Independent component analysis of correlated neuronal responses in area MT

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Abstract— **This work explores independent components of correlated firing in area MT. The pair-wise time-varying firing rate of two neighboring MT neurons in response to the same stimulus is estimated by the spline approximation to averaged spike trains over trials, and processed by the PottsICA algorithm for recovering independent sources. Numerical results show independent component analysis of correlated firing able to retrieve the effective source whose behaviors are highly consistent with variation of the stimulus.**

Keywords— **mutual information, population code, correlated firing, MT neurons, blind source separation**

I. Introduction

Correlated firing of neighboring neurons in areas from retina to visual cortex have been reported in [1]-[3]. Correlated neuronal activities have been considered as crucial materials for exploring population encoding of the stimulus. In this work, we employ independent component analysis[4][5][6] to retrieve independent sources of correlated neuronal responses and examine the consistency between behaviors of the extracted independent source and variation of the stimulus.

The data that we analysis was published by the authors of the work[2]. The data, filed as emu084 in the homepage[7], contains pair-wise spike trains measured from two neighboring MT neurons in response to stochastic motions. For each trial, the stimulus realized by stochastic motions of tremendous dots on a video screen is characterized by a coherence parameter whose sign specifies two-alternative moving directions of coherent dots and whose absolute scalar corresponds to the number of coherent dots. Every 45 milliseconds during a trial, according to the absolute value of the coherence parameter, a portion of dots on the screen are randomly selected as coherent dots, all of which are programmed to move along the direction specified the sign of the coherence parameter, and the remains are considered as random dots, each of which moves along a random direction by a small displacement.

In the previous work[2], it has been shown that each MT neuron has its own preference to the coherent direction of stochastic motions. The experimenter can thus select two neighboring MT neurons which have similar preferred directions, and measure their correlated firing in response to a variety of stimuli characterized by variant coherence parameters. In the experiment[2], the two possible signs of the coherence parameter respectively denote the preferred and anti-preferred directions of the monitored MT neurons.

II. Materials and methods

The data contains experimental results of 420 trials. For each trial j, the pair-wise spike train can be represented by $\{\mathbf{x_j[t]}\}_{\mathbf{t=1}}^{\mathbf{N}}$, where N denotes the number of total time steps during a trial, $\mathbf{x_j[t]} = (x_{j1}[t]\ x_{j2}[t])^{\mathbf{T}}$ denotes the response of the two neurons at the tth time step, and $x_{jk}[t] \in \{0, 1\}$ for all j, k, t. By the representation, a neuron generates at most one spike at each time step.

The experiment uses 15 possible coherence parameters, denoted by $C = \{c_i\}_{i=1}^{15}$, for 420 trials. Let $c(j) \in C$ denote the coherence parameter used at trial j. The averaged pair-wise spike train over all trials with coherence parameters identical to c_i is expressed as follows,

$$\xi_i[t] = \frac{1}{n_i} \sum_{j:c(j)=c_i} \mathbf{x_j}[t],$$

where n_i denotes the size of the set $\{j|c(j) = c_i\}$.

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The averaged pair-wise spike train, $\{\boldsymbol{\xi}_\mathbf{i}[\mathbf{t}]\}_\mathbf{t}$, is further processed by the spline interpolation to form the time-varying firing rate, $\{\boldsymbol{\lambda}_\mathbf{i}[\mathbf{t}]\}_{\mathbf{t}=1}^{\mathbf{N}}$, of the two neurons in response to the stimulus correspondent to c_i, where $\boldsymbol{\lambda}_\mathbf{i}[\mathbf{t}] = (\boldsymbol{\lambda}_{\mathbf{i1}}[\mathbf{t}], \boldsymbol{\lambda}_{\mathbf{i2}}[\mathbf{t}])^\mathbf{T}$. The spline approximation is carried out by the built-in spline tools in the MATLAB package. The time-varying firing rate of the two neurons is shown in figure 1, where the plot in figure 1a or 1b contains 15 curves. A curve in figure 1a represents the firing rate of one neuron, $\{\lambda_{i1}[t]\}_{t=1}^{N}$, and in figure 1b represents the firing rate of the other neuron, $\{\lambda_{i2}[t]\}_{t=1}^{N}$, in response to c_i.

Independent component analysis is further employed to retrieve independent sources from observations, $\{\boldsymbol{\lambda}_\mathbf{i}[\mathbf{t}]\}_{\mathbf{t}=1}^{\mathbf{N}}$, separately for each i. Since the ICA process is the same for all i, the subindex i is omitted in the following presentation. Assume that the time-varying firing rate of the two neurons in responding to the stimulus correspondent to $c \in C$ are linear mixtures of two independent sources, represented by

$$\boldsymbol{\lambda}[\mathbf{t}] = \mathbf{A}\mathbf{s}[\mathbf{t}], \tag{1}$$

where $\mathbf{A}$ denotes the unknown mixing matrix and $\{\mathbf{s}[\mathbf{t}]\}_{\mathbf{t}=1}^{\mathbf{N}}$ denotes the unknown independent sources. An ICA algorithm aims to search for an effective demixing matrix $\mathbf{W}$, by which the linear transformation,

$$\boldsymbol{\gamma}[\mathbf{t}] = \mathbf{W}\boldsymbol{\lambda}[\mathbf{t}], \tag{2}$$

could recover independent sources.

Let $\boldsymbol{\gamma} = (\gamma_1, \gamma_2)^\mathbf{T}$ denote a random vector that characterizes estimated sources $\{\boldsymbol{\gamma}[\mathbf{t}]\}_{\mathbf{t}=1}^{\mathbf{N}}$. The effectiveness of $\mathbf{W}$ can be quantified by the KL-divergence between the joint pdf of $\boldsymbol{\gamma}$ and the product of marginal pdfs of components in $\boldsymbol{\gamma}$, expressed by

$$KL(\boldsymbol{\gamma}) = \int_{R^2} p(\boldsymbol{\gamma}) \ln \frac{p(\boldsymbol{\gamma})}{p_1(\gamma_1)\, p_2(\gamma_2)} d\boldsymbol{\gamma}$$

where p denotes the joint pdf of $\boldsymbol{\gamma}$ and p_k denotes the marginal pdf of γ_k. If the two components in $\boldsymbol{\gamma}$ are independent, $KL(\boldsymbol{\gamma})$ reduces to zero. Since $KL(\boldsymbol{\gamma})$ must be non-negative, it measures the mutual information or dependency among components in $\boldsymbol{\gamma}$. The transformation in equation (2) will make a difference between $KL(\boldsymbol{\lambda})$ and $KL(\boldsymbol{\gamma})$, expressed by

$$D(\mathbf{W}) \triangleq \mathrm{KL}(\boldsymbol{\lambda}) - \mathrm{KL}(\boldsymbol{\gamma}),$$

which quantifies the reduced dependency caused by the transformation (2). Following the fact that the pdf of $\boldsymbol{\gamma}$ is the product of the pdf of $\boldsymbol{\lambda}$ and $|\det(\mathbf{W})|^{-1}$, the difference can be rewritten as follows,

$$D(\mathbf{W}) = \sum_{\mathbf{k}} \mathrm{H}(\boldsymbol{\lambda}_\mathbf{k}) + \ln|\det(\mathbf{W})| - \sum_{\mathbf{k}} \mathrm{H}(\boldsymbol{\gamma}_\mathbf{k}), \tag{3}$$

where the marginal entropy of univariate λ is defined by

$$H(\gamma_k) = -\int_R p_k(\gamma_k) \ln p_k(\gamma_k)\, d\gamma_k, \tag{4}$$

and $\det(\mathbf{W})$ denotes the determinant of $\mathbf{W}$.

Here we use the PottsICA algorithm[6] to estimate $\mathbf{W}$ and $D(\mathbf{W})$. The PottsICA algorithm uses the normalized histogram to represent the marginal pdf of each γ_k. By the marginal pdf representation, minimization of $KL(\boldsymbol{\gamma})$ with respect to $\mathbf{W}$ turns tractable and can be realized by neural relaxation based on a hybrid of mean field annealing and gradient descent methods. The marginal entropies in $D(\mathbf{W})$ can be also estimated based on the representation of normalized histograms for marginal pdfs. So we can estimate $D(\mathbf{W})$ for the demixing matrix obtained by the PottsICA algorithm.

III. Numerical results and Discussions

For each coherence parameter c_i in C, the time-varying firing rate of the monitored MT neurons, represented by $\{\boldsymbol{\lambda}_\mathbf{i}[\mathbf{t}]\}_{\mathbf{t}=1}^{\mathbf{N}}$, is processed by the PottsICA algorithm, and is then transformed to the independent firing rate, $\{\boldsymbol{\gamma}_\mathbf{i}(\mathbf{t})\}_{\mathbf{t}=1}^{\mathbf{N}}$, by the obtained demixing matrix $\mathbf{W}_\mathbf{i}$. The quantity $D(\mathbf{W}_\mathbf{i})$ that measures the reduced dependency by the demixing transformation is shown in figure 2 for all i, where the horizontal axis measures the coherence parameter.

It is observed that the scale of the reduced dependency appears graded when the coherence parameter is respectively set high negative, low negative, low positive and high positive. The evidence for dependent firing of the two MT neurons becomes stronger as the the coherence parameter increases. For high negative coherence, the obtained D value is zero; the two MT neurons tend to have independent firing. This is because the coherent direction specified by negative coherence is the anti-preferred direction of the monitored MT neurons.

For low negative coherence, the evidence for dependent firing is still weak. However when the coherence parameter is set low positive, the evidence for dependent firing becomes stronger relative to the case with negative coherence. With positive coherence, the coherent direction of stochastic motions is close to the preferred direction of the two MT neurons. In the occasion, both the individual firing rates of the two MT neurons are encoded with informations about the positive stimulus. An ICA algorithm helps to achieve two independent sources, one of which is expected to carry with most informations encoded within the two individual firing rates.

Two independent sources extracted from the time-varying firing rate of the two MT neurons are shown in figure 3. Since the PottsICA algorithm possesses the order-preserving property[6], we can relate the first component of $\boldsymbol{\gamma}_\mathbf{i}$ to one source and the second component of $\boldsymbol{\gamma}_\mathbf{i}$ to the other source for all i. The response of two independent sources to each c_i is shown in figure 3. A curve in figure 3a displays the sequence of $\{\lambda_{i1}[t]\}_{t=1}^N$ for approximating the response of one source, and in figure 3b draws the sequence of $\{\lambda_{i2}[t]\}_{t=1}^N$ for approximating the response of the other source to c_i.

From figure 3a, it is observed that the 15 curves within the time interval at about $[200, 350]$ form four clusters, respectively corresponding to high negative, low negative, low positive and high positive coherence. The response of this source to low positive coherence becomes distinguishable from that to low negative coherence. The response of the source in figure 3a is significantly consistent with variation of the stimulus. The same consistency can not be found in figure 1a and 1b, where observed firing rate of the two MT neurons is displayed. The extracted source in figure 3a have been shown to carry with most informations encoded within the two individual firing rates.

By independent component analysis of correlated firing of the two MT neurons, we have estimated an effective source that encodes most informations within the firing rate of pair-wise neurons for distinguishing variation of the stimulus. The signals of neuronal activities that encode the stimulus are partially contained by the measured individual spike train. The ICA algorithm plays a role of extracting significant signals from correlated firing of neighboring neurons following the assumption of linear mixtures. In the near future, we will explore independent component analysis for more paired MT neurons for further investigation to population codes of neuronal activities.

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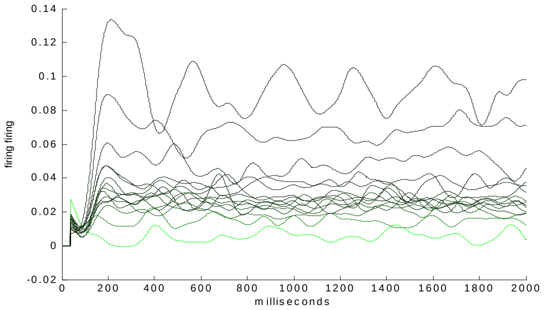
Figure list

Fig. 1 The pair-wise time-varying firing rate of two neighboring MT neurons estimated by the spline approximation to the averaged spike trains over trials.

Fig. 2 The dependency reduced by the demixing matrix.

Fig. 3 Independent components of the pair-wise time-varying firing rate of two MT neurons.



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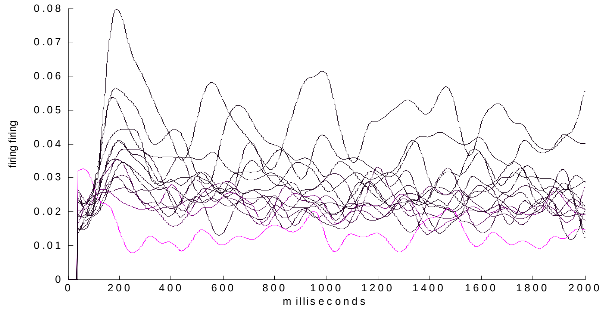

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Figure 1

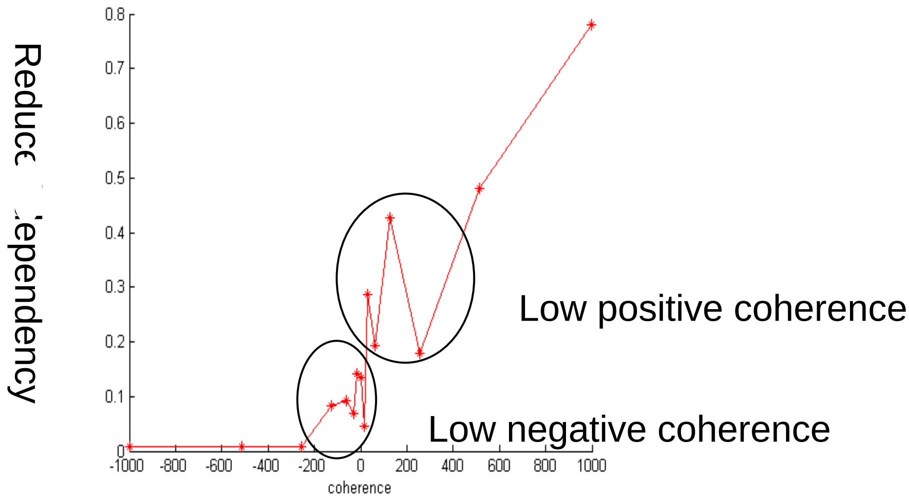
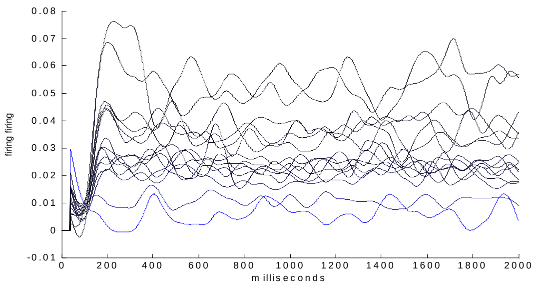


Figure 2



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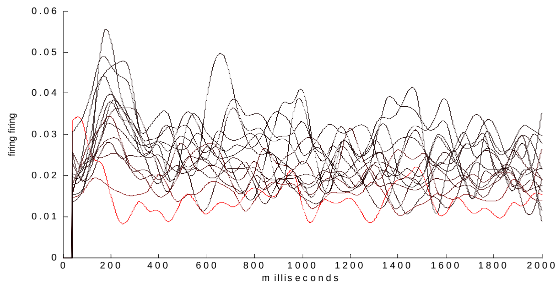

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Figure 3