Quantum Teleportation
&
Super Dense Coding

- Entanglement
- Quantum Teleportation
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Entanglement

$$| b_{00} >= \frac{1}{\sqrt{2}} | 0,0 > + \frac{1}{\sqrt{2}} | 1,1 >$$

$$| b_{01} >= \frac{1}{\sqrt{2}} | 0,1 > + \frac{1}{\sqrt{2}} | 1,0 >$$

$$| b_{10} >= \frac{1}{\sqrt{2}} | 0,0 > - \frac{1}{\sqrt{2}} | 1,1 >$$

$$| b_{11} >= \frac{1}{\sqrt{2}} | 0,1 > - \frac{1}{\sqrt{2}} | 1,0 >$$

Entanglement Correlation

Entanglement as a channel

- Classical:

1 bit

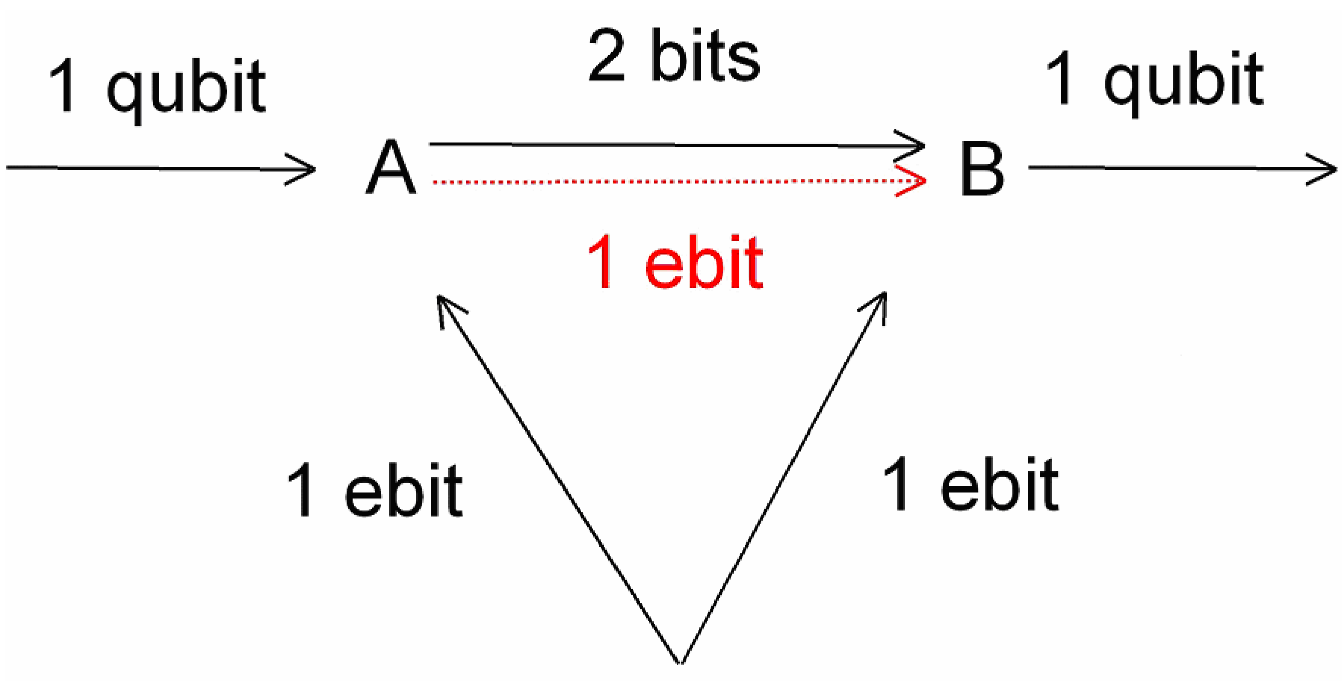
A ————————————————→ B

- Quantum:

1 ebit

A · >B

Quantum Teleportation



Quantum Teleportation

A wanna teleport $|a> = a|0>_A + b|1>_A$ through classical channel

- Give the system an entangled pair:

$$|y_0> = |a>|b_{00}>$$

$$= \frac{1}{\sqrt{2}}|a>(|00> + |11>)$$

$$= \frac{1}{\sqrt{2}}(a|000>_{AAB} + b|100>_{AAB} + a|011>_{AAB} + b|111>_{AAB})$$

- Apply **CNOT** on first two bits:

$$|y_1\rangle = \frac{1}{\sqrt{2}}(a|000\rangle_{AAB} + b|110\rangle_{AAB} + a|011\rangle_{AAB} + b|101\rangle_{AAB})$$

- Apply **H** on first bit:

$$|y_2\rangle = \frac{1}{2}[a|0\rangle_A + |1\rangle_A)|00\rangle_{AB} + b(|0\rangle_A - |1\rangle_A)|10\rangle_{AB}$$
$$+ a(|0\rangle_A + |1\rangle_A)|11\rangle_{AB} + b(|0\rangle_A - |1\rangle_A)|01\rangle_{AB}]$$
$$= \frac{1}{2}[a|000\rangle_{AAB} + a|100\rangle_{AAB} + b|010\rangle_{AAB} - b|110\rangle_{AAB}$$
$$+ a|011\rangle_{AAB} + a|111\rangle_{AAB} + b|001\rangle_{AAB} - b|101\rangle_{AAB}]$$
$$= \frac{1}{2}[|00\rangle_{AA}(a|0\rangle_B + b|1\rangle_B) + |10\rangle_{AA}(a|0\rangle_B - b|1\rangle_B)$$
$$+ |01\rangle_{AA}(a|1\rangle_B + b|0\rangle_B) + |11\rangle_{AA}(a|1\rangle_B - b|0\rangle_B)]$$

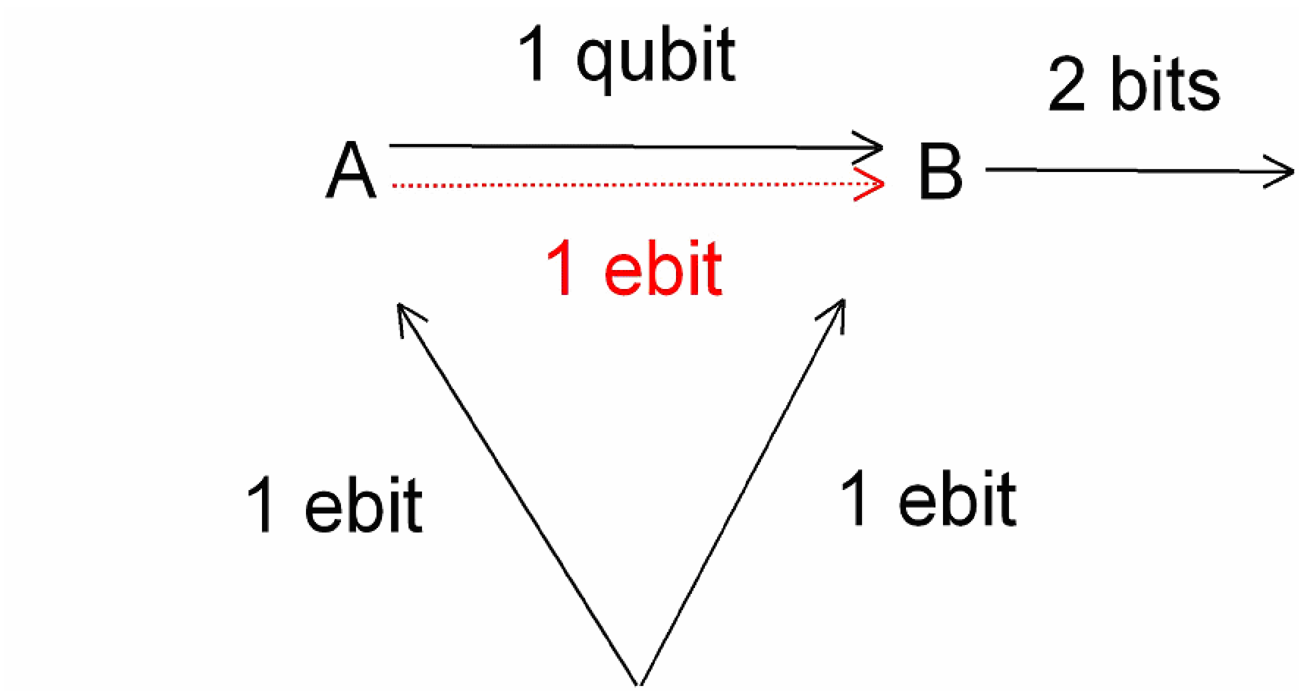
A measure first two bits:

A:	B:
$\|00\rangle$	$(a\|0\rangle+b\|1\rangle)/\sqrt{2}$
$\|01\rangle$	$(a\|0\rangle-b\|1\rangle)/\sqrt{2}$
$\|10\rangle$	$(a\|1\rangle+b\|0\rangle)/\sqrt{2}$
$\|11\rangle$	$(a\|1\rangle-b\|1\rangle)/\sqrt{2}$

Then B can get $|\alpha>$ according to A's measurement:

A:	B's action	After action
$\|00>$ ⟶	I ⟶	$(a\|0>+b\|1>)/\sqrt{2}$
$\|01>$ ⟶	Z ⟶	$(a\|0>+b\|1>)/\sqrt{2}$
$\|10>$ ⟶	X ⟶	$(a\|0>+b\|1>)/\sqrt{2}$
$\|11>$ ⟶	Y ⟶	$(a\|0>+b\|1>)/\sqrt{2}$

Super Dense Coding



Super Dense Coding

- A & B share an entangled pair:

$$|\psi_0\rangle = |\beta_1\rangle = \frac{1}{\sqrt{2}}(|0,0\rangle + |1,1\rangle)$$

- Then A apply an action {I,Z,X,Y} to the first bit:

$$|\psi_1\rangle = I\,|\beta_1\rangle = \frac{1}{\sqrt{2}}(|0,0\rangle_{AB} + |1,1\rangle_{AB}) = |\beta_1\rangle$$

$$|\psi_1\rangle = Z\,|\beta_1\rangle = \frac{1}{\sqrt{2}}(|0,0\rangle_{AB} - |1,1\rangle_{AB}) = |\beta_2\rangle$$

$$|\psi_1\rangle = X\,|\beta_1\rangle = \frac{1}{\sqrt{2}}(|0,1\rangle_{AB} + |1,0\rangle_{AB}) = |\beta_3\rangle$$

$$|\psi_1\rangle = Y\,|\beta_1\rangle = \frac{1}{\sqrt{2}}(|0,1\rangle_{AB} - |1,0\rangle_{AB}) = |\beta_4\rangle$$

- A send her qubit to B and B apply a **CNOT** on this two qubits:

$$|y_2\rangle = CNOT\,|b_1\rangle = \frac{1}{\sqrt{2}}(|0\rangle_B + |1\rangle_B)\,|0\rangle_B$$

$$|y_2\rangle = CNOT\,|b_2\rangle = \frac{1}{\sqrt{2}}(|0\rangle_B - |1\rangle_B)\,|0\rangle_B$$

$$|y_2\rangle = CNOT\,|b_3\rangle = \frac{1}{\sqrt{2}}(|0\rangle_B + |1\rangle_B)\,|1\rangle_B$$

$$|y_2\rangle = CNOT\,|b_4\rangle = \frac{1}{\sqrt{2}}(|0\rangle_B - |1\rangle_B)\,|1\rangle_B$$

- Hence B can discern $\{b_1, b_2\}$ from $\{b_3, b_4\}$ by measuring the second bit and the system state becomes

$$|y_3\rangle^{b1}_{b2} = \frac{1}{\sqrt{2}}(|0\rangle_B \pm |1\rangle_B) \quad \text{or} \quad |y_3\rangle^{b3}_{b4} = \frac{1}{\sqrt{2}}(|0\rangle_B \pm |1\rangle_B)$$

- Then B applies a **H** on $|y_3>$ and takes the measurement will discern b_1/b_3 from b_2/b_4:

$$H|y_3>_{b2}^{b1} = \begin{cases} |0>_B \\ |1>_B \end{cases}$$

or

$$H|y_3>_{b4}^{b3} = \begin{cases} |0>_B \\ |1>_B \end{cases}$$

- Thus B can exactly know what action A taked, and A can encode 2 bits by this action set.