922 M0520 Compuation Theory and Algorithms **National Taiwan University**
Fall 2004 **Department of CSIE**

Homework 1

September 29, 2004 Due date: October 13, 2004

1. (10%) Find the error in the following proof that $2 = 1$.
   Consider the equation $a = b$. Multiply both sides by a to obtain $a^2 = ab$. Subtract b^2 from both sides to get $a^2 - b^2 = ab - b^2$. Now factor each side, $(a + b)(a - b) = b(a - b)$, and divide each side by $(a - b)$, to get $a + b = b$. Finally, let a and b equal 1, which shows that $2 = 1$.

2. (10%) Design a DFA recognizing the following language over $\Sigma = \{0, 1\}$, $A = \{w | w \text{ ends in } 00\}$. Show both the state diagram and the formal definition.

3. (10%) Design a DFA recognizing the following language over $\Sigma = \{0, 1, \cdots, 9\}$, $B = \{w | w \text{ is a multiple of } 3, \text{ when } w \text{ is treated as a decimal number }\}$. For example, 621 is in B ($621\%3 = 0$) but not 761 ($761\%3 = 2$). You can show your design in either the state diagram or the formal definition.

4. (10%) Prove that $A - B$ is a regular language if both A and B are regular languages.

5. (10%) Convert the NFA in Figure 1 to an equivalent DFA.

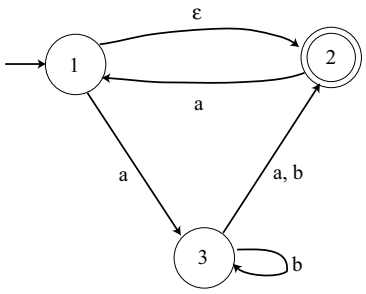

Figure 1

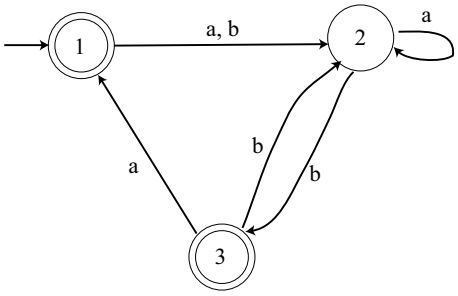

Figure 2

6. (10%) Convert the regular expression, $(0 + 1)^*000(0 + 1)^*$, to an equivalent NFA.

7. (10%) Convert the finite automata in Figure 2 to a regular expression.

8. (10%) Show that the language, $C = \{www | w \in \{a, b\}^*\}$, is not regular.

9. (10%) Show that the language, $D = \{0^m 1^n | m \neq n\}$, is not regular.

10. (10%) For any string $w = w_1 w_2 \cdots w_n$, the reverse of w, written w^R, is the string w in reverse order, $w = w_n \cdots w_2 w_1$. For any language A, let $A^R = \{w^R | w \in A\}$. Show that if A is regular, so is A^R.