CSIE 1000 Introduction to Computers **National Taiwan University**
Fall 2023 **Department of CSIE**

Homework 1

October 17, 2023 Due date: October 31, 2023

1. (12%) What are the binary representations of the following hexadecimal numbers? What are the decimal numbers they represent when interpreted as unsigned and signed integers (with 2's complement representations)?
   a. 3B
   b. A5

2. (8%) What are the 8-bit 2's complement representations of the following decimal numbers? Please give both their binary and hexadecimal representations.
   a. 24
   b. -59

3. (10%)Show that (a) $A + B(A + C) + AC = A + BC$ and (b) $(A + B)(\overline{A} + B) = B$ using algebraic rules/theorems.

4. (10%) (a) Find the minimal Boolean expression for the following truth table. (b) Draw a circuit for the simplified Boolean expression.

A	B	C	D	Y
0	0	0	0	0
0	0	0	1	1
0	0	1	0	X
0	0	1	1	X
0	1	0	0	0
0	1	0	1	X
0	1	1	0	X
0	1	1	1	X
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	X
1	1	1	1	1

5. (10%) Design a 4-bit *leading-1* circuit to report the location where the first 1 appears in the input. The circuit accepts a 4-bit input $X = X_3X_2X_1X_0$ as the input. It outputs a 1-bit flag P and a 2-bit output $Z = Z_1Z_0$ indicating that X_Z is the leading 1. The flag $P = 1$ if there is at least one 1 in the input. For example, if $X = 0110$, $P = 1$ and $Z = 10$ because the leading 1 appears at X_2. As another example, if $X = 1010$, $Z = 11$ because X_3 is the leading 1. When the input contains only 0's, $P = 0$. When $P = 0$, Z is irrelevant and can be anything.

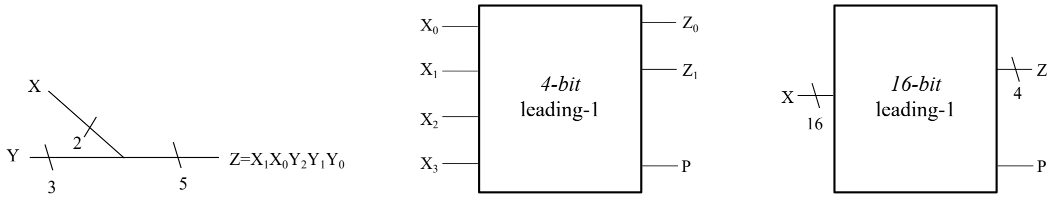
6. (10%) Design a 16-bit *leading-1* circuit. The circuit accepts a 16-bit input $X = X_{15}X_{14}\cdots X_0$ as the input. It outputs a 1-bit flag P and a 4-bit output $Z = Z_3Z_2Z_1Z_0$ indicating that X_Z is the leading 1. Note that, if you need to combine two buses into one, you can use the notation in Figure 1(a).

7. (10%) For representing real numbers in TOY, one could use the fixed-point format or the floating-point format. For simplicity, we only consider positive real numbers.

   For the fixed-point notation, we put the binary point between the 8-th bit and the 7-th bit. Thus, the first 8 bits are for the integer and the last 8 bits are fractional. For example, the word 0000 0101 0110 0000 represents 00000101.01100000 which equals $2^2 + 2^0 + 2^{-2} + 2^{-3} = 5.375$ in decimal.

For the floating-point format, we allocate the first 12 bits for the mantissa and the last 4 bits for the exponent. Note that the exponent ranges from +8 to -7. To avoid using 2's complement, we add 7 to it for making it within the range [15..0] and store the sum. For the above example 5.375_{10}, its floating-point representation can be derived as the following. We first find the leading 1 and shift the binary point right before it, that is, $00000101.01100000 = 0.10101100000 \times 2^3$. Then, we store the fractional part 101011000000 as the mantissa and $(3 + 7)_{10} = 10_{10} = 1010_2$ as the exponent. Thus, its floating-point representation is 1010 1100 0000 1010. Note that, for 0, its floating-point representation is 0000 0000 0000 0000.

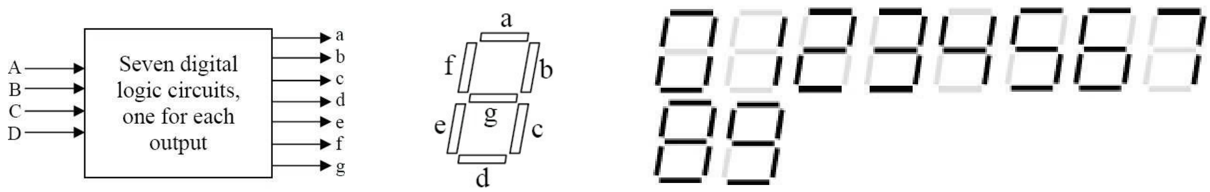
Design a circuit to convert a fixed-point number $N = N_{15}N_{14} \cdots N_0$ into a floating-point number $F = F_{15}F_{14} \cdots F_0$ in the formats specified above. You will need a 16-bit left shifter which accepts a 16-bit X as the number to be shifted and a 4-bit S as the shift amount. Note that you can use the sub-bus notation. For example, you can use $Y[15..4]$ to specify the 12-bit sub-bus $Y_{15}Y_{14} \cdots Y_4$ of a 16-bit number Y.



(a) combining two buses (b) 4-bit leading-1 circuit (c) 16-bit leading-1 circuit

Figure 1: The leading-1 circuits.

8. (20%) Design a 7-segment display driver which accepts a 4-bit input (ABCD where A is the MSB) and outputs 7 bits, which controls the on/off status of a 7-segment display, as shown in Figure 2. Assume that the input ABCD encodes a number ranging within [0..9]. (a) List the truth table for a,b,c,d,e,f,g in the driver. Use X to indicate "don't care". (b) Write down the Boolean expressions for all segments. You should simplify them if possible.



(a) 7-segment display driver (b) names of segments (c) Configurations for the 10 possible inputs

Figure 2: The 7-segment display driver.

9. (10%) In Hack ALU, the following configurations of inputs are used for $x|y$ and $y - x$. Explain why they work.

zx	nx	zy	ny	f	no	out
0	1	0	1	0	1	$x\|y$
0	0	0	1	1	1	$y - x$