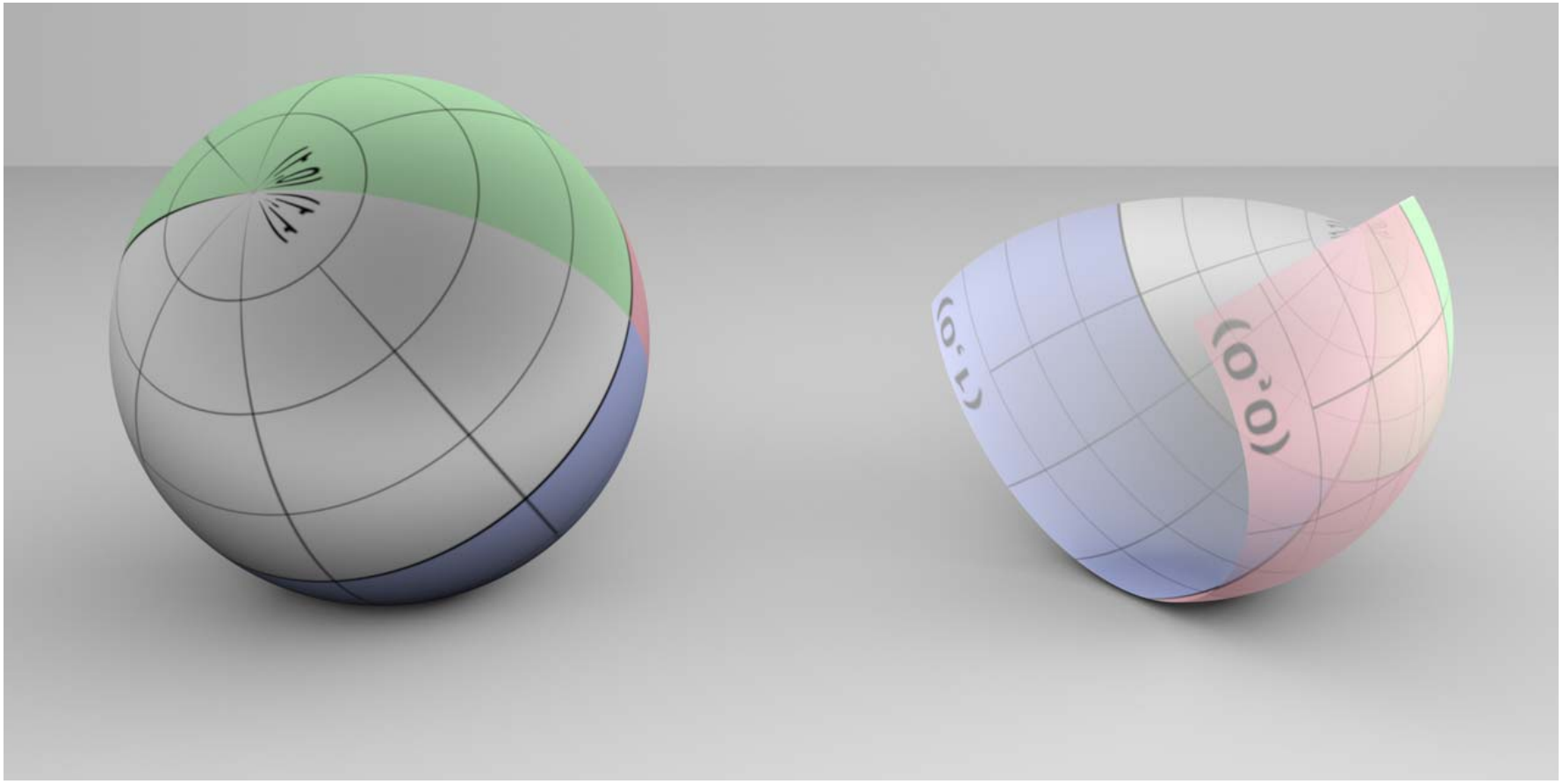
$\phi=u\,\phi_{max}$

$\theta=\theta_{min}+v(\theta_{max}-\theta_{min})$

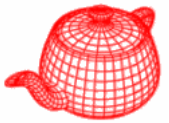
useful for texture mapping



Sphere



Sphere (construction)



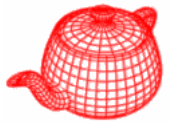
```
class Sphere: public Shape {
……
private:
   float radius;
   float phiMax;
   float zmin, zmax;
   float thetaMin, thetaMax;
}
```

thetas are derived from z

```
 Sphere(const Transform &o2w, bool ro,
        float rad, float zmin, float zmax,
        float phiMax);
```

- Bounding box for sphere, only z clipping

Intersection (algebraic solution)



- Perform in object space, **`WorldToObject(r, &ray)`**
- Assume that ray is normalized for a while

$$x^2 + y^2 + z^2 = r^2$$

$$(o_x + td_x)^2 + (o_y + td_y)^2 + (o_z + td_z)^2 = r^2$$

$$At^2 + Bt + C = 0$$

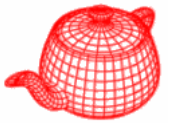
Step 1

$$A = d_x^2 + d_y^2 + d_z^2$$

$$B = 2(d_x o_x + d_y o_y + d_z o_z)$$

$$C = o_x^2 + o_y^2 + o_z^2 - r^2$$

Algebraic solution



$$t_0 = \frac{-B - \sqrt{B^2 - 4AC}}{2A} \qquad t_1 = \frac{-B + \sqrt{B^2 - 4AC}}{2A}$$

Step 2

If (B^2-4AC<0) then the ray misses the sphere

Step 3

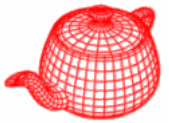
Calculate t_0 and test if t_0<0

Step 4

Calculate t_1 and test if t_1<0

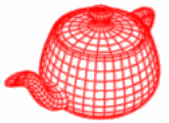
check the real source code in sphere.cpp

Quadric (in pbrt.h)



```
inline bool Quadratic(float A, float B, float C,
                      float *t0, float *t1) {
  // Find quadratic discriminant
  float discrim = B * B - 4.f * A * C;
  if (discrim < 0.) return false;
  float rootDiscrim = sqrtf(discrim);
  // Compute quadratic _t_ values
  float q;
  if (B < 0) q = -.5f * (B - rootDiscrim);
  else       q = -.5f * (B + rootDiscrim);
  *t0 = q / A;
  *t1 = C / q;
  if (*t0 > *t1) swap(*t0, *t1);
  return true;
}
```

Why?



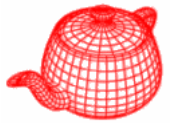
- Cancellation error: devastating loss of precision when small numbers are computed from large numbers by addition or subtraction.

```
double x1 = 10.000000000000004;
double x2 = 10.000000000000000;
double y1 = 10.000000000000004;
double y2 = 10.000000000000000;
double z = (y1 - y2) / (x1 - x2); // 11.5
```

$$t_0 = \frac{q}{A} \qquad t_1 = \frac{C}{q}$$

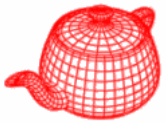
$$q = \begin{cases} \frac{1}{2}\left(B - \sqrt{B^2 - 4AC}\right) & \text{if } B < 0 \\ \frac{1}{2}\left(B + \sqrt{B^2 - 4AC}\right) & \text{otherwise} \end{cases}$$

Range checking

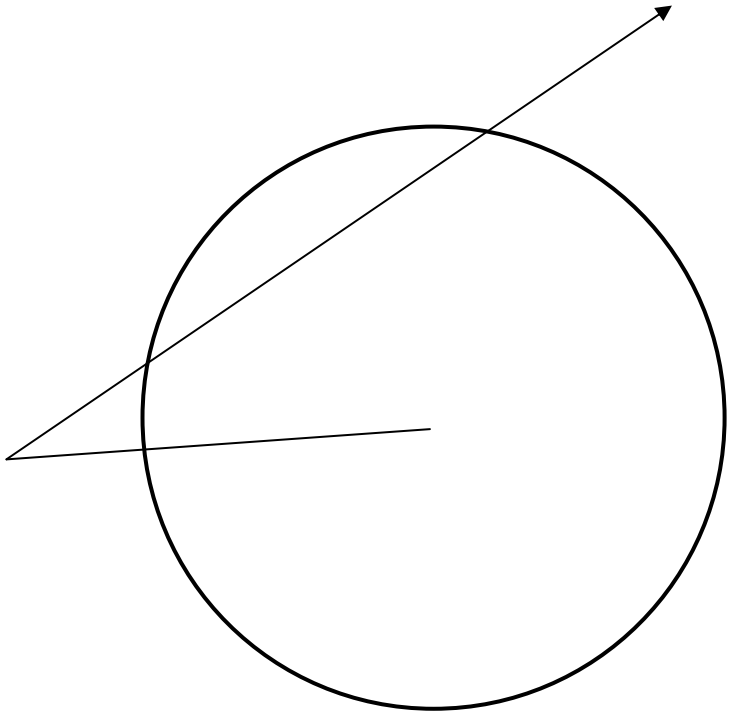


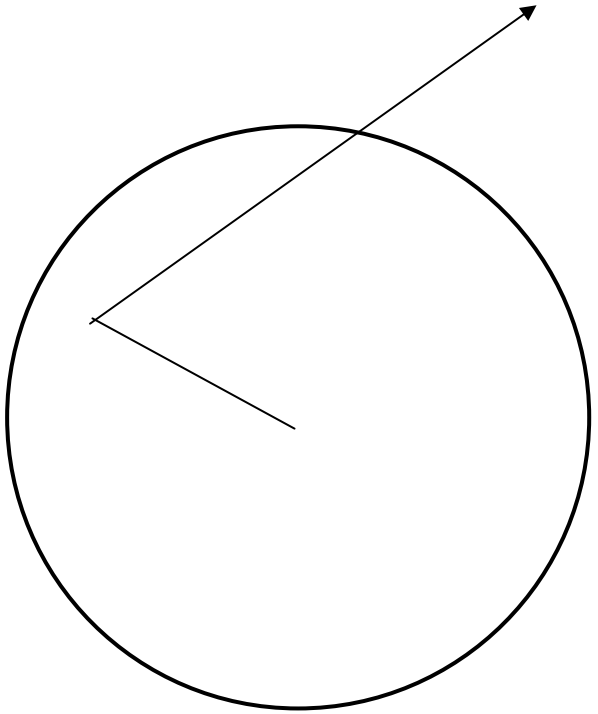
```
if (t0 > ray.maxt || t1 < ray.mint) return false;
float thit = t0;
if (t0 < ray.mint) {
    thit = t1;
    if (thit > ray.maxt) return false;
}
...
phit = ray(thit);
phi = atan2f(phit.y, phit.x);
if (phi < 0.) phi += 2.f*M_PI;
// Test sphere intersection against clipping
parameters
if (phit.z < zmin || phit.z > zmax || phi > phiMax)
{
  ...
}
```

Geometric solution

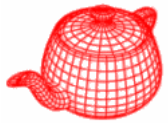


1. Origin inside? $o_x^2 + o_y^2 + o_z^2 > r^2$



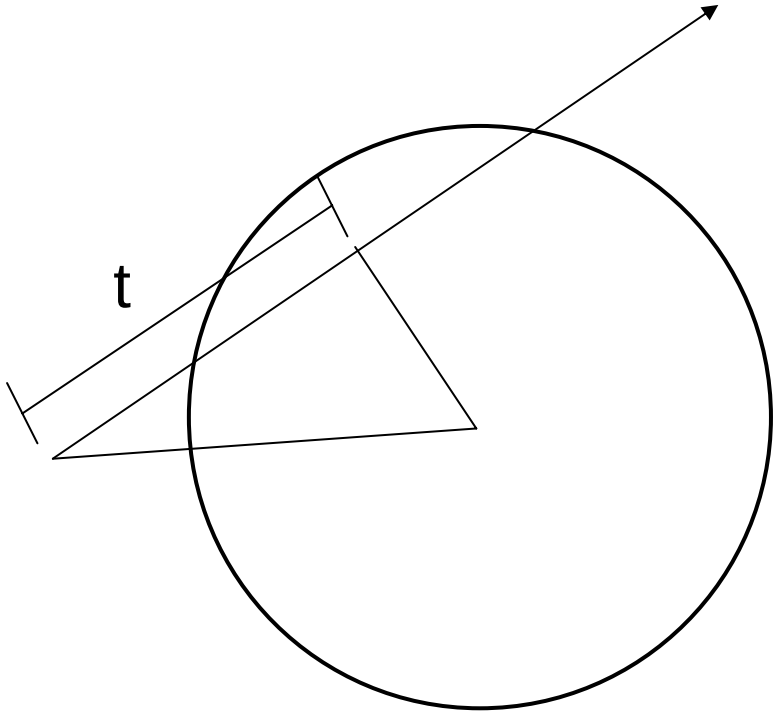


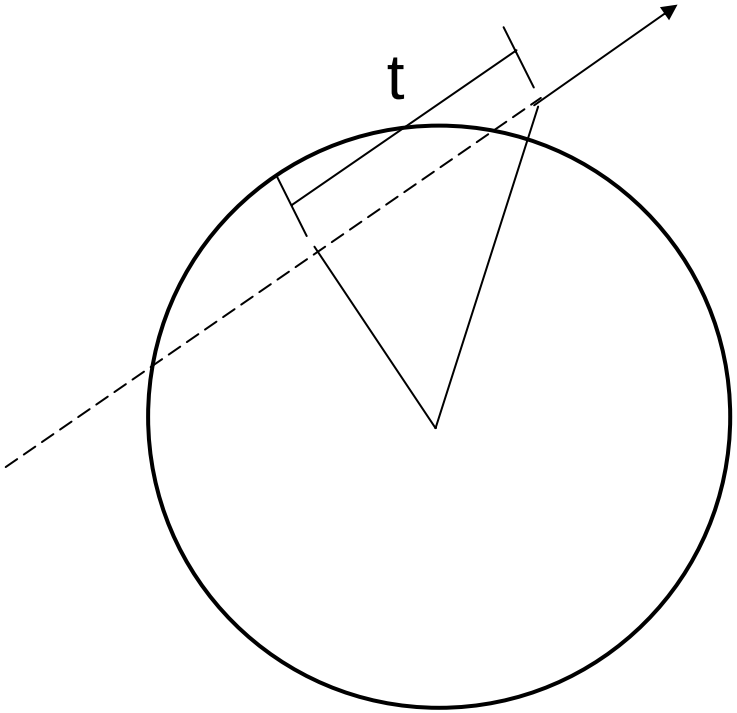
Geometric solution



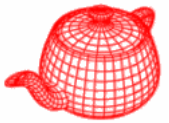
2. find the closest point, t=-O • D D is normalized

   if t<0 and O outside return false



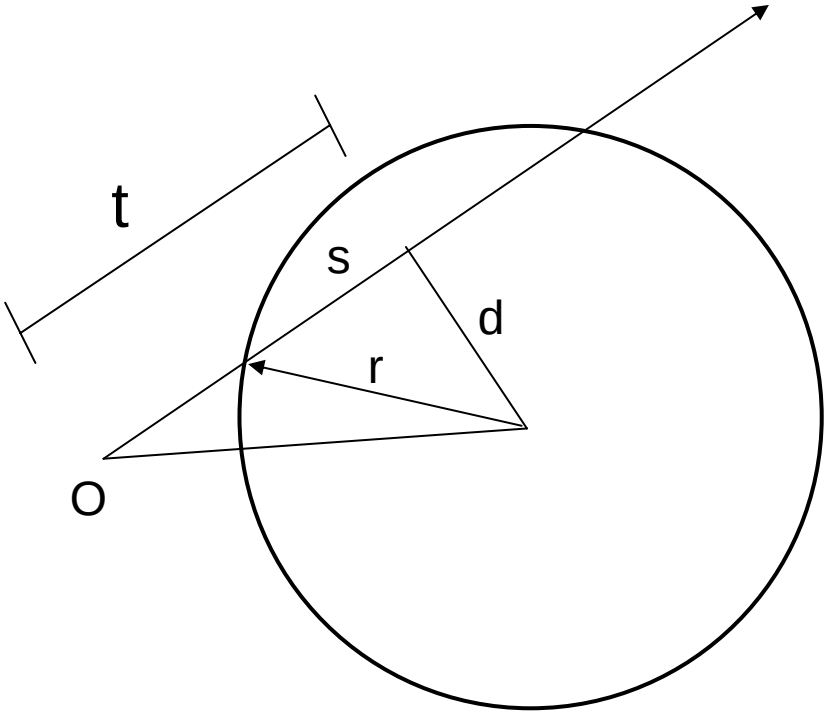


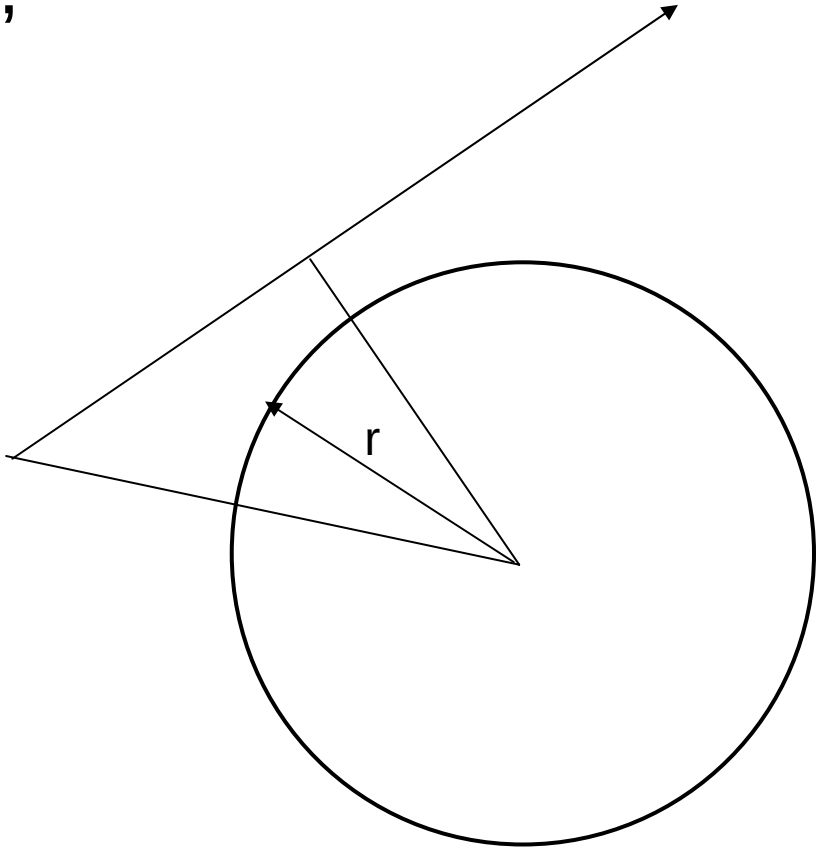
Geometric solution



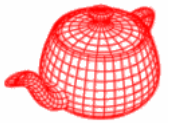
3. find the distance to the origin, $d^2=O^2-t^2$

   if $s^2=r^2-d^2<0$ return false;





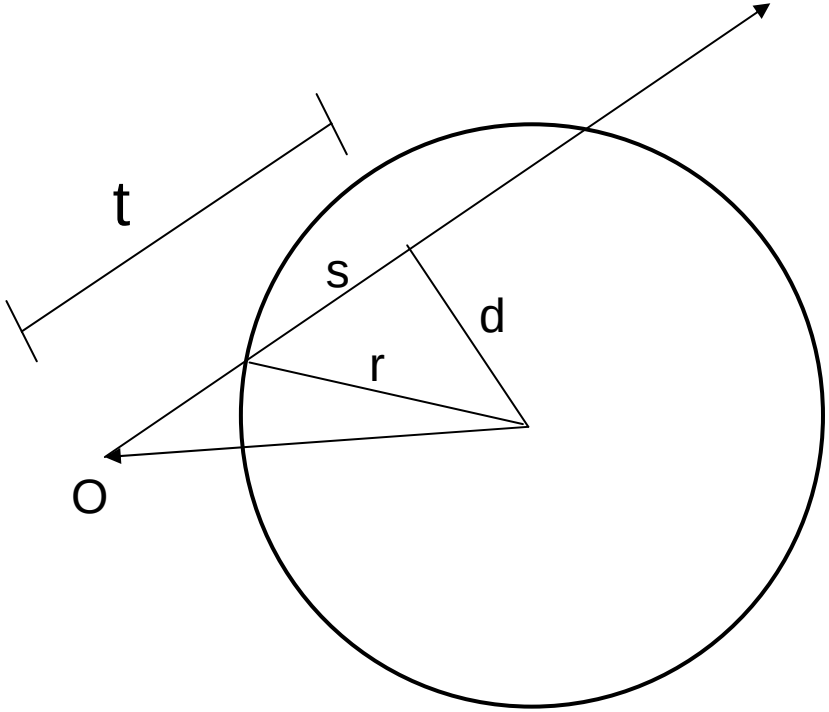
Geometric solution

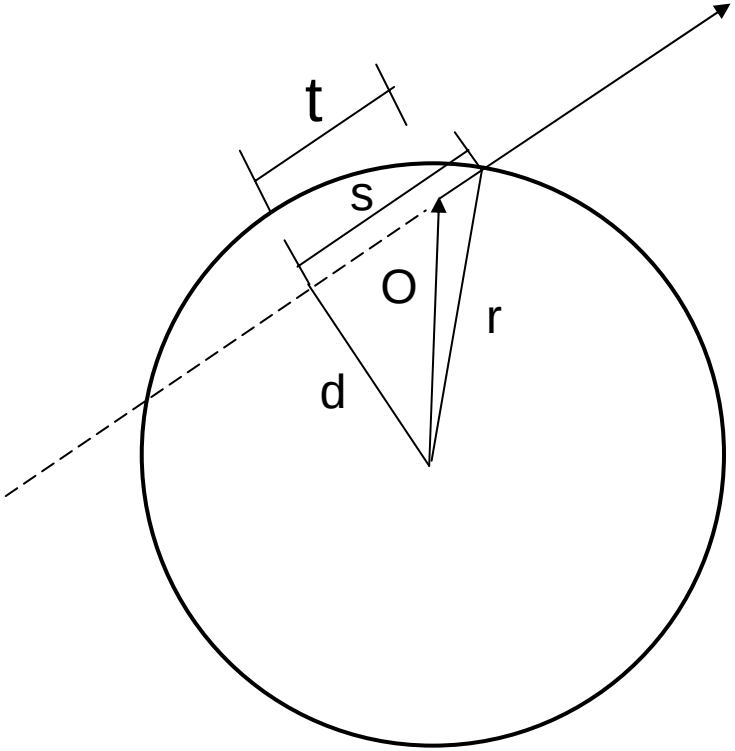


4. calculate intersection distance,

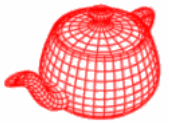
   if (origin outside) then t-s

   else t+s



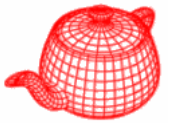


Sphere



- Have to test sphere intersection against clipping parameters
- Fill in information for **`DifferentialGeometry`**
  - Position
  - Parameterization (u,v)
  - Parametric derivatives
  - Surface normal
  - Derivatives of normals
  - Pointer to shape

Partial sphere



$u = \phi / \phi_{max}$

$v = (\theta - \theta_{min}) / (\theta_{max} - \theta_{min})$

- Partial derivatives (pp103 of textbook)

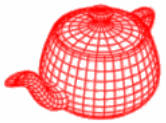
$$\frac{\partial p}{\partial u} = (-\phi_{\max} y, \phi_{\max} x, 0)$$

$$\frac{\partial p}{\partial v} = (\theta_{\max} - \theta_{\min})(z \cos\phi, z \sin\phi, -r \sin\theta)$$

- Area (pp107)

$$A = \phi_{\max} r (z_{\max} - z_{\min})$$

Cylinder

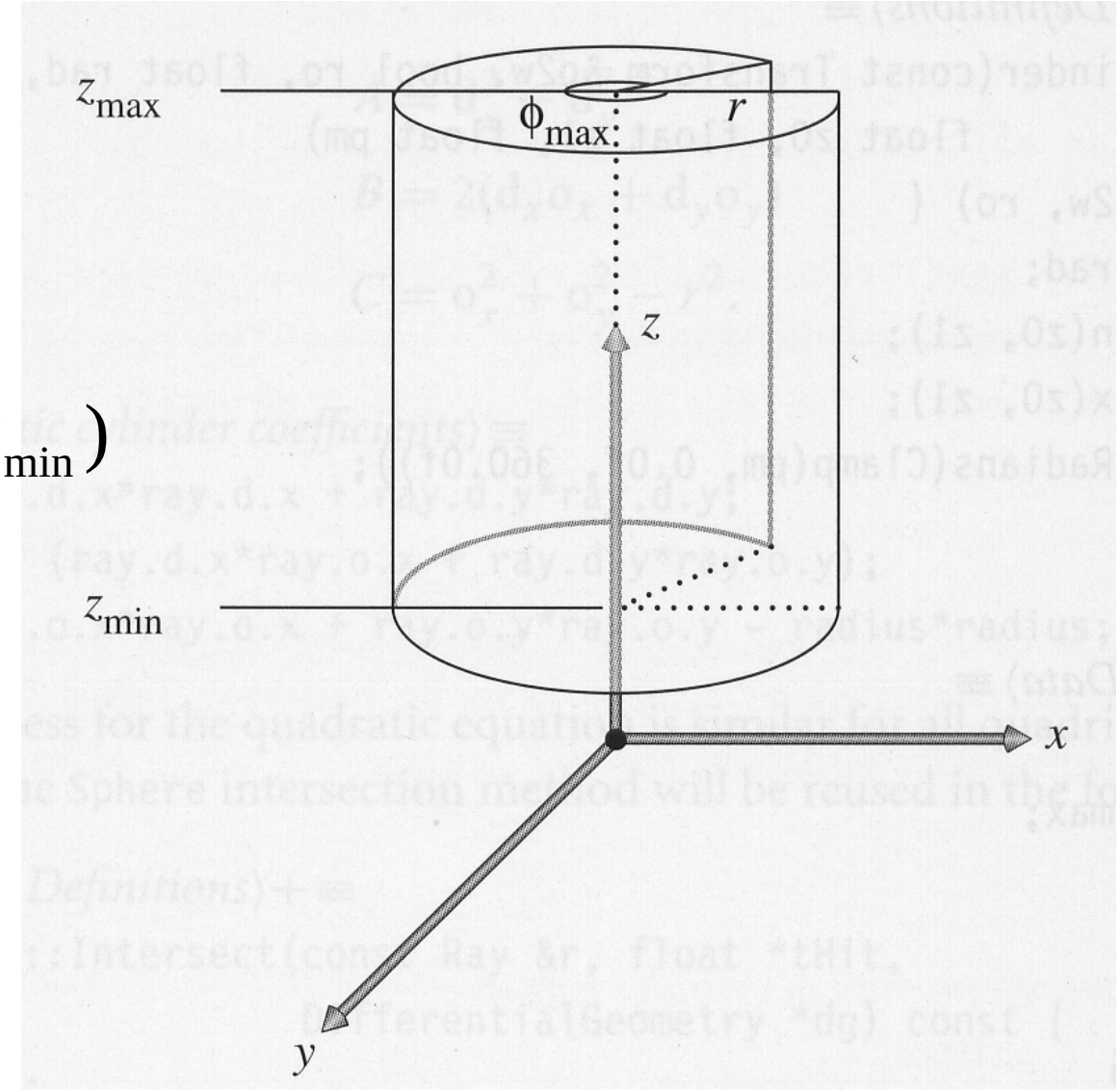


$$\phi = u\phi_{\max}$$

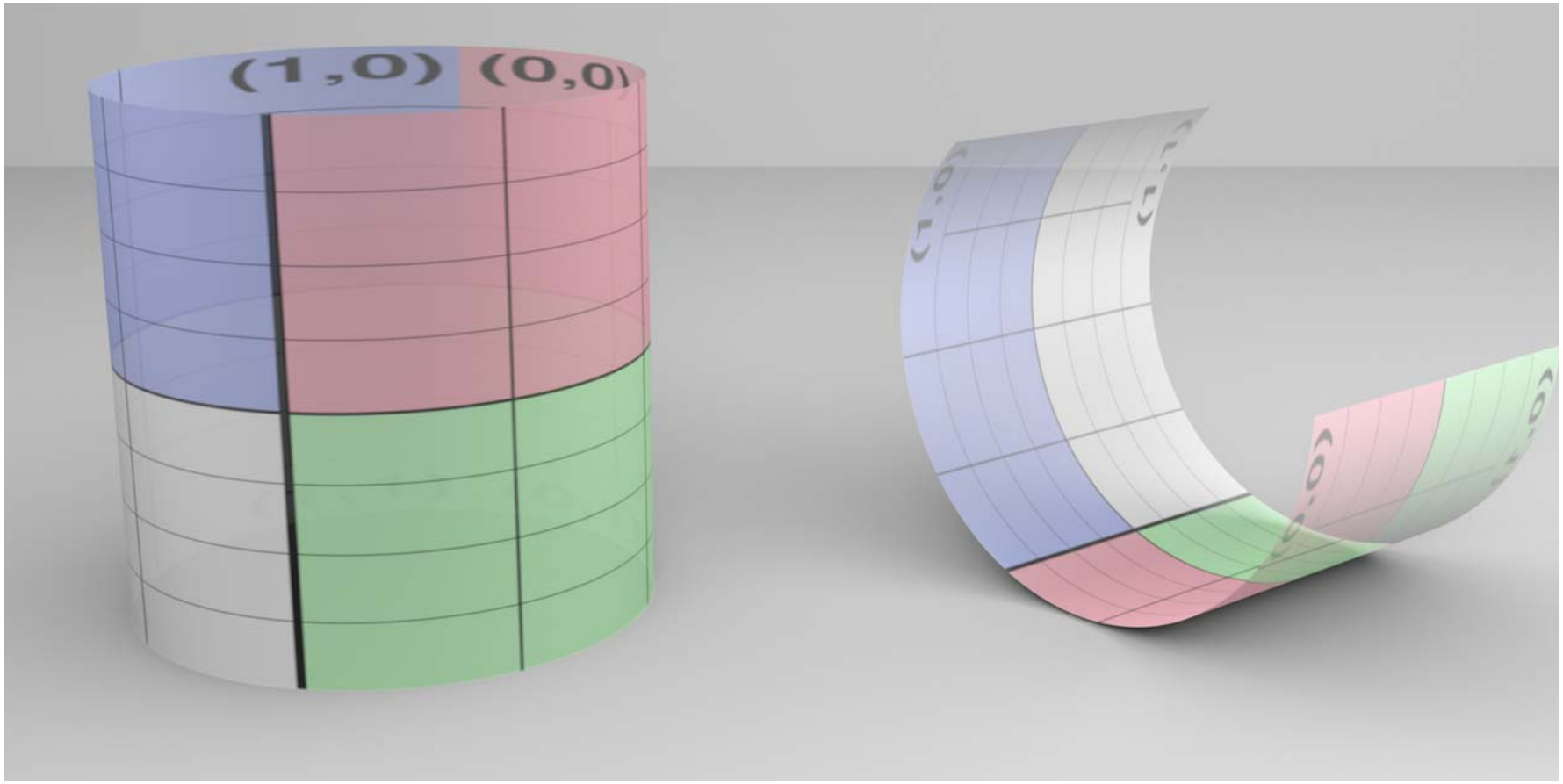
$$x = r\cos\phi$$

$$y = r\sin\phi$$

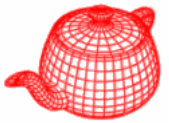
$$z = z_{\min} + v(z_{\max} - z_{\min})$$



Cylinder



Cylinder (intersection)



$$x^2 + y^2 = r^2$$

$$(o_x + td_x)^2 + (o_y + td_y)^2 = r^2$$

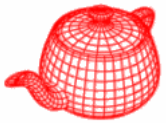
$$At^2 + Bt + C = 0$$

$$A = d_x^2 + d_y^2$$

$$B = 2(d_x o_x + d_y o_y)$$

$$C = o_x^2 + o_y^2 - r^2$$

Disk

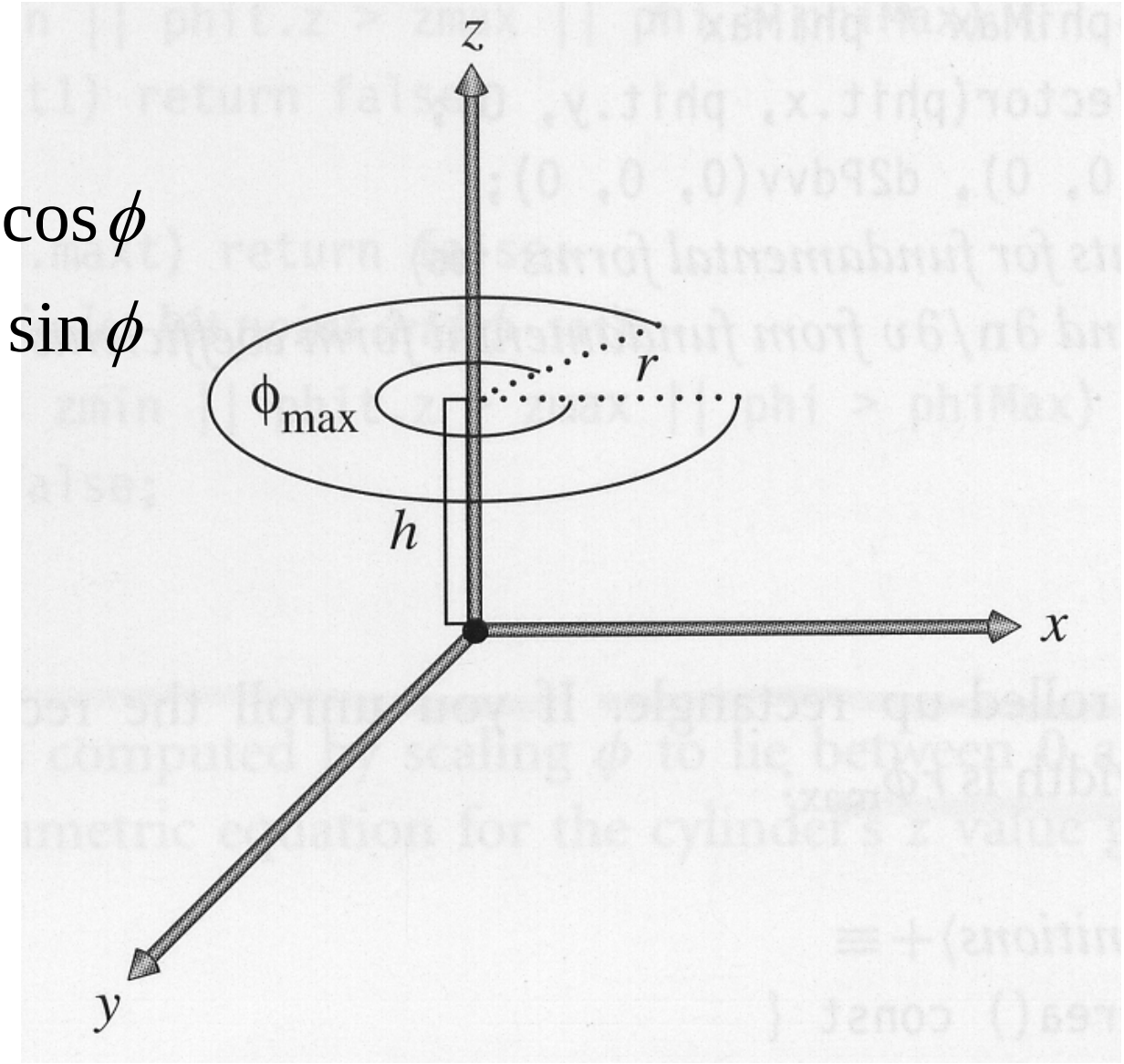


$$\phi = u\phi_{\max}$$

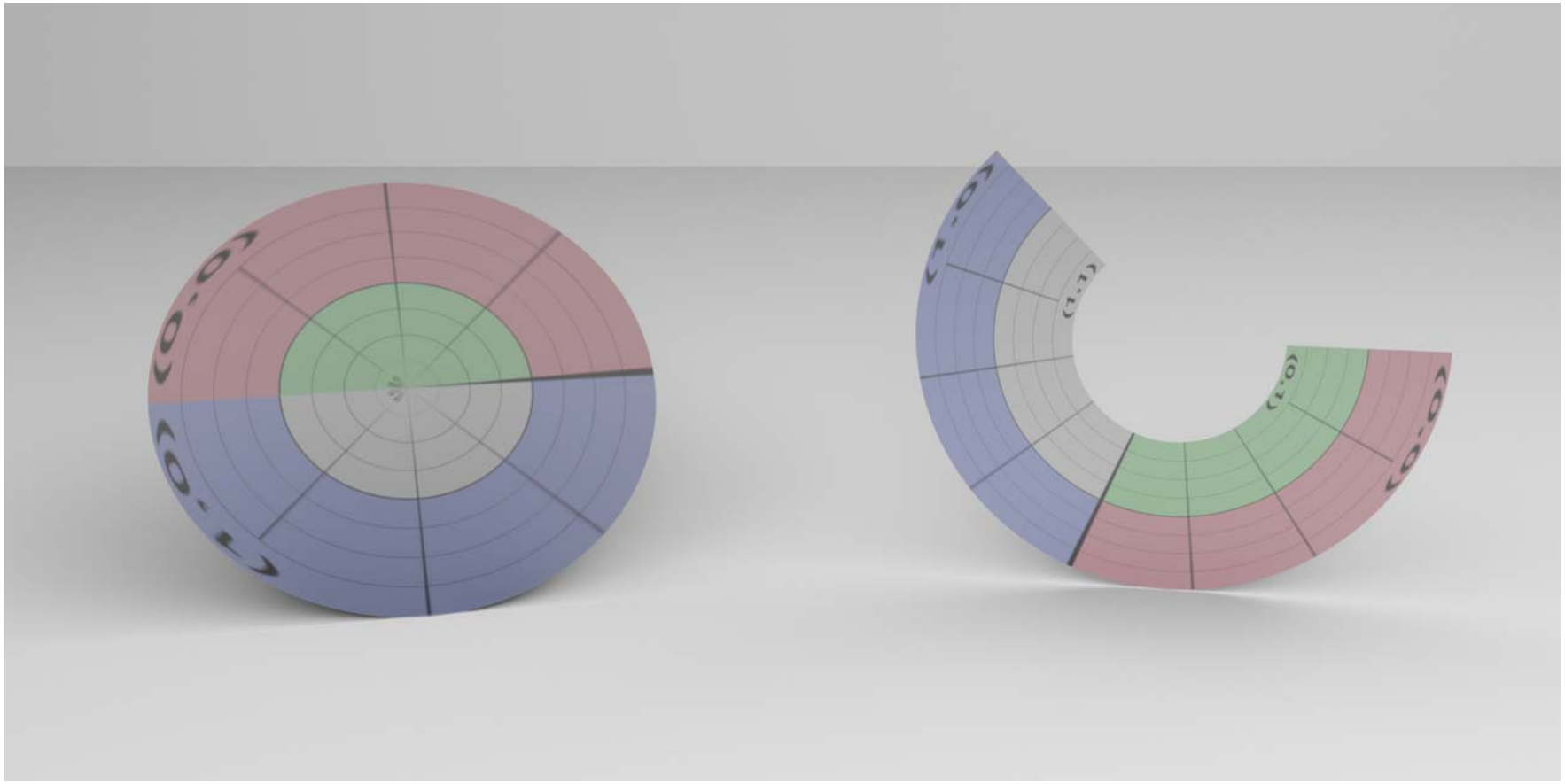
$$x = ((1-v)r_i + vr)\cos\phi$$

$$y = ((1-v)r_i + vr)\sin\phi$$

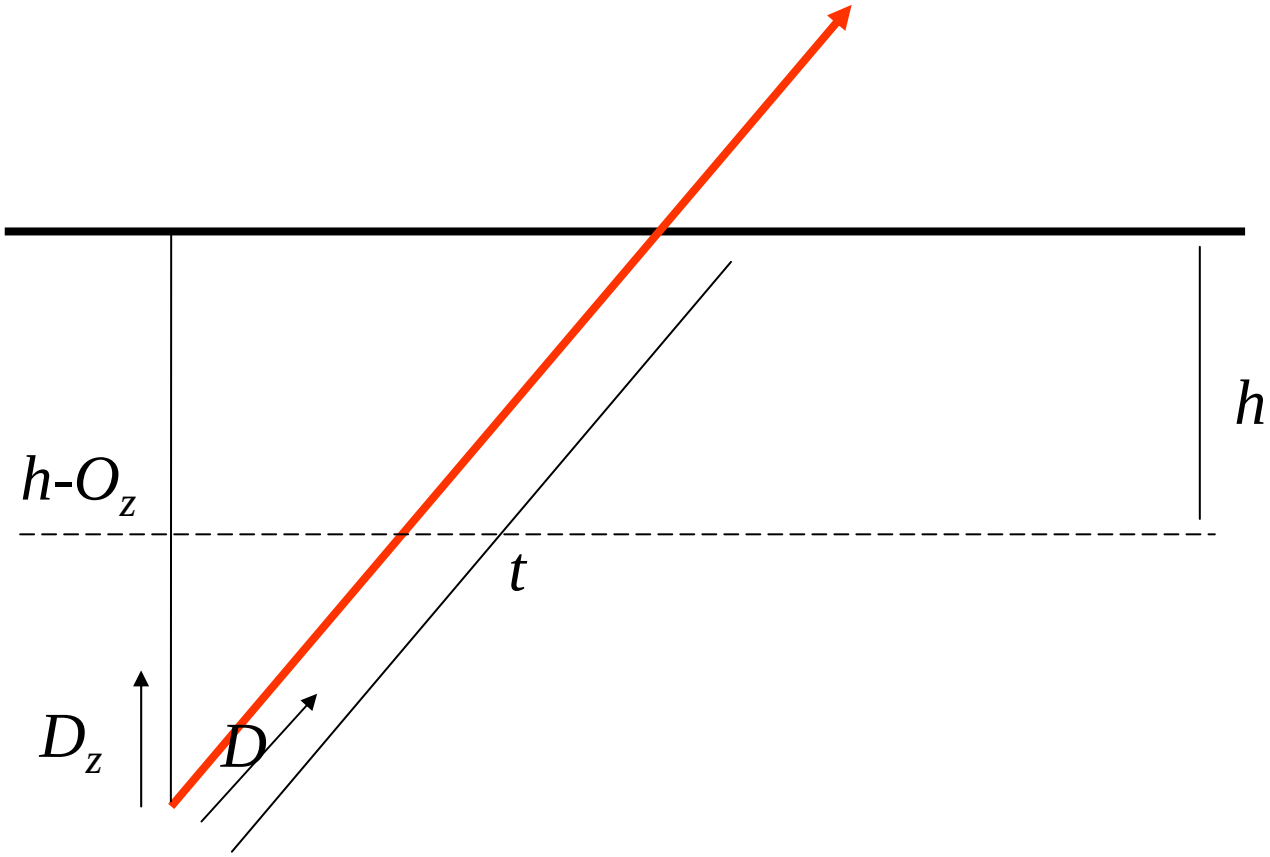
$$z = h$$



Disk



Disk (intersection)

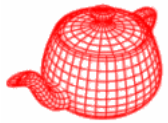


$$\frac{|D|}{D_z} = \frac{t}{h - O_z}$$

$$t = \frac{|D|}{D_z}(h - O_z)$$

$$t' = \frac{t}{|D|} = \frac{h - O_z}{D_z}$$

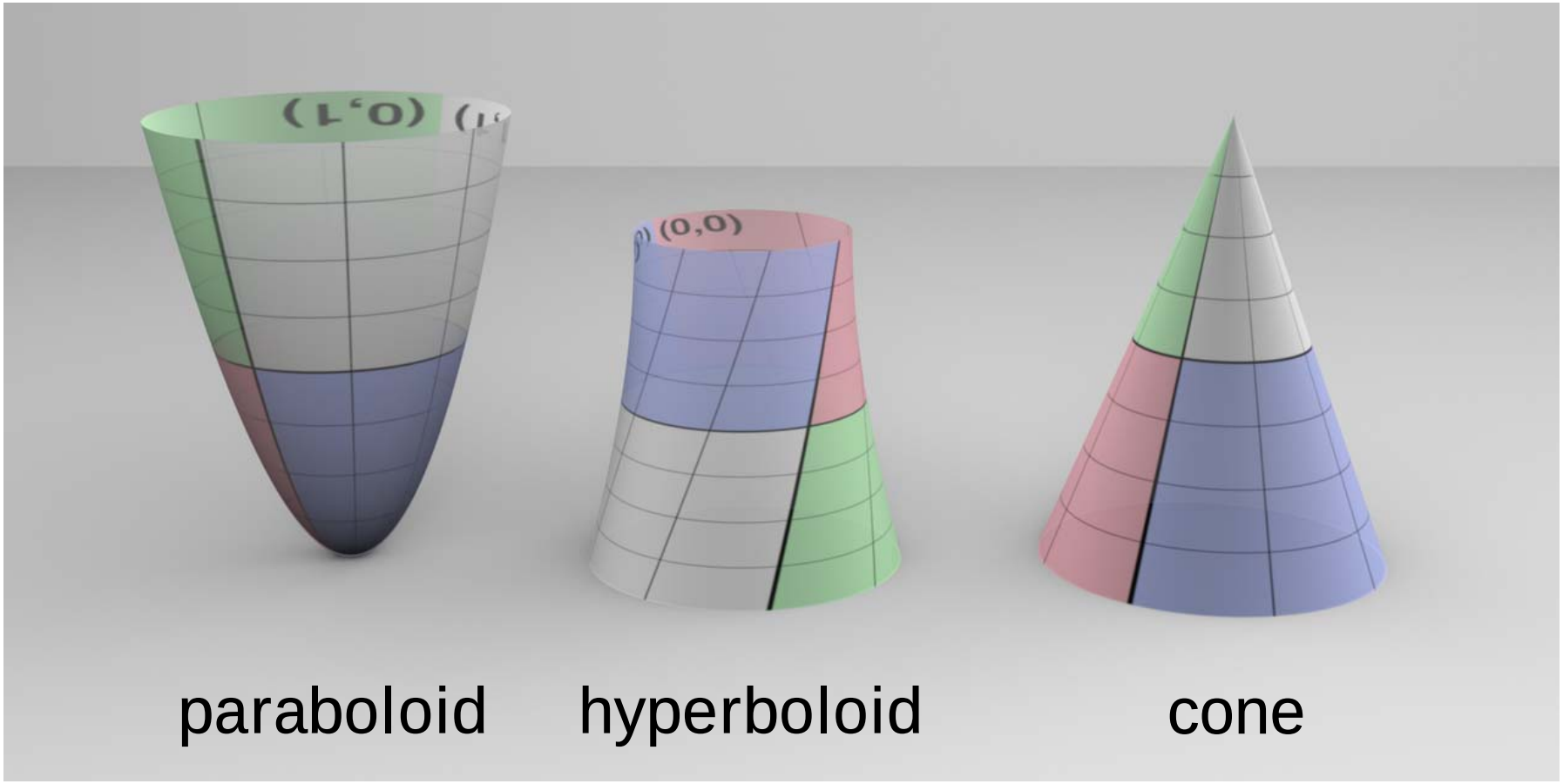
Other quadrics



$$\frac{hx^2}{r^2}+\frac{hy^2}{r^2}-z=0$$

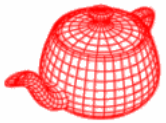
$$x^2+y^2-z^2=-1$$

$$\left(\frac{hx}{r}\right)^2+\left(\frac{hy}{r}\right)^2-(z-h)^2=0$$

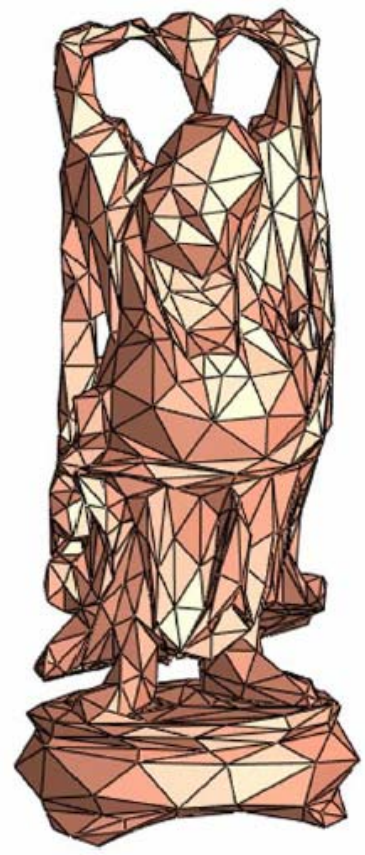


paraboloid hyperboloid cone

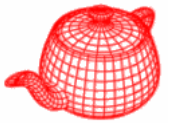
Triangle mesh



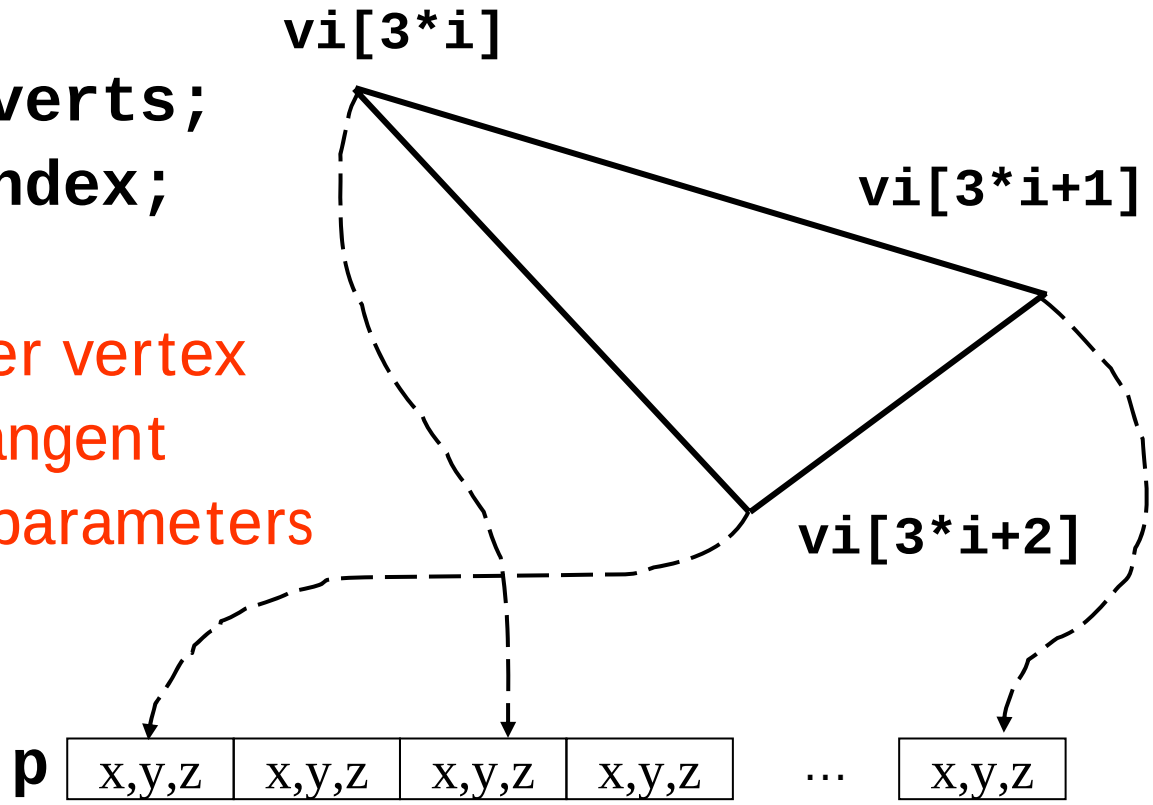
The most commonly used shape. In `pbrt`, it can be supplied by users or tessellated from other shapes.



Triangle mesh

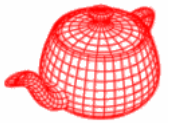


```
class TriangleMesh : public Shape {
…
  int ntris, nverts;
  int *vertexIndex;
  Point *p;
  Normal *n; per vertex
  Vector *s; tangent
  float *uvs; parameters
}
```



Note that **p** is stored in world space to save transformations. **n** and **s** are in object space.

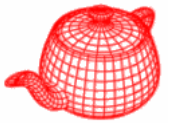
Triangle mesh



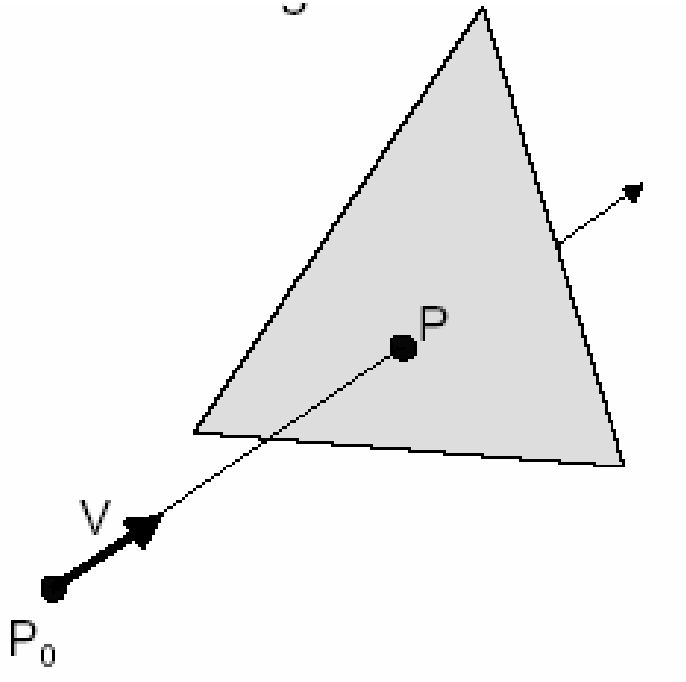
Pbrt calls Refine() when it encounters a shape that is not intersectable.

```
Void TriangleMesh::Refine(vector<Reference<Shape>>
                          &refined)
{
  for (int i = 0; i < ntris; ++i)
    refined.push_back(new Triangle(ObjectToWorld,
    reverseOrientation, (TriangleMesh *)this, i));
}
```

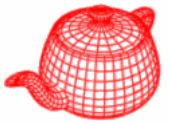
Ray triangle intersection



1. Intersect ray with plane
2. Check if point is inside triangle



Ray plane intersection



$$Ray : P = P_0 + tV$$

$$Plane : P \cdot N + d = 0$$

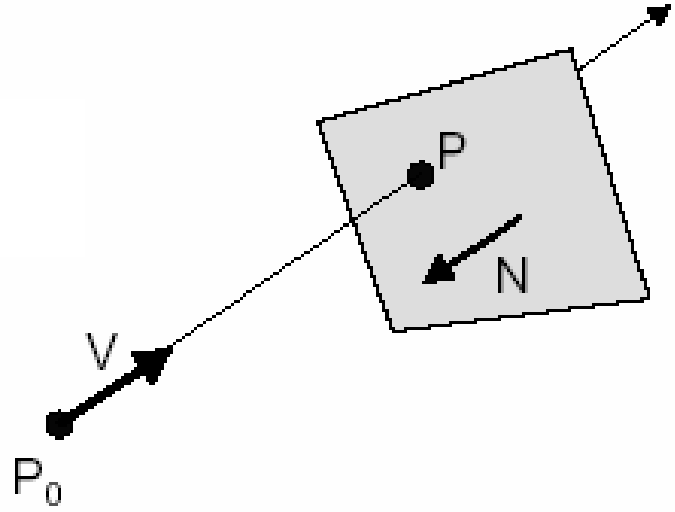
Algebraic Method

Substituting for P, we get:

$$(P_0 + tV) \cdot N + d = 0$$

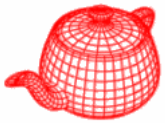
Solution:

$$t = \frac{-(P_0 \cdot N + d)}{(V \cdot N)}$$



$$P = P_0 + tV$$

Ray triangle intersection I



Algebraic Method

For each side of triangle:

$$V_1 = T_1 - P_0$$

$$V_2 = T_2 - P_0$$

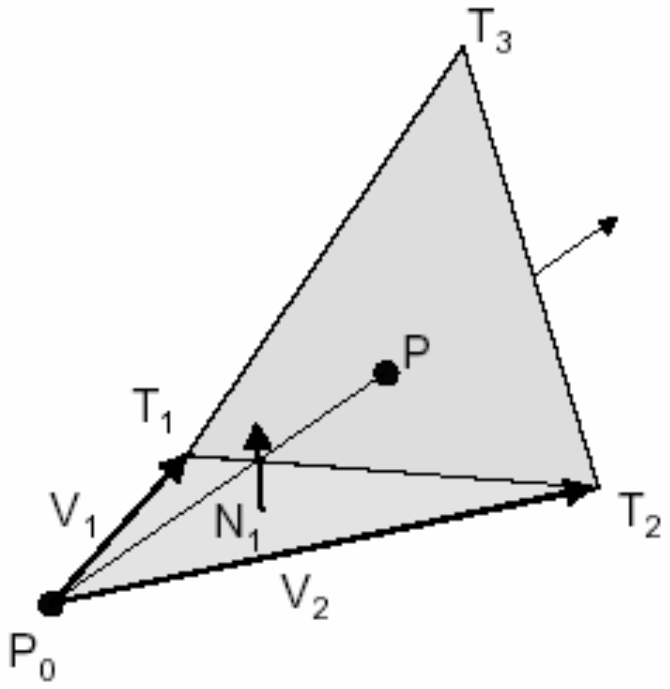
$$N_1 = V_1 \times V_2$$

$$Normalize\ N_1$$

$$d_1 = -P_0 \cdot N_1$$

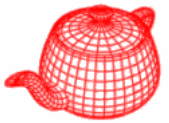
$$if\ \left(\left(P \cdot N_1 + d_1\right) < 0\right)$$

$$return\ FALSE$$



end

Ray triangle intersection II



Parametric Method

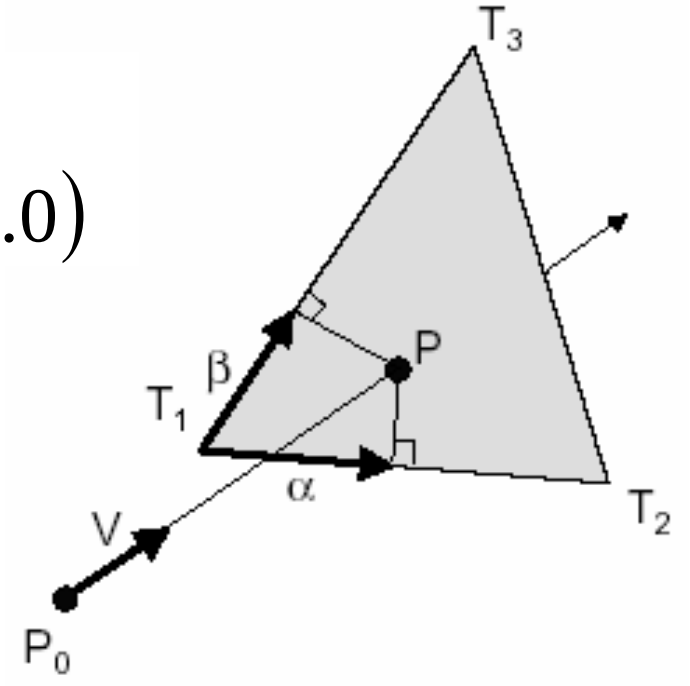
Compute α, β :

$$P = \alpha(T_2 - T_1) + \beta(T_3 - T_1)$$

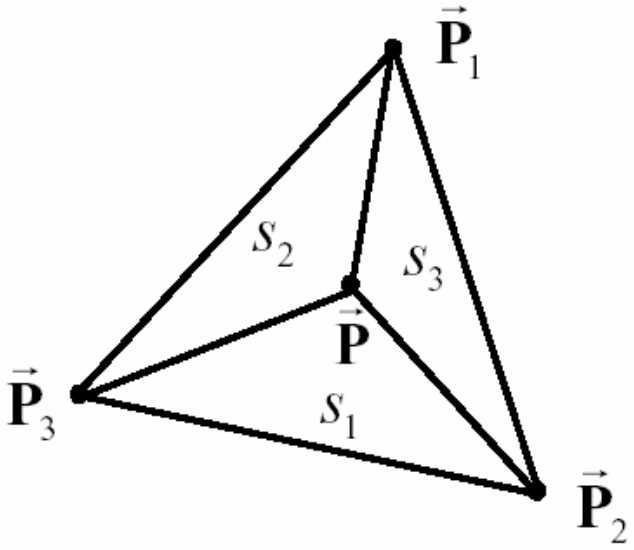
if $(0.0 \le \alpha \le 1.0)$ *and* $(0.0 \le \beta \le 1.0)$

and $(\alpha + \beta \le 1.0)$

then P *is inside triangle*



Ray triangle intersection III



$$s_1 = \text{area}(\Delta \mathbf{P}\mathbf{P}_2\mathbf{P}_3)$$

$$s_2 = \text{area}(\Delta \mathbf{P}\mathbf{P}_3\mathbf{P}_1)$$

$$s_3 = \text{area}(\Delta \mathbf{P}\mathbf{P}_1\mathbf{P}_2)$$

Barycentric coordinates

$$\vec{\mathbf{P}} = s_1\vec{\mathbf{P}}_1 + s_2\vec{\mathbf{P}}_2 + s_3\vec{\mathbf{P}}_3$$

Inside criteria

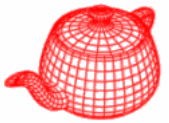
$$0 \le s_1 \le 1$$

$$0 \le s_2 \le 1$$

$$0 \le s_3 \le 1$$

$$s_1 + s_2 + s_3 = 1$$

Fast minimum storage intersection

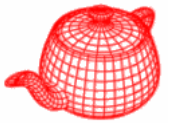


$$O + tD = (1-u-v)V_0 + uV_1 + vV_2$$

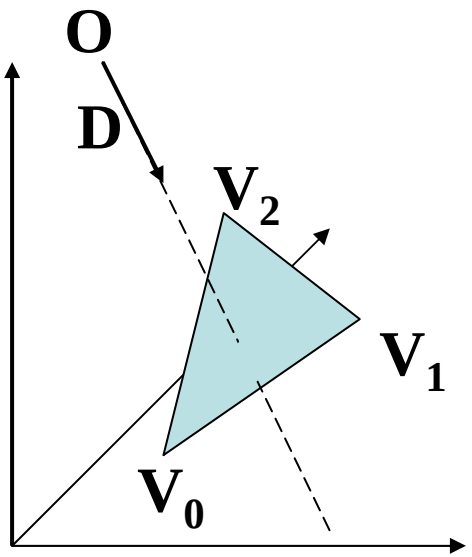
$$u, v \geq 0 \text{ and } u + v \leq 1$$

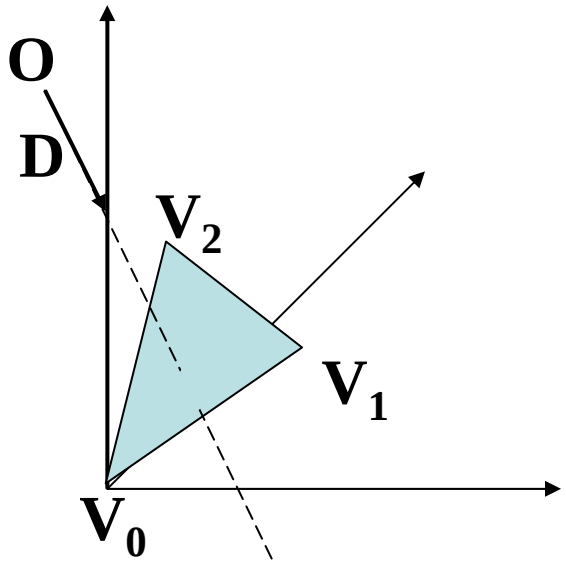
$$\begin{bmatrix} -D & V_1 - V_0 & V_2 - V_0 \end{bmatrix} \begin{bmatrix} t \\ u \\ v \end{bmatrix} = O - V_0$$

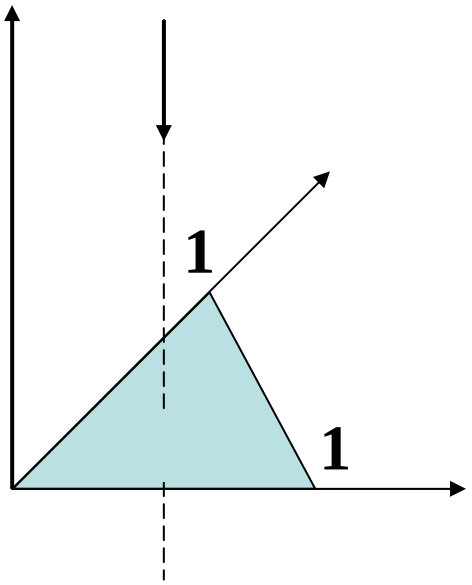
Fast minimum storage intersection



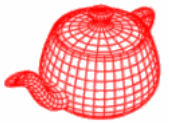
$$\begin{bmatrix} -D & V_1 - V_0 & V_2 - V_0 \end{bmatrix} \begin{bmatrix} t \\ u \\ v \end{bmatrix} = O - V_0$$







Fast minimum storage intersection

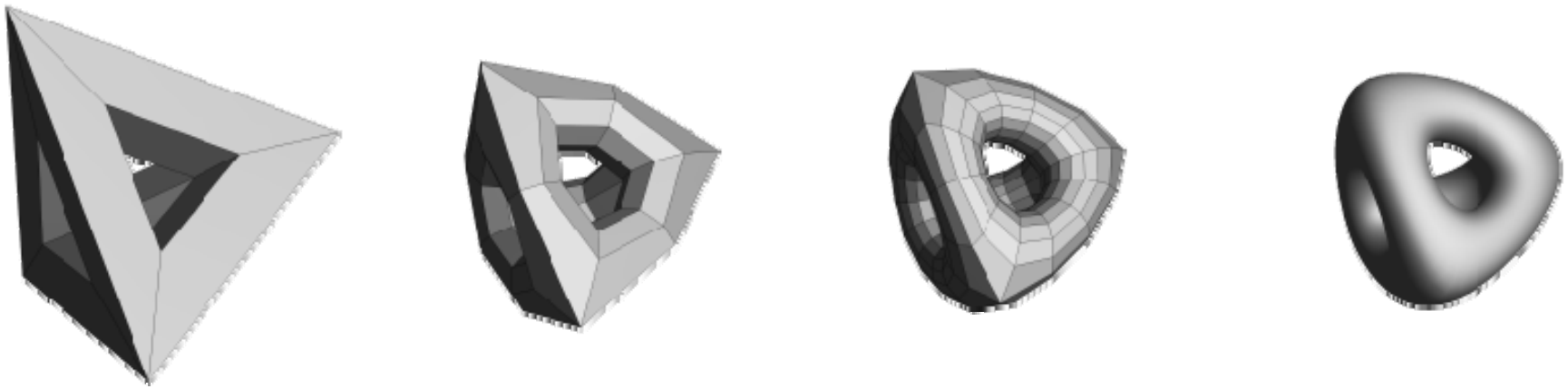


$$E_1 = V_1 - V_0 \qquad E_2 = V_2 - V_0 \qquad T = O - V_0$$

$$P = D \times E_2 \qquad Q = T \times E_1$$

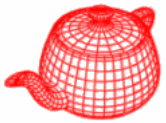
$$\begin{bmatrix} t \\ u \\ v \end{bmatrix} = \frac{1}{P \cdot E_1} \begin{bmatrix} Q \cdot E_2 \\ P \cdot T \\ Q \cdot D \end{bmatrix}$$

Subdivision surfaces

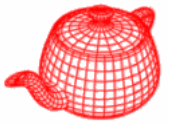


http://www.subdivision.org/demos/demos.html

Subdivision surfaces



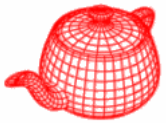
Advantages of subdivision surfaces



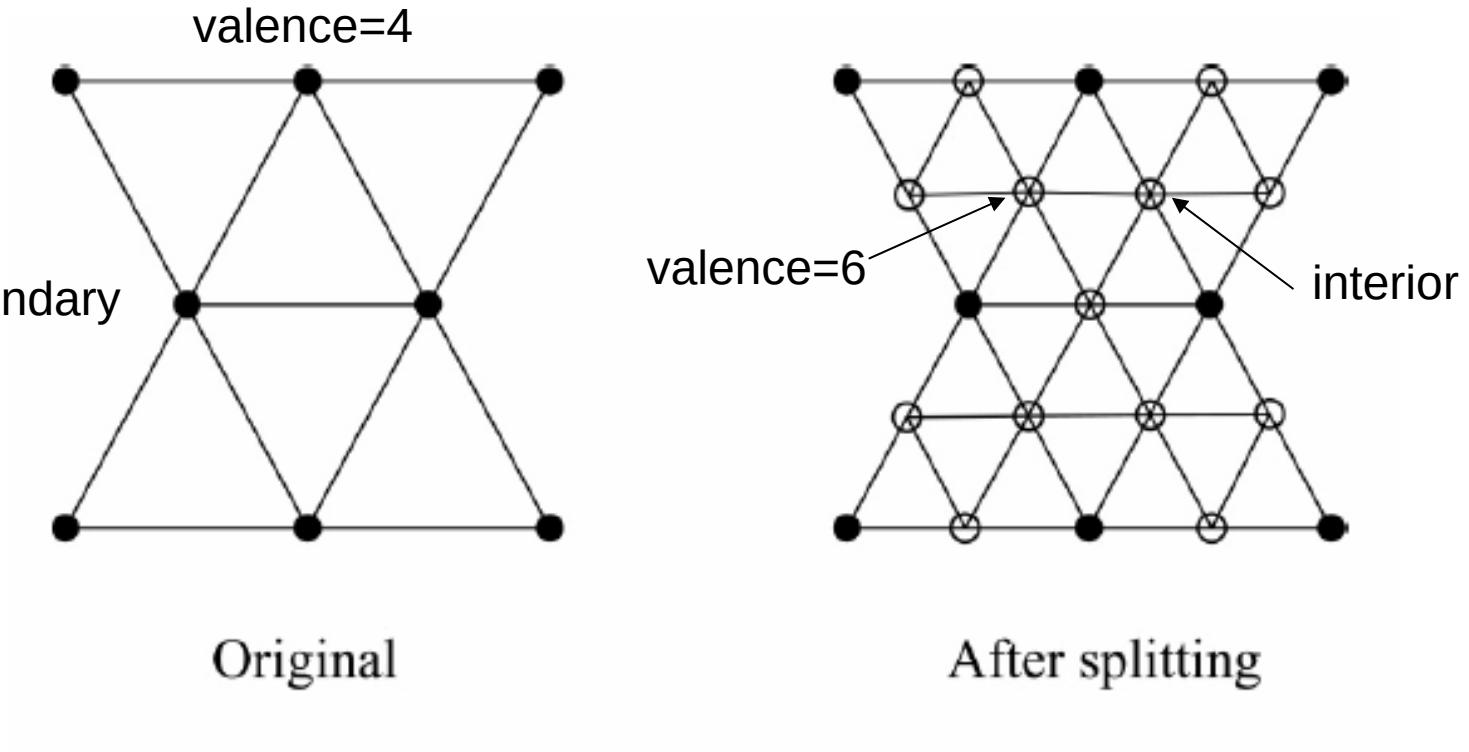
- Smooth surface
- Existing polygon modeling can be retargeted
- Well-suited to describing objects with complex topology
- Easy to control localized shape
- Level of details

Geri’s game

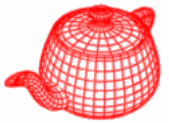
Loop Subdivision Scheme



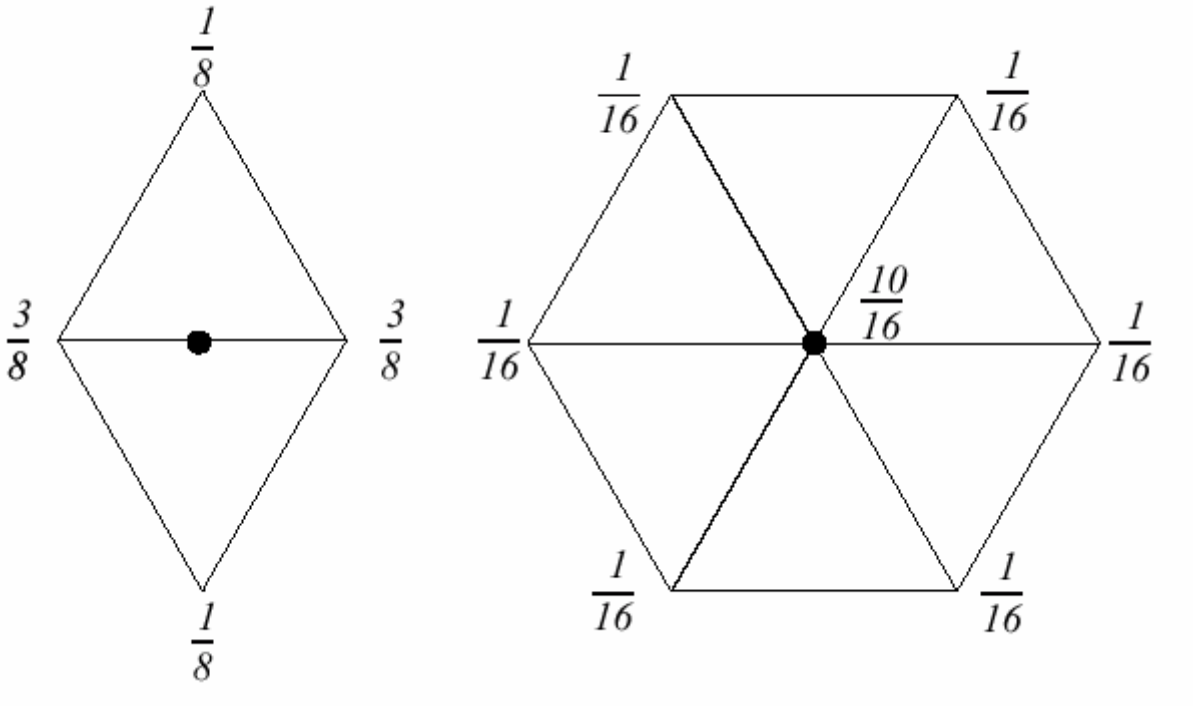
- Refine each triangle into 4 triangles by splitting each edge and connecting new vertices



Loop Subdivision Scheme

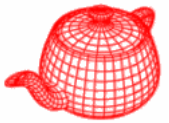


- Where to place new vertices?
  - Choose locations for new vertices as weighted average of original vertices in local neighborhood

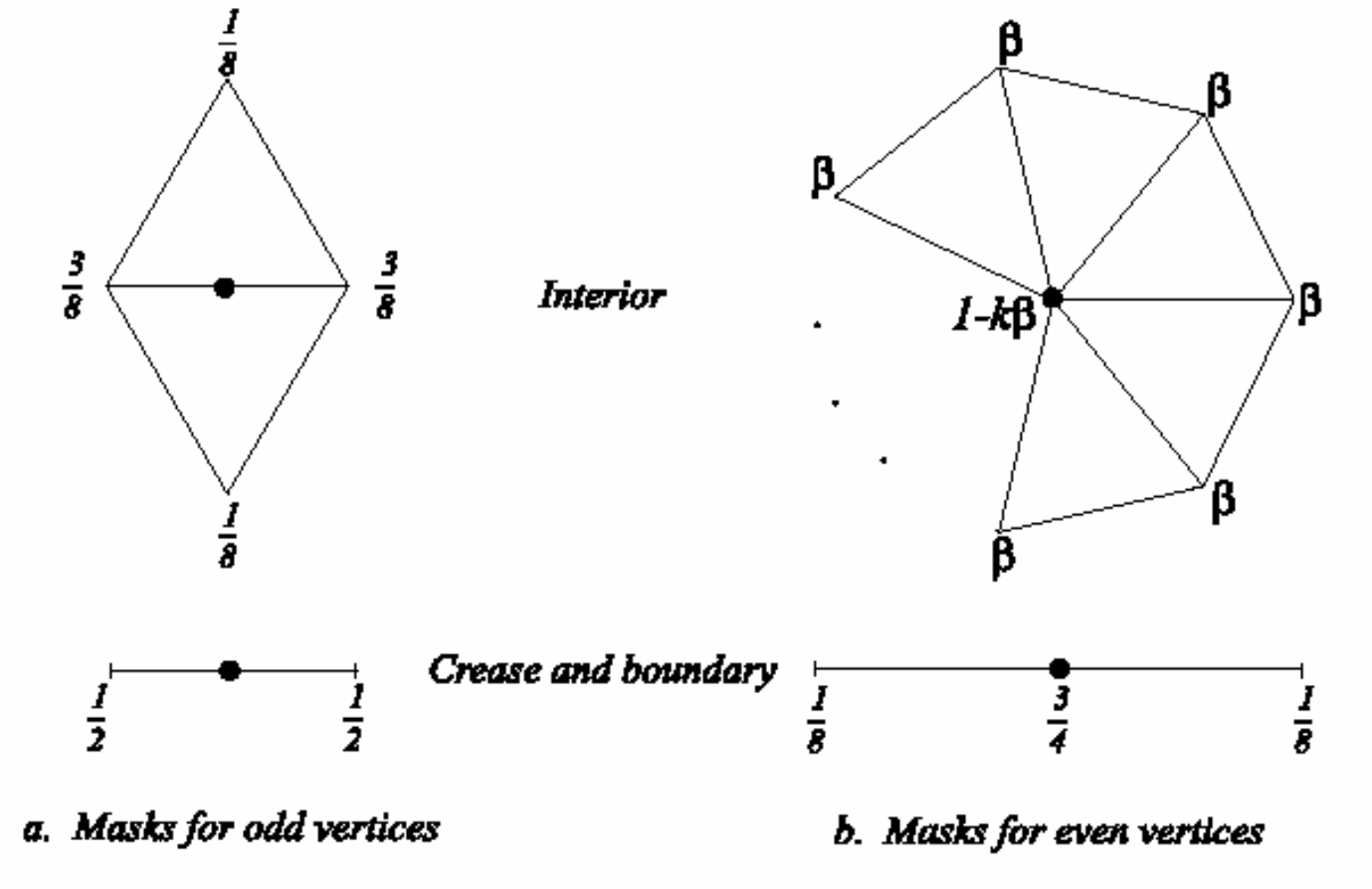


odd vertices even vertices

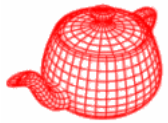
Loop Subdivision Scheme



- Where to place new vertices?
  - Rules for *extraordinary vertices* and *boundaries*:



Butterfly subdivision



- Interpolating subdivision: larger neighborhood