922 M0520 Compuation Theory and Algorithms **National Taiwan University**
Fall 2004 **Department of CSIE**

Homework 2

October 20, 2004 Due date: November 3, 2004

1. (15%) [Sipser 2.2] Given the languages $A = \{a^m b^n c^n | m, n \geq 0\}$ and $C = \{a^n b^n c^m | m, n \geq 0\}$,
   a. Design pushdown automata to recognize each of the languages A and C.
   b. Use the languages A and C together with Example 2.20 (in which, we prove that $B = \{a^n b^n c^n | n \geq 0\}$ is not context free) to show that the class of context-free languages is not closed under intersection.
   c. Use part (b) and DeMorgan's law ($\overline{A \cup B} = \overline{A} \cap \overline{B}$) to show that the class of context-free languages is not closed under complementation.

2. (15%) [Sipser 2.4] Give context-free grammars that generate the following languages over $\{0, 1\}$.
   a. $\{w | w$ contains at least three 1s$\}$
   b. $\{w|$ the length of w is odd and its middle symbol is a 0 $\}$
   c. $\{w | w = w^R\}$

3. (10%) Convert the following CFG to an equivalent PDA,
   $A \rightarrow BAB|B|\epsilon$
   $B \rightarrow 00|\epsilon$

4. (10%) [Sipser 2.18b] Using the pumping lemma to show that the language over $\{0, \#\}$, $\{0^n \# 0^{2n} \# 0^{3n} | n \geq 0\}$, is not context free.

5. (12%) [Sipser 3.5] Examine the formal definition of a Turing machine to answer the following questions, and explain your reasoning.
   a. Can a Turing machine ever write the blank symbol $\sqcup$ on its tape?
   b. Can the tape alphabet Γ be the same as the input alphabet Σ?
   c. Can a Turing machine's head *ever* be in the same location in two successive steps?
   d. Can a Turing machine contain just a single state?

6. (20%) [Sipser 3.2] Given TM M_1 whose description and state diagram appear in Example 3.5 of the textbook (the Turing machine shown in the class to recognize the language $\{w\#w | w \in \{0, 1\}^*\}$), for each of the following inputs, show the sequence of configurations that M_1 enters.
   a. 11
   b. 1#1
   c. 1##1
   d. 10#11
   e. 10#10

7. (18%) [Sipser 3.8] Give implementation-level descriptions of Turing Machines that decide the following languages over the alphabet $\{0, 1\}$:
   a. $\{w | w$ contains an equal number of 0s and 1s$\}$
   b. $\{w | w$ contains twice as many 0s as 1s$\}$ (that is, (the number of 0s)=2*(the number of 1s))
   c. $\{w | w$ does not contain twice as many 0s as 1s$\}$