Homework 1

October 15, 2024   Due date: October 29, 2024

1. (12%) What are the binary representations of the following hexadecimal numbers? What are the decimal numbers they represent when interpreted as 8-bit unsigned and signed integers (with 2's complement representations)?
   a. 6A
   b. D7

2. (8%) What are the 8-bit 2's complement representations of the following decimal numbers? Please give both their binary and hexadecimal representations.
   a. 107
   b. -99

3. (10%) Show that $(A + B)(A + C) = A + BC$ using (a) the truth table and (b) Boolean algebra rules.

4. (10%) Show that NOR is universal.

5. (10%) (a) Find the minimal Boolean expression for the following truth table. (b) Draw a circuit for the simplified Boolean expression.

A	B	C	D	Y
0	0	0	0	X
0	0	0	1	X
0	0	1	0	X
0	0	1	1	0
0	1	0	0	0
0	1	0	1	X
0	1	1	0	0
0	1	1	1	X
1	0	0	0	1
1	0	0	1	0
1	0	1	0	X
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	X
1	1	1	1	1

6. (10%) A circuit has four inputs and two outputs, P and D. The input $X = X_3 X_2 X_1 X_0$ represents an unsigned integer from 0 to 15. The output P should be 1 if the input is prime (note: 0 and 1 are not prime). Otherwise, P should be 0. The output D should be 1 if the number is divisible by 4 (0 is divisible by 5). Otherwise, D should be 0. Provide the truth tables for both P and D. What are their simplified Boolean expressions?

7. (10%) Design a 2-bit multiplier (MUL) circuit that accepts two 2-bit inputs, $X = X_1 X_0$ and $Y = Y_1 Y_0$, and outputs their 4-bit product $P = P_3 P_2 P_1 P_0$, where $P = X \times Y$. Simply provide the simplified Boolean expressions for each bit of the output. It is not required to draw the circuit.

8. (20%) Design a 7-segment display driver which accepts a 4-bit input (ABCD where A is the MSB) and outputs 7 bits, which controls the on/off status of a 7-segment display, as shown in Figure 1. Assume that the input ABCD encodes a number ranging within $[0..9]$. (a) List the truth table for a,b,c,d,e,f,g in the driver. Use X to indicate "don't care". (b) Write down the Boolean expressions for all segments. You should simplify them if possible.
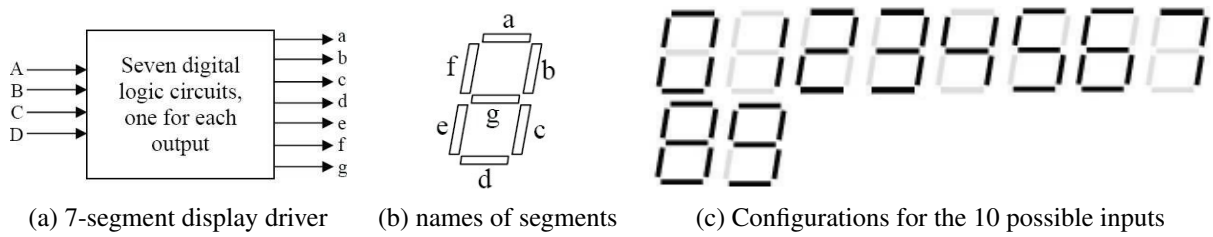
(a) 7-segment display driver   (b) names of segments   (c) Configurations for the 10 possible inputs

Figure 1: The 7-segment display driver.

9. (10%) In Hack ALU, the following configurations of inputs are used for $x - y$ and $x + 1$. Explain why they work.

zx	nx	zy	ny	f	no	out
0	1	0	0	1	1	$x - y$
0	1	1	1	1	1	$x + 1$