Computer Graphics

Computer Science & Information Technology

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Introduction

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- Grading: exam on the final exam week

What is computer graphics ?

- Definition
  - the pictorial *synthesis* of real or imaginary objects from their computer-based models

INPUT \ OUTPUT	descriptions	images
descriptions		Computer Graphics
images	Computer Vision	Image Processing

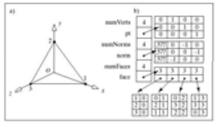
Computer graphics

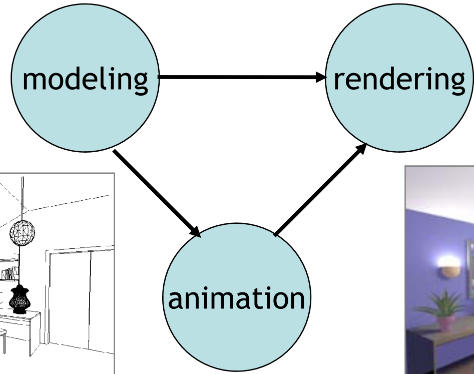
- Create a 2D image/animation of a 3D world

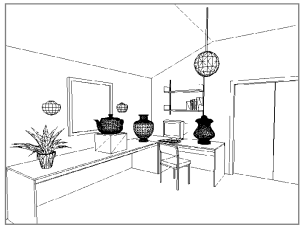
Applications

- Movies
- Interactive entertainment
- Industrial design
- Architecture
- Culture heritage

Computer graphics

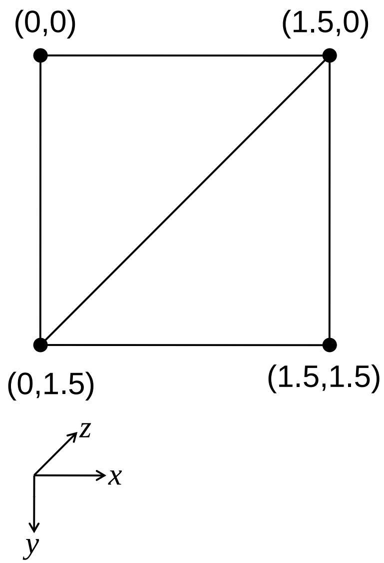






Modeling

A simple example

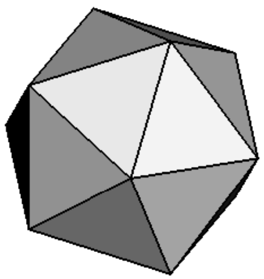


\# vertices
0.0, 0.0, 0.0
1.5, 0.0, 0.0
0.0, 1.5, 0.0
1.5, 1.5, 0.0

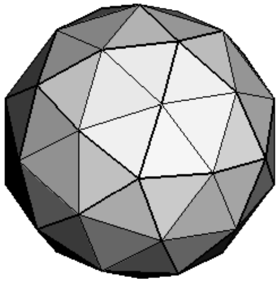
\# triangles
0, 2, 1
1, 2, 3

The power of triangles

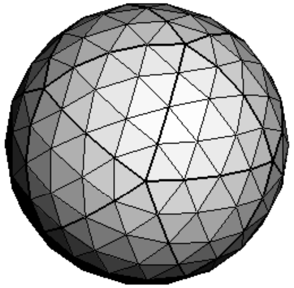
- Every thing can be represented by triangles to a degree of precision.



20 triangles



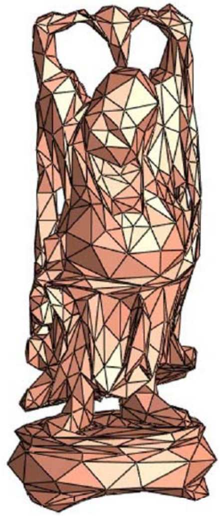
80 triangles



320 triangles

More complex examples

a real buddha



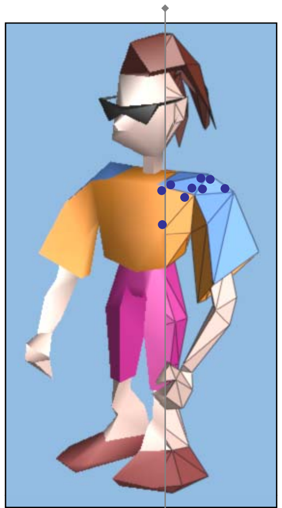
4K mesh

rendered 2.4M mesh

Modeling

- The position of the model can be acquired by 3D scanner or made by artists using modeling tools.
- There are other ways for representing geometric objects, but triangles have many advantages.

Triangle meshes



$\{f_1\} : \{ v_1 , v_2 , v_3 \}$
$\{f_2\} : \{ v_3 , v_2 , v_4 \}$
…
connectivity

$\{v_1\} : (x,y,z)$
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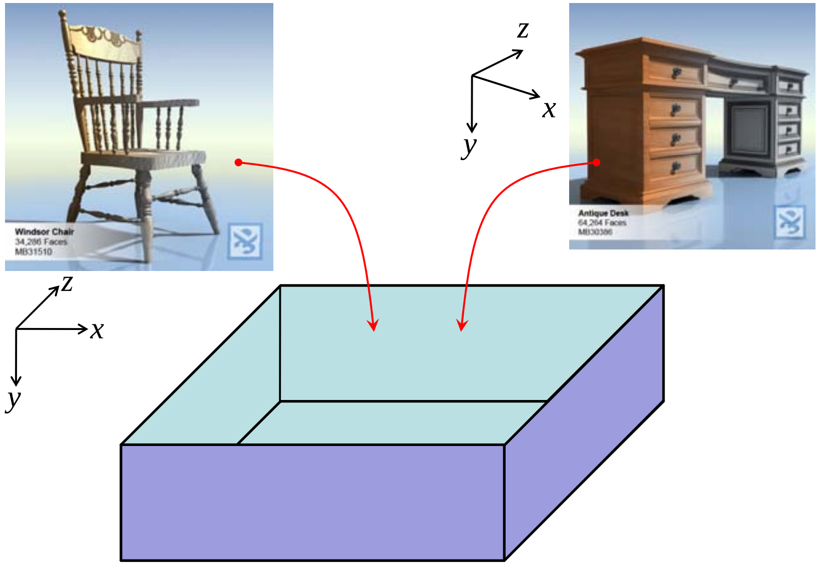
…
geometry

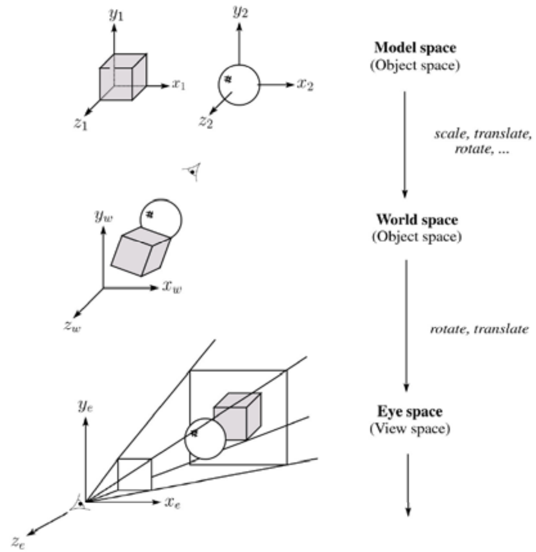
$\{f_1\}$: *"skin material"*
$\{f_2\}$: *"brown hair"*
…
face attributes

$\{v_2,f_1\} : (n_x,n_y,n_z)\ (u,v)$
$\{v_2,f_2\} : (n_x,n_y,n_z)\ (u,v)$
…
corner attributes

Composition of a scene



Graphics pipeline



Transformations

Representation

We can represent a **point**, **p** = (x,y) in the plane

- as a column vector $\begin{bmatrix} x \\ y \end{bmatrix}$
- as a row vector $\begin{bmatrix} x & y \end{bmatrix}$

Representation

We can represent a **2-D transformation** *M* by a matrix

$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

If **p** is a column vector, *M* goes on the left:

$$\mathbf{p'} = M\mathbf{p}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D transformations

Here's all you get with a 2 x 2 transformation matrix *M*:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

So:

$$x' = ax + by$$
$$y' = cx + dy$$

We will develop some intimacy with the elements *a, b, c, d*…

Identity

Suppose we choose *a=d=1, b=c=0:*

- Gives the **identity** matrix:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- Doesn't move the points at all

Scaling

Suppose we set *b=c=0*, but let *a* and *d* take on any *positive* value:

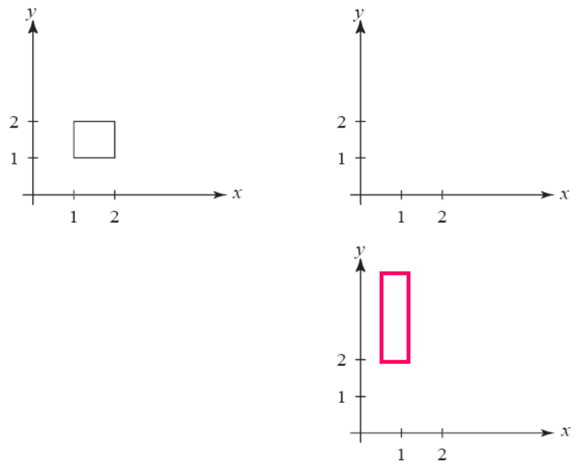
- Gives a **scaling** matrix:

$$\begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}$$

- Provides **differential scaling** in *x* and *y*:

$$x' = ax$$
$$y' = dy$$

Scaling



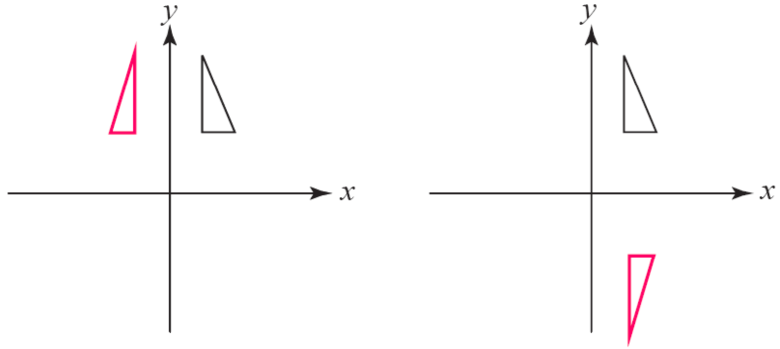
$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1/2 & 0 \\ 0 & 2 \end{bmatrix}$$

Reflection

Examples:

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

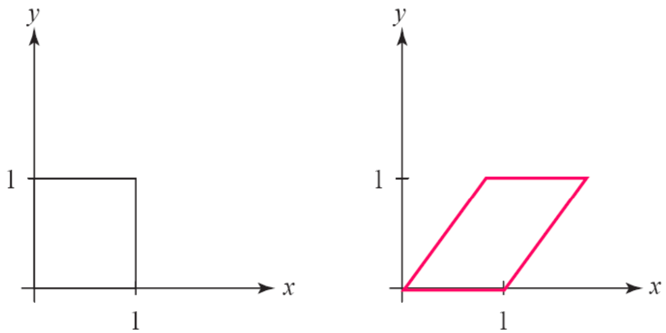


Shearing

The matrix $\begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix}$

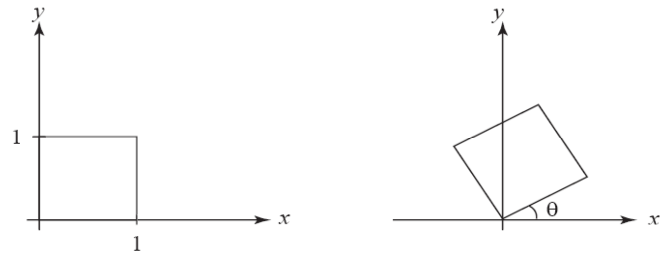
gives:

$$x' = x + by$$
$$y' = y$$



$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

Rotation



$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} -\sin(\theta) \\ \cos(\theta) \end{bmatrix}$$

$$M = R(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

Limitations of a 2X2 matrix

- Scaling
- Rotation
- Reflection
- Shearing

- What do we miss?

Homogeneous coordinate

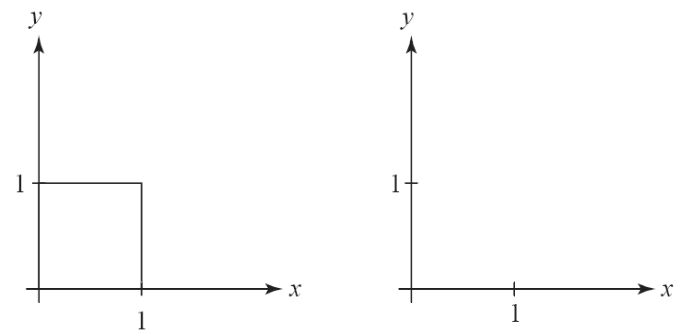
Idea is to loft the problem up into 3-space, adding a third component to every point:

$$\begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

And then transform with a 3 x 3 matrix:

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = T(\mathbf{t}) \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translation



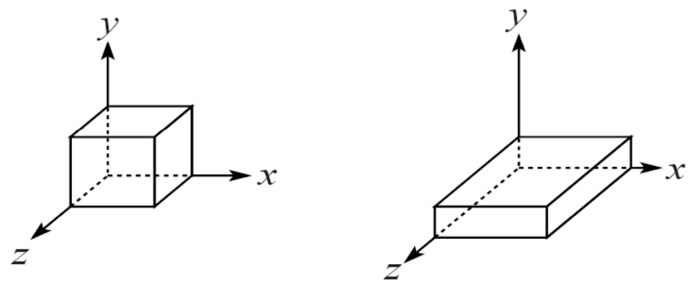
$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1/2 \\ 0 & 0 & 1 \end{bmatrix}$$

3D scaling

Some of the 3-D transformations are just like the 2-D ones.

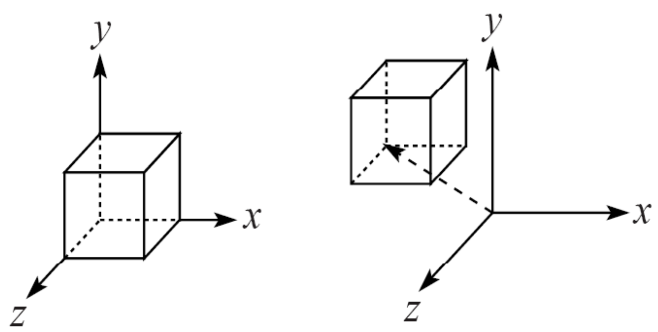
For example, <u>scaling</u>:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



3D translation

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



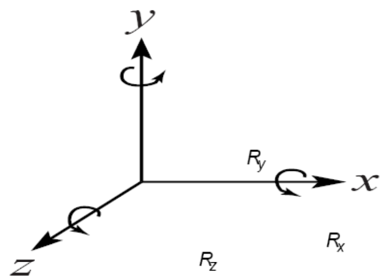
3D rotation

Rotation now has more possibilities in 3D:

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

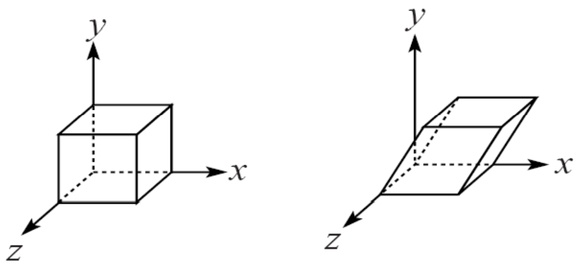


Use right hand rule

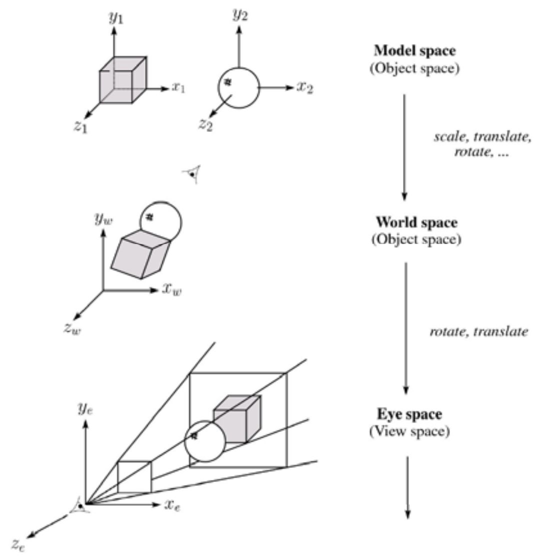
3D shearing

Shearing is also more complicated. Here is one example:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & b & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

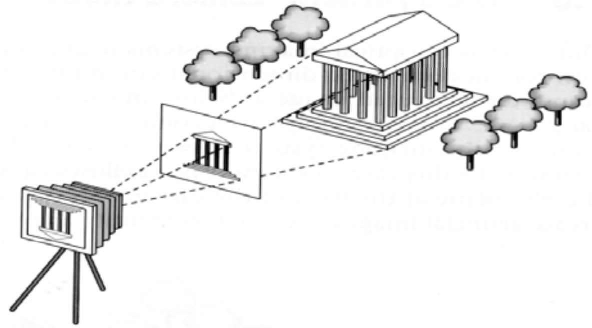


Graphics pipeline



Projections

Imaging with the synthetic camera

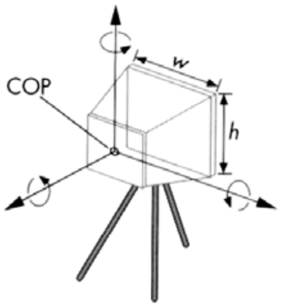


The image is rendered onto an **image plane** or **projection plane** (usually in front of the camera).

Projectors emanate from the **center of projection** (COP) at the center of the lens (or pinhole).

The image of an object point *P* is at the intersection of the projector through *P* and the image plane.

Specifying a viewer



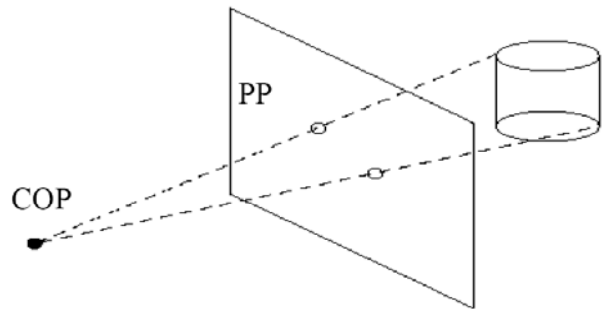
Camera specification requires four kinds of parameters:

- *Position:* the COP.
- *Orientation:* rotations about axes with origin at the COP.
- *Focal length:* determines the size of the image on the film plane, or the **field of view**.
- *Film plane:* its width and height, and possibly orientation.

Projections

Projections transform points in *n*-space to *m*-space, where $m < n$.

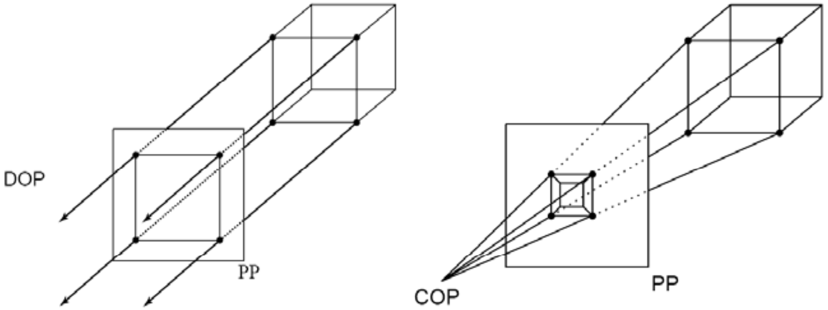
In 3D, we map points from 3-space to the **projection plane (PP)** along **projectors** emanating from the **center of projection (COP)**.



There are two basic types of projections:

- **Perspective** - distance from COP to PP finite
- **Parallel -** distance from COP to PP infinite

Parallel and perspective projections



orthographic perspective

Orthographic transformation

For parallel projections, we specify a **direction of projection** (DOP) instead of a COP.

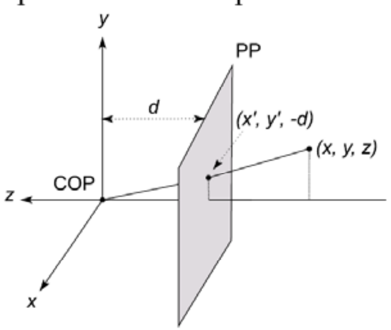
We can write orthographic projection onto the *z=0* plane with a simple matrix.

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

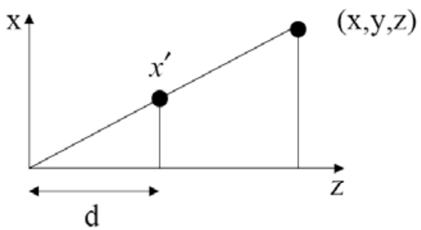
Normally, we do not drop the z value right away. Why not?

Perspective projection

Q: How do we perform the perspective projection from eye space into screen space?



Using similar triangles gives:



Perspective transform

We can write this transformation in matrix form:

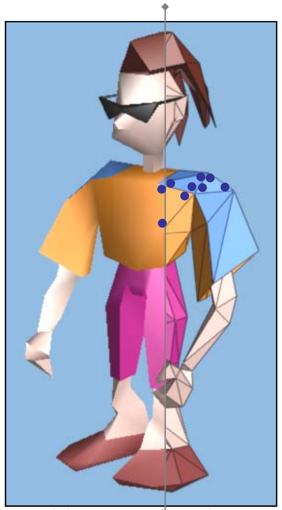
$$\begin{bmatrix} X \\ Y \\ Z \\ W \end{bmatrix} = MP = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix}$$

Perspective divide:

$$\begin{bmatrix} X/W \\ Y/W \\ Z/W \\ W/W \end{bmatrix} = \begin{bmatrix} \frac{x}{z/d} \\ \frac{y}{z/d} \\ d \\ 1 \end{bmatrix}$$

Graphics pipeline review

Triangle meshes



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$\{f_2\} : \{ v_3 , v_2 , v_4 \}$
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connectivity

$\{v_1\} : (x,y,z)$
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geometry

$\{f_1\}$: *"skin material"*
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…

face attributes

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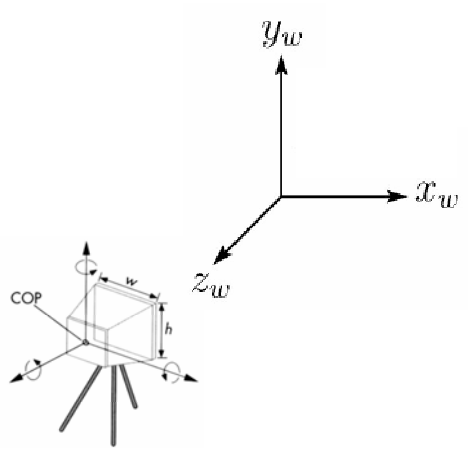
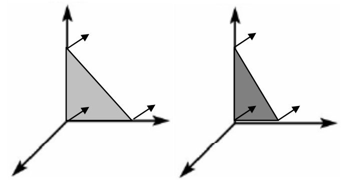
$\{v_2,f_2\} : (n_x,n_y,n_z)\ (u,v)$
…

corner attributes

Review of graphics pipeline

Transformation



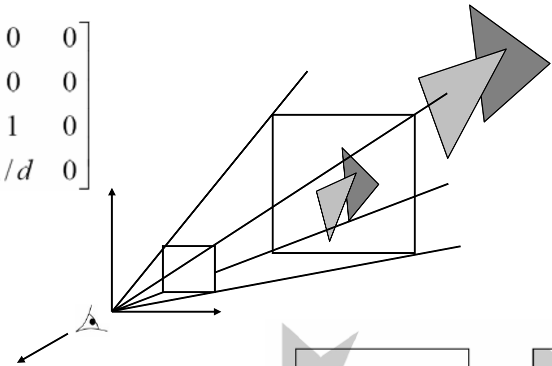


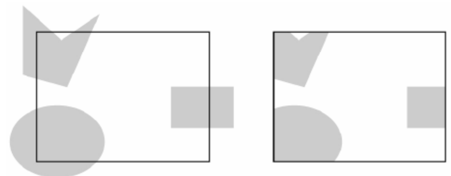
$$\begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Review of graphics pipeline

Projection & clipping

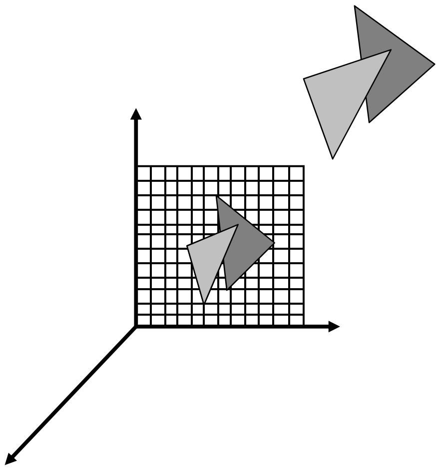
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix}$$





Review of graphics pipeline

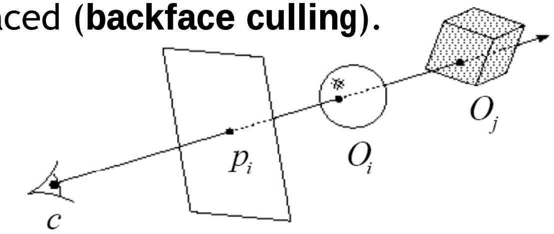
- Rasterization
- Visibility



Visibility (Hidden surface removal)

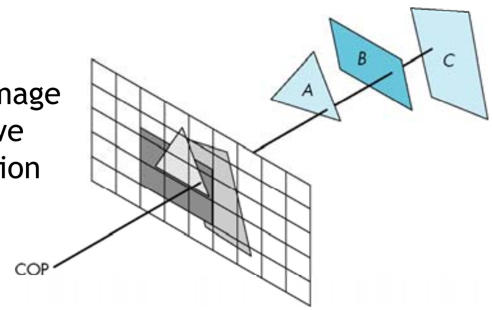
Hidden surface removal

- Determining what to render at each pixel.
- A point is visible if there exists a direct line-of-sight to it, unobstructed by another other objects (**visible surface determination**).
- Moreover, some objects may be invisible because there are behind the camera, outside of the field-of-view, too far away (**clipping**) or back faced (**backface culling**).



Hidden surfaces: why care?

- **Occlusion**: Closer (opaque) objects along same viewing ray obscure more distant ones
- Reasons for removal
  - Efficiency: As with clipping, avoid wasting work on invisible objects
  - Correctness: The image will look wrong if we don’t model occlusion properly

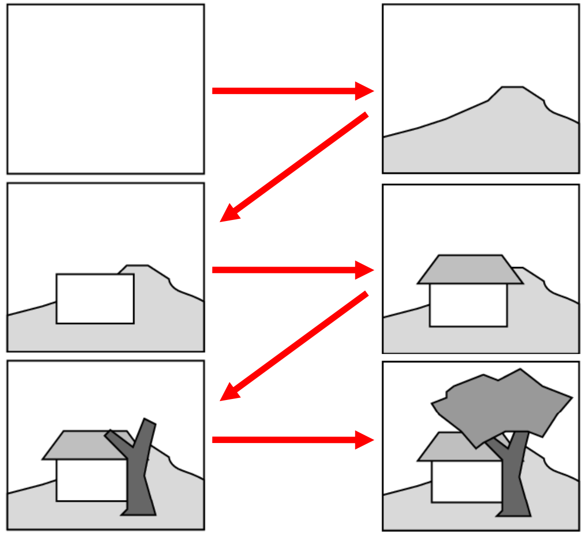


Hidden surface removal algorithms

- Painter's algorithm
- Binary space partitioning
- Z-buffer
- Ray casting
- And many others

Painter's algorithm

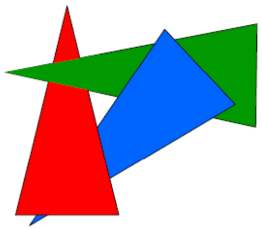
Draw primitives from back to front to avoid need for depth comparisons



from Shirley

Painter's algorithm

- Idea: Sort primitives by minimum depth, then rasterize from furthest to nearest
- When there are depth overlaps, do more tests of bounding areas, etc. to see one actually occludes the other
- Cyclical overlaps are a problem

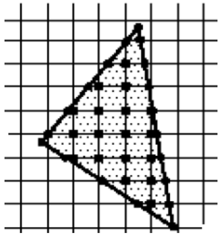


Z-buffer algorithm

- Resolve depths at the pixel level
- Idea: add Z to frame buffer, when a pixel is drawn, check whether it is closer than what's already in the framebuffer
- Proposed by Ed Catmull in 1975, widely used today, especially in hardware.

- Z-buffer, texture, subdivsion surface, RenderMan
- Co-founder of Pixar
- 3 Oscars (1993, 1996, 2001), SIGGRAPH Steven Coons Award (1993)

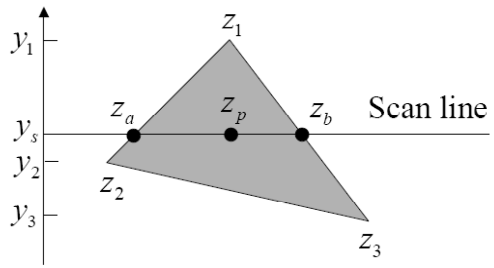
Z-buffer algorithm



```
for each pixel pi
{
        Z-buffer[ pi ] = FAR
        Fb[ pi ] = BACKGROUND_COLOR
}

for each polygon P
{
        for each pixel pi in the projection of P
        {
                Compute depth z and shade s of P at pi
                if z < Z-buffer[ pi ]
                {
                        Z-buffer[ pi ] = z
                        Fb[ pi ] = s
                }
        }
}
```

Z-buffer algorithm



$$z_a = z_1 - (z_1 - z_2)\frac{y_1 - y_s}{y_1 - y_2}$$

$$z_b = z_1 - (z_1 - z_3)\frac{y_1 - y_s}{y_1 - y_3}$$

$$z_p = z_b - (z_b - z_a)\frac{x_b - x_p}{x_b - x_a}$$

The z-Buffer Algorithm

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

\+

5	5	5	5	5	5	5
5	5	5	5	5	5	
5	5	5	5	5		
5	5	5	5			
5	5	5				
5	5					
5						

=

5	5	5	5	5	5	5	0
5	5	5	5	5	5	0	0
5	5	5	5	5	0	0	0
5	5	5	5	0	0	0	0
5	5	5	0	0	0	0	0
5	5	0	0	0	0	0	0
5	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

5	5	5	5	5	5	5	0
5	5	5	5	5	5	0	0
5	5	5	5	5	0	0	0
5	5	5	5	0	0	0	0
5	5	5	0	0	0	0	0
5	5	0	0	0	0	0	0
5	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

\+

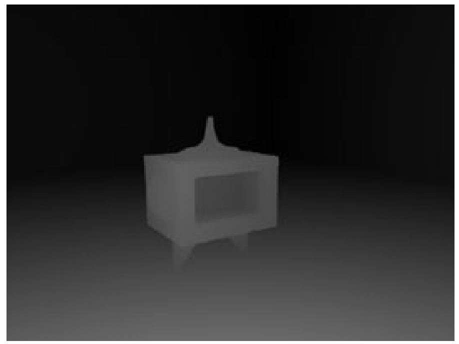
3					
4	3				
5	4	3			
6	5	4	3		
7	6	5	4	3	
8	7	6	5	4	3

=

5	5	5	5	5	5	5	0
5	5	5	5	5	5	0	0
5	5	5	5	5	0	0	0
5	5	5	5	0	0	0	0
6	5	5	3	0	0	0	0
7	6	5	4	3	0	0	0
8	7	6	5	4	3	0	0
0	0	0	0	0	0	0	0

Z-buffer: example

color buffer

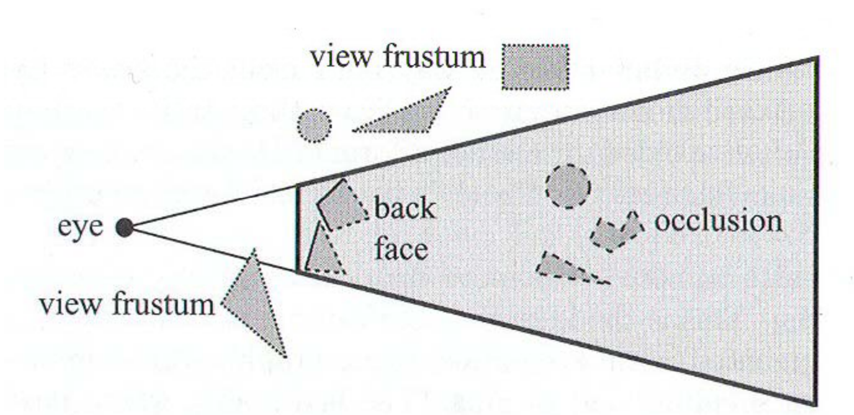


depth buffer

Z-Buffer

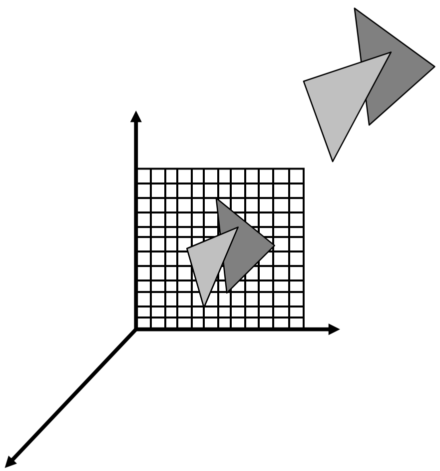
- Benefits
  - Easy to implement
  - Works for any geometric primitive
  - Parallel operation in hardware (independent of order of polygon drawn)
- Limitations
  - Memory required for depth buffer
  - Quantization and aliasing artifacts
  - Overfill
  - Transparency does not work well

Clipping (view frustum culling)



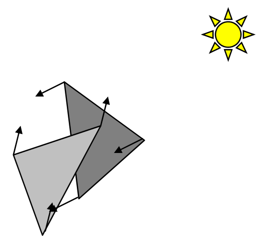
Review of graphics pipeline

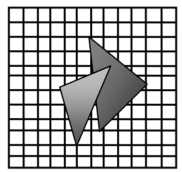
- Rasterization
- Visibility



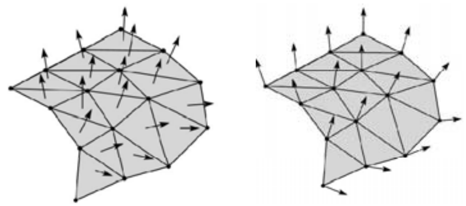
Review of graphics pipeline

- Shading





$$I = k_e + k_a I_a + \sum_i f(d_i) I_{li}\left[k_d (\mathbf{N}\cdot\mathbf{L}_i)_+ + k_s (\mathbf{V}\cdot\mathbf{R})_+^{n_s}\right]$$



Shading

Z-buffer algorithm

```
for each pixel p_i
{
        Z-buffer[ p_i ] = FAR
        Fb[ p_i ] = BACKGROUND_COLOR
}

for each polygon P
{
        for each pixel p_i in the projection of P
        {
                Compute depth z and shade s of P at p_i
                if z < Z-buffer[ p_i ]
                {
                        Z-buffer[ p_i ] = z
                        Fb[ p_i ] = s
                }
        }
}
```

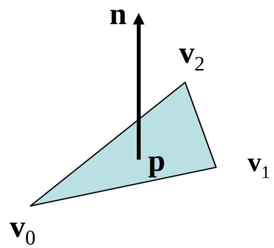
What is normal?

Normal for a triangle

plane $\mathbf{n} \cdot (\mathbf{p} - \mathbf{v}_0) = 0$

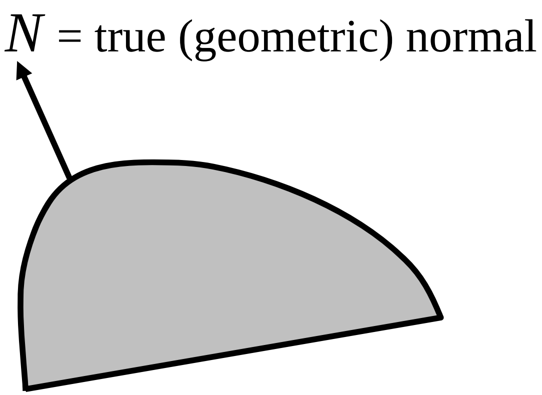
$\mathbf{n} = (\mathbf{v}_2 - \mathbf{v}_0) \times (\mathbf{v}_1 - \mathbf{v}_0)$

normalize $\mathbf{n} \leftarrow \mathbf{n} / |\mathbf{n}|$

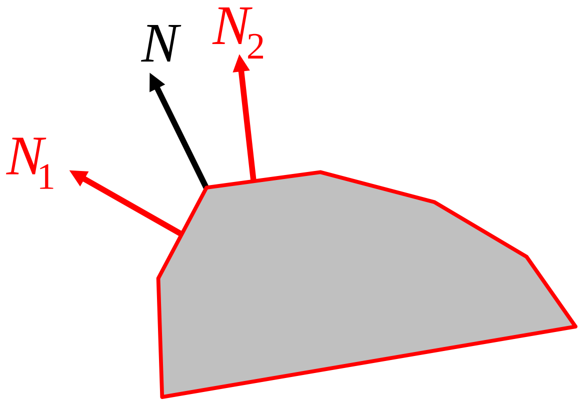


Note that right-hand rule determines outward face

Using average normals

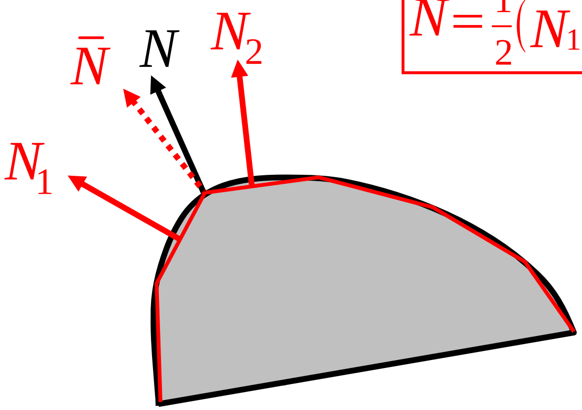


Using average normals



Using average normals

$$\bar{N} = \frac{1}{2}(N_1 + N_2)$$

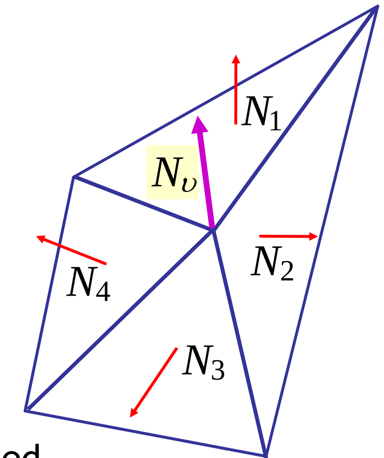


Using average normals

$$N_\upsilon = \frac{(N_1 + N_2 + N_3 + N_4)}{\|N_1 + N_2 + N_3 + N_4\|}$$

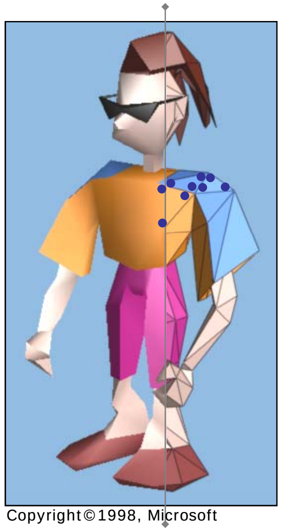
More generally,

$$N_\upsilon = \frac{\sum_{i=1}^{n} N_i}{\left| \sum_{i=1}^{n} N_i \right|}$$



It can also be area-weighted.

Definitions of Triangle Meshes



$\{f_1\} : \{ v_1 , v_2 , v_3 \}$
$\{f_2\} : \{ v_3 , v_2 , v_4 \}$
…

connectivity

$\{v_1\} : (x,y,z)$
$\{v_2\} : (x,y,z)$
…

geometry

$\{f_1\}$: *"skin material"*
$\{f_2\}$: *"brown hair"*
…

face attributes

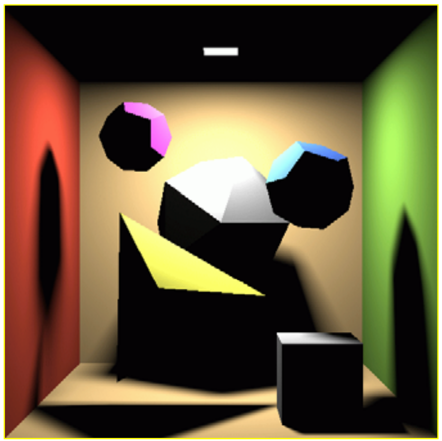
$\{v_2,f_1\} : (n_x,n_y,n_z)\ (u,v)$
$\{v_2,f_2\} : (n_x,n_y,n_z)\ (u,v)$
…

corner attributes

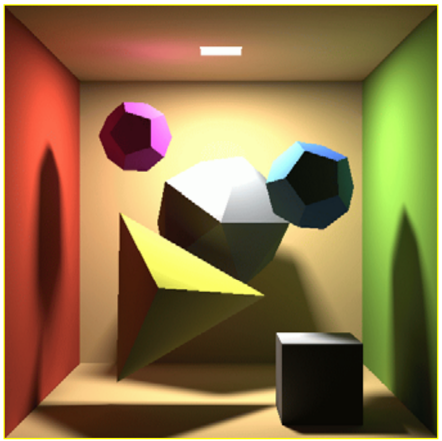
Illumination (shading) models

- Interaction between light sources and objects in scene that results in perception of intensity and color at eye
- Local vs. global models
  - Local: perception of a particular primitive only depends on light sources **directly** affecting that one primitive
    - Geometry
    - Material properties
    - Shadows cast (global?)
  - Global: also take into account **indirect** effects on light of other objects in the scene
    - Light reflected/refracted
    - Indirect lighting

Local vs. global models

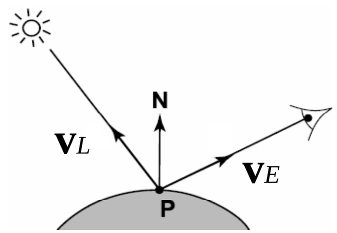


Direct lighting



Indirect lighting

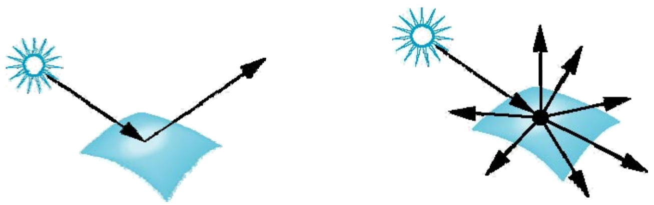
Setup



- Point **P** on a surface through a pixel **p**
- Normal **N** at **P**
- Lighting direction $\mathbf{v}_L$
- Viewing direction $\mathbf{v}_E$
- Compute color **L** for pixel **p**

Surface types

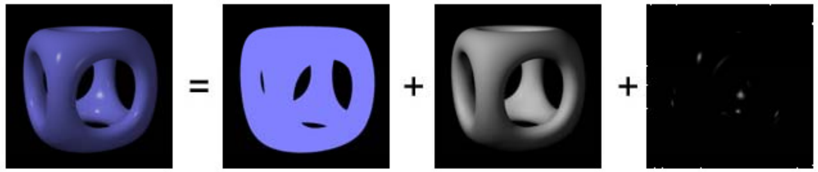
- The smoother a surface, the more reflected light is concentrated in the direction a perfect mirror would reflected the light
- A very rough surface scatters light in all directions



smooth surface　　rough surface

Basics of local shading

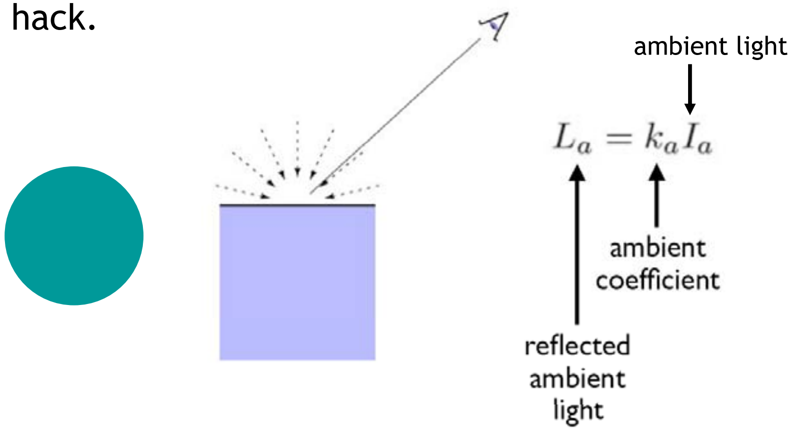
- Diffuse reflection
  - light goes everywhere; colored by object color
- Specular reflection
  - happens only near mirror configuration; usually white
- Ambient reflection
  - constant accounted for other source of illumination



color and ambient　　diffuse　　specularity

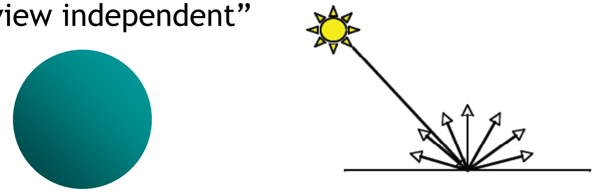
Ambient shading

- add constant color to account for disregarded illumination and fill in black shadows; a cheap hack.

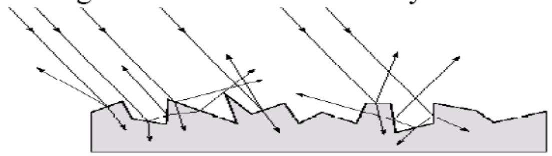


Diffuse shading

- Assume light reflects equally in all directions
  - Therefore surface looks same color from all views; "view independent"

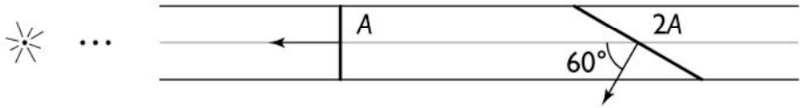


Picture a rough surface with lots of tiny **microfacets:**



Diffuse shading

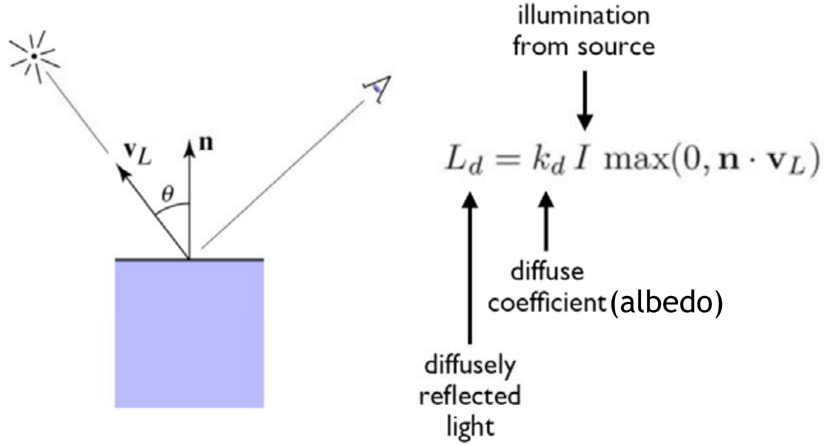
- Illumination on an oblique surface is less than on a normal one (Lambertian cosine law)



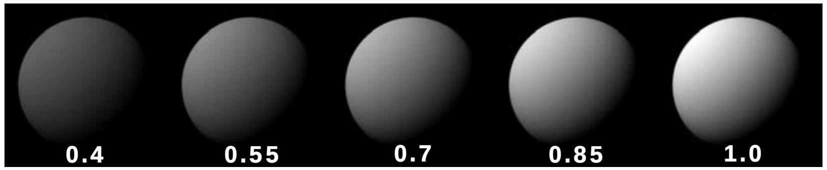
  - Generally, illumination falls off as cosθ

Diffuse shading (Gouraud 1971)

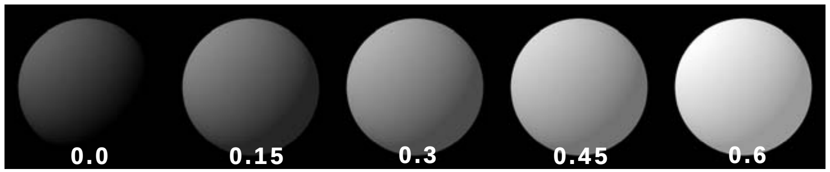
- Applies to *diffuse*, *Lambertian* or *matte* surfaces



Diffuse shading



diffuse-reflection model with different k_d

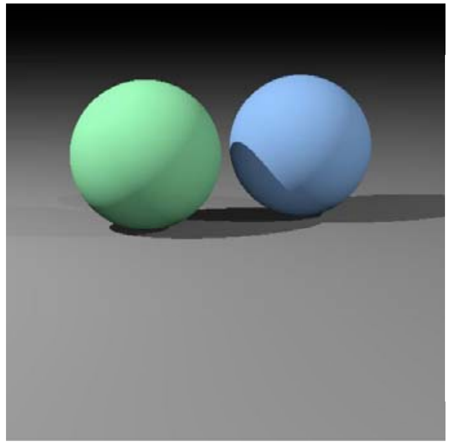


ambient and diffuse-reflection model with different k_a

and $I_a = I_p = 1.0, k_d = 0.4$

Diffuse shading

For color objects, apply the formula for each color channel separately

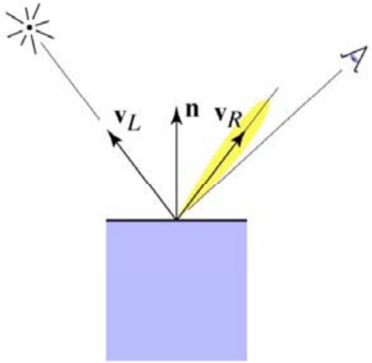


Specular shading

- Some surfaces have highlights, mirror like reflection; view direction dependent; especially for smooth shinny surfaces

Specular shading (Phong 1975)

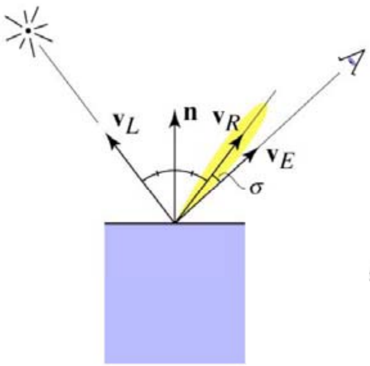
- Also known as *glossy*, *rough specular* and *directional diffuse* reflection



$$\mathbf{v}_R = \mathbf{v}_L + 2((\mathbf{n}\cdot\mathbf{v}_L)\mathbf{n} - \mathbf{v}_L)$$
$$= 2(\mathbf{n}\cdot\mathbf{v}_L)\mathbf{n} - \mathbf{v}_L$$

Specular shading

- Fall off gradually from the perfect reflection direction



$$\mathbf{v}_R = \mathbf{v}_L + 2((\mathbf{n}\cdot\mathbf{v}_L)\mathbf{n} - \mathbf{v}_L)$$
$$= 2(\mathbf{n}\cdot\mathbf{v}_L)\mathbf{n} - \mathbf{v}_L$$

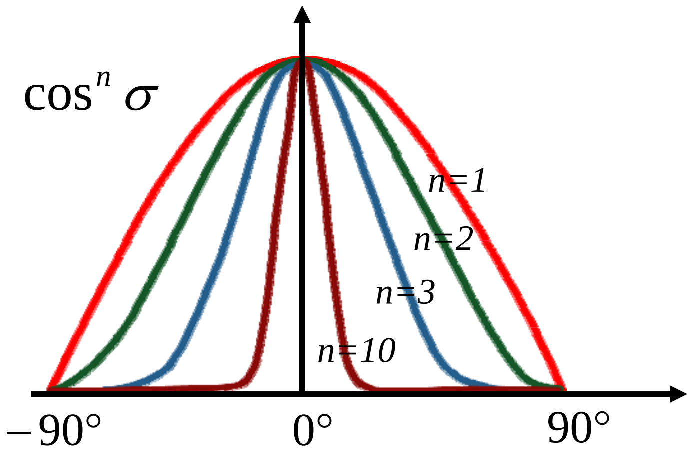
$$L_s = k_s\, I \max(0, \cos\sigma)^n$$
$$= k_s\, I \max(0, \mathbf{v}_E\cdot\mathbf{v}_R)^n$$

specularly reflected light

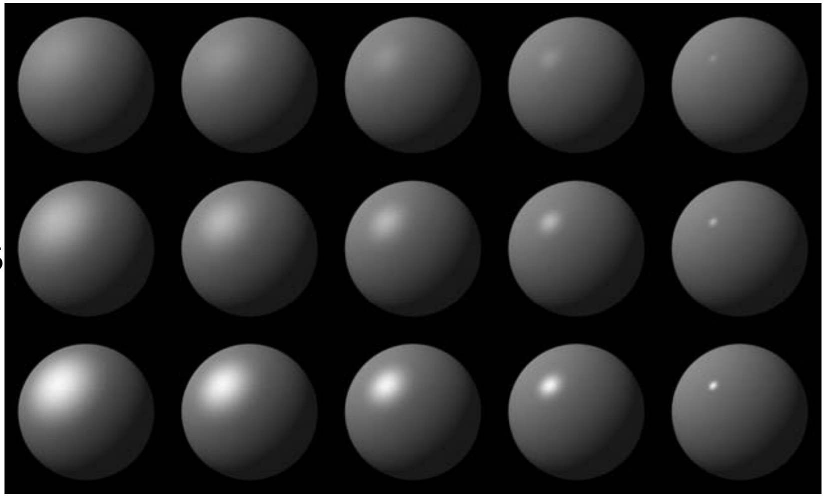
specular coefficient

Specular shading

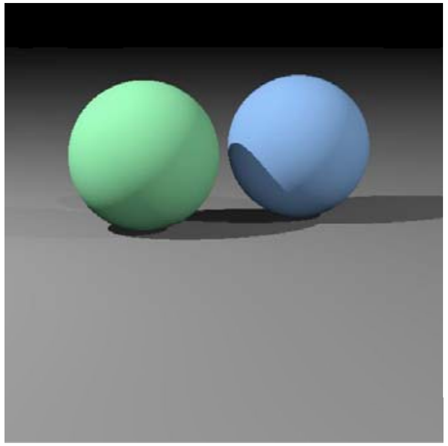
- Increasing n narrows the lobe

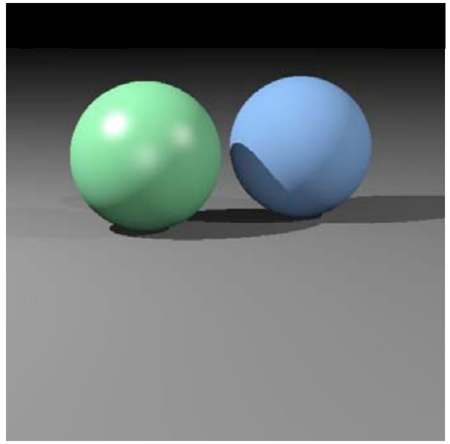


Specular shading



Specular shading


diffuse


diffuse + specular

Put it all together

- Include ambient, diffuse and specular

$$L = L_a + L_d + L_s$$
$$= k_a I_a + I\left(k_d \max(0, \mathbf{n} \cdot \mathbf{v}_L) + k_s \max(0, \mathbf{n} \cdot \mathbf{v}_H)^n\right)$$

- Sum over many lights

$$L = L_a + \sum_i (L_d)_i + (L_s)_i$$
$$= k_a I_a + \sum_i I_i\left(k_d \max(0, \mathbf{n} \cdot (\mathbf{v}_L)_i) + k_s \max(0, \mathbf{n} \cdot (\mathbf{v}_H)_i)^n\right)$$

Choosing the parameters

n_s in the range [0,100]

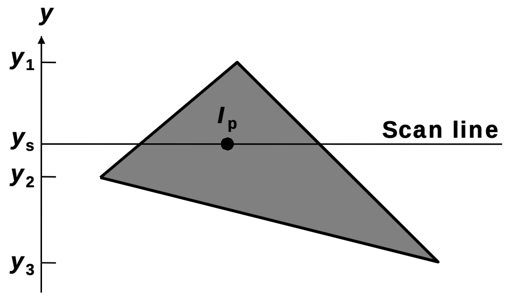
Try $k_a + k_d + k_s \le 1$

Use a small k_a (~0.1)

	n_s	k_d	k_s
Metal	Large	Small, color of metal	Large, color of metal
Plastic	Medium	Medium, color of plastic	Medium, white
Planet	0	Varying	0

Computing lighting at each pixel

- Most accurate approach: Compute component illumination at each pixel with individual positions, light directions, and viewing directions
- But this could be expensive...



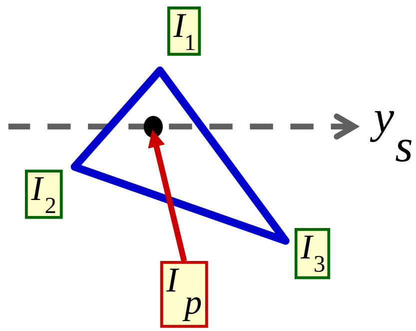
Shading models for polygons

- Flat Shading
  - Faceted Shading
  - Constant Shading
- Gouraud Shading
  - Intensity Interpolation Shading
  - Color Interpolation Shading
- Phong Shading
  - Normal-Vector Interpolation Shading

Flat Shading

- Compute constant shading function, over each polygon
- Same normal and light vector across whole polygon
- Constant shading for polygon

$$I_p = I$$

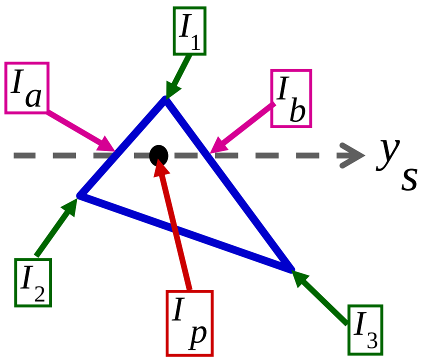


Intensity Interpolation (Gouraud)

$$I_a = I_1 \frac{y_s - y_2}{y_1 - y_2} + I_2 \frac{y_1 - y_s}{y_1 - y_2}$$

$$I_b = I_1 \frac{y_s - y_3}{y_1 - y_3} + I_3 \frac{y_1 - y_s}{y_1 - y_3}$$

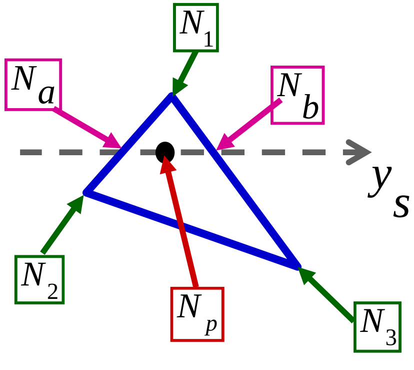
$$I_p = I_a \frac{x_b - x_p}{x_b - x_a} + I_b \frac{x_p - x_a}{x_b - x_a}$$



Normal Interpolation (Phong)

$$N_a = N_1 \frac{y_s - y_2}{y_1 - y_2} + N_2 \frac{y_1 - y_s}{y_1 - y_2}$$

$$N_b = N_1 \frac{y_s - y_3}{y_1 - y_3} + N_3 \frac{y_1 - y_s}{y_1 - y_3}$$



Normal Interpolation (Phong)

$$\tilde{N}_p = \frac{N_a}{\|N_a\|}\left[\frac{x_b - x_p}{x_b - x_a}\right] + \frac{N_b}{\|N_b\|}\left[\frac{x_p - x_a}{x_b - x_a}\right]$$

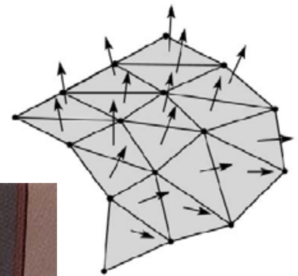
$$N_p = \frac{\tilde{N}_p}{\left\|\tilde{N}_p\right\|}$$

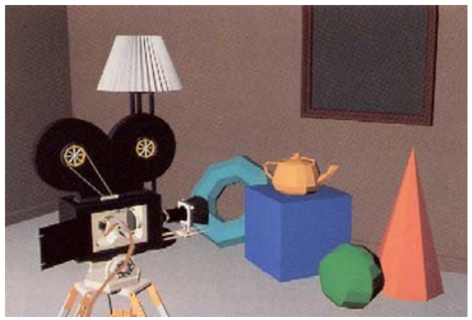
Normalizing makes this a unit vector

Gouraud v.s. Phong Shading

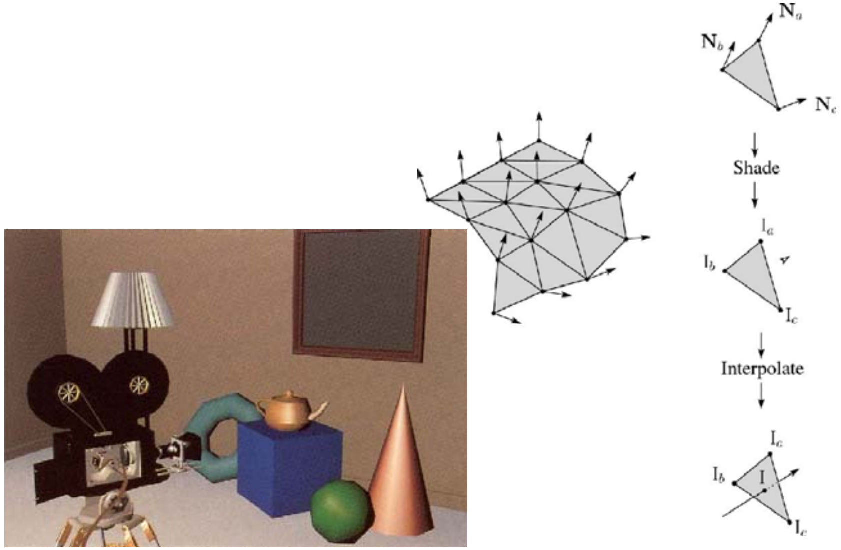
Gouraud Phong Gouraud Phong

Flat shading

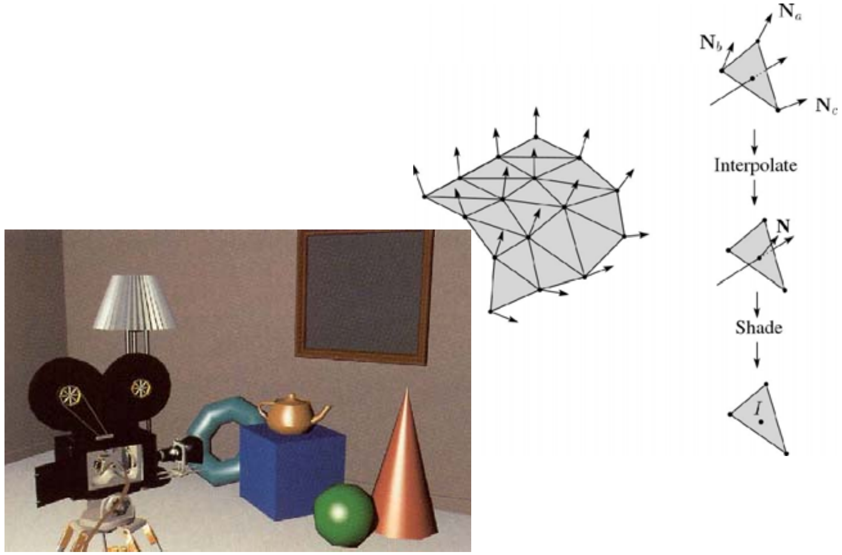




Gouraud shading

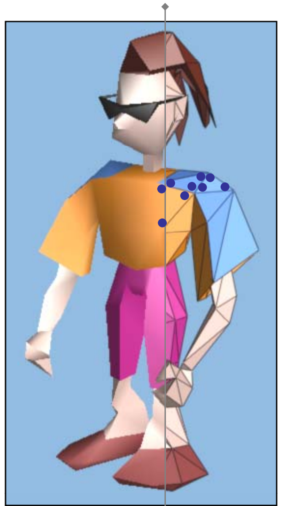


Phong shading



Graphics Pipeline

Triangle meshes



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$\{f_1\} : \{ v_1 , v_2 , v_3 \}$
$\{f_2\} : \{ v_3 , v_2 , v_4 \}$
…

connectivity

$\{v_1\} : (x,y,z)$
$\{v_2\} : (x,y,z)$
…

geometry

$\{f_1\}$: *“skin material”*
$\{f_2\}$: *“brown hair”*
…

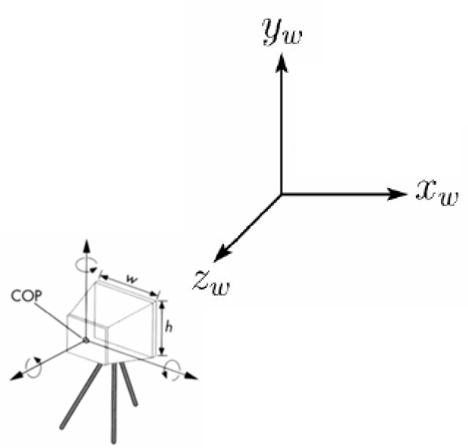
face attributes

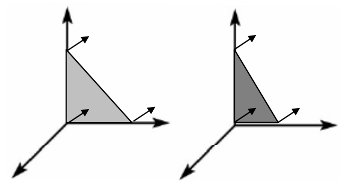
$\{v_2,f_1\} : (n_x,n_y,n_z)\ (u,v)$
$\{v_2,f_2\} : (n_x,n_y,n_z)\ (u,v)$
…

corner attributes

Review of graphics pipeline

Transformation



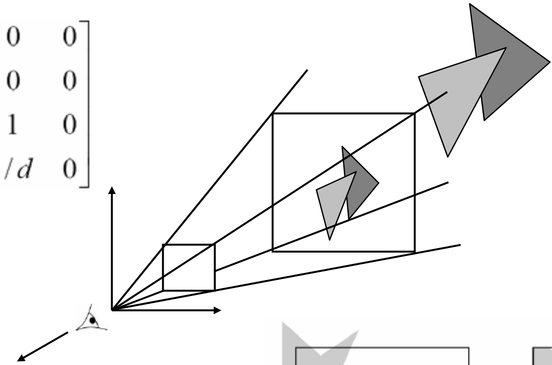


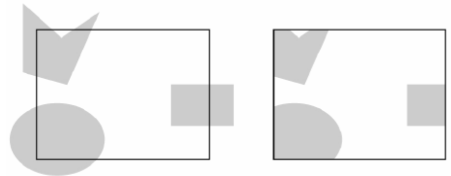
$$\begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Review of graphics pipeline

Projection & clipping

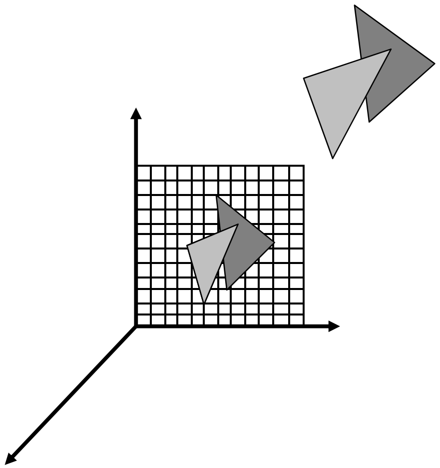
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix}$$





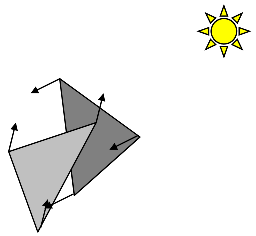
Review of graphics pipeline

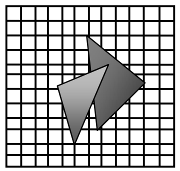
- Rasterization
- Visibility



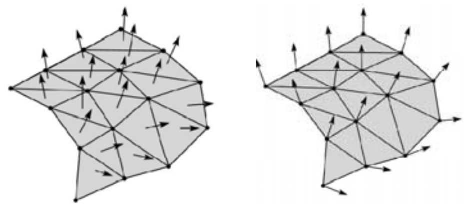
Review of graphics pipeline

- Shading





$$I = k_e + k_a I_a + \sum_i f(d_i) I_{li} \left[k_d (\mathbf{N} \cdot \mathbf{L}_i)_+ + k_s (\mathbf{V} \cdot \mathbf{R})_+^{n_s} \right]$$

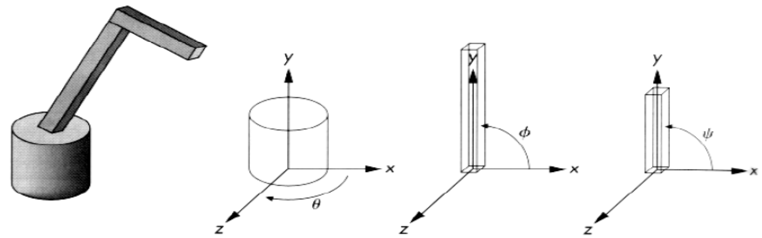


Animation

Hierarchical modeling: a robot arm

Consider this robot arm with 3 degrees of freedom:

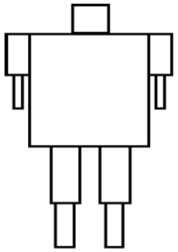
- Base rotates about its vertical axis by θ
- Lower arm rotates in its *xy*-plane by ϕ
- Upper arm rotates in its *xy*-plane by ψ

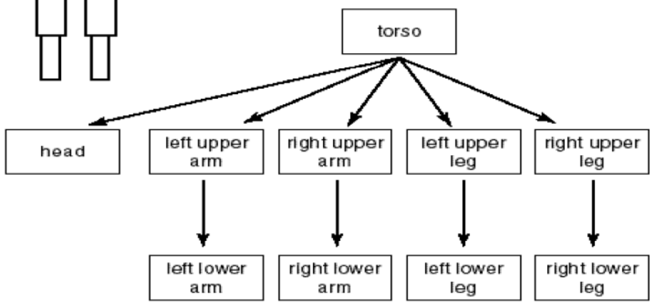


Hierarchical modeling

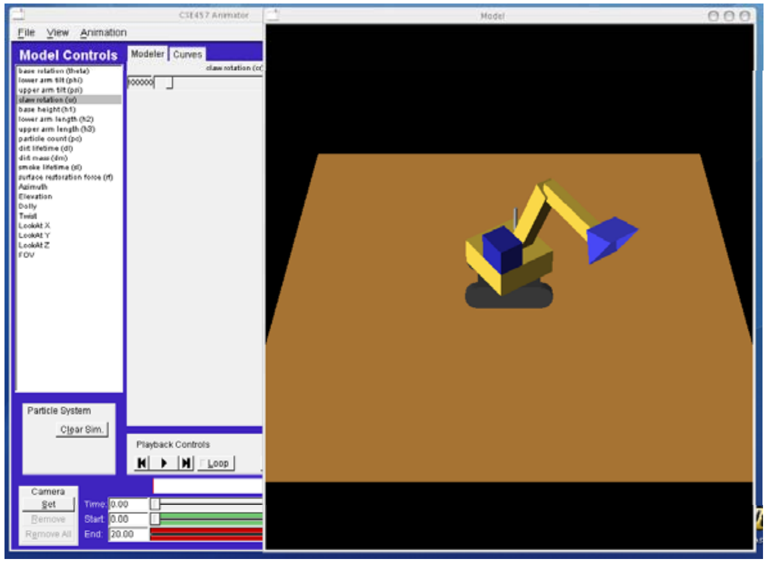
Hierarchical models can be composed of instances using trees or DAGs:

- edges contain geometric transformations
- nodes contain geometry (and possibly drawing attributes)





Animator demos



Videos

- TigerWang
- Racing

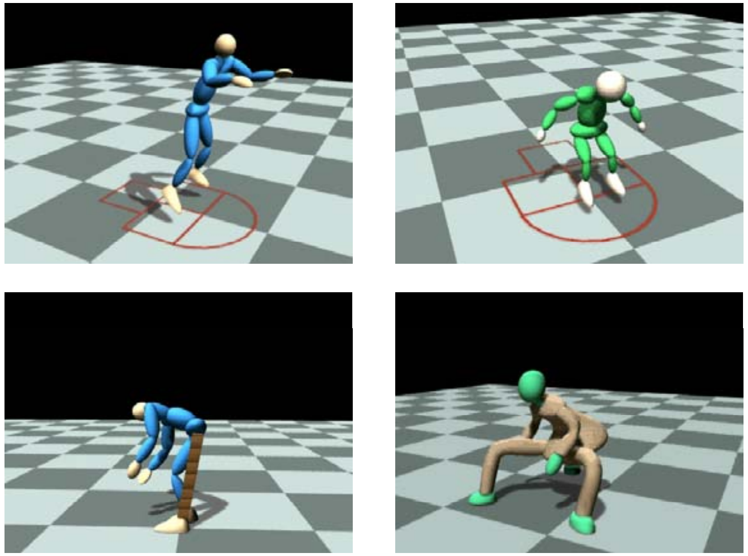
Advanced topics

Global illumination

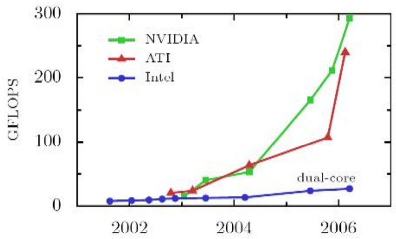
$$L_o(\mathbf{x},\omega,\lambda,t) = L_e(\mathbf{x},\omega,\lambda,t) + \int_\Omega f_r(\mathbf{x},\omega',\omega,\lambda,t) L_i(\mathbf{x},\omega',\lambda,t)(-\omega'\cdot\mathbf{n})d\omega'$$

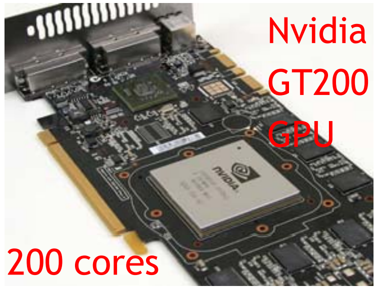
Complex materials

Realistic motion



Graphics hardware





Animation production

Animation production pipeline

story

text treatment

storyboard

voice

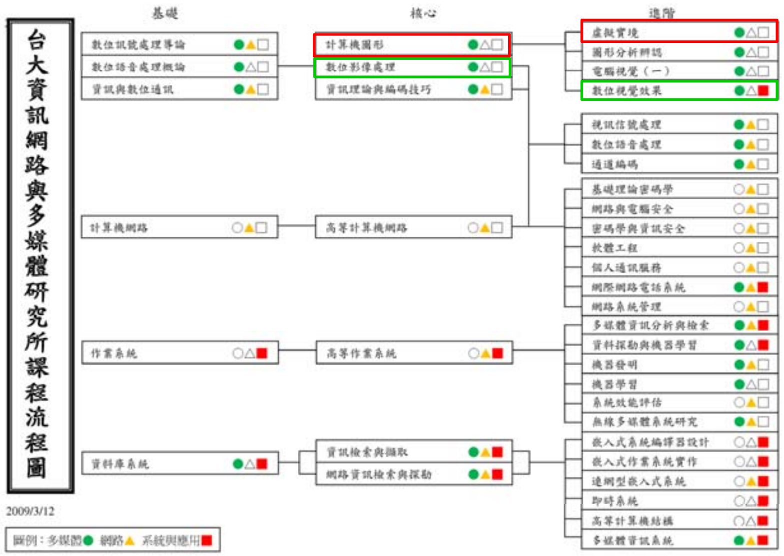
storyreal

look and feel

Animation production pipeline

What's next?

Related courses



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