Quantum Reincarnate

投胎 轉世 化身

The state of particle 1 to be sent to Bob.

$$|\phi\rangle = a|\uparrow_1\rangle + b|\downarrow_1\rangle = \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \end{pmatrix}$$

Alice create two entangled particles. The state for these two entangled particles:

$$\frac{1}{\sqrt{2}}(|\uparrow_2\rangle \otimes |\downarrow_3\rangle. - |\downarrow_2\rangle \otimes |\uparrow_3\rangle)$$

Combined wave function for the 3-particle system as a direct product state

$$|\Psi\rangle = |\phi\rangle \otimes \frac{1}{\sqrt{2}}(|\uparrow_2\rangle \otimes |\downarrow_3\rangle. - |\downarrow_2\rangle \otimes |\uparrow_3\rangle)$$

or, equivalently,

$$|\Psi\rangle = \frac{a}{\sqrt{2}}(|\uparrow_1\rangle \otimes |\uparrow_2\rangle \otimes |\downarrow_3\rangle. - |\uparrow_1\rangle \otimes |\downarrow_2\rangle \otimes |\uparrow_3\rangle) +$$

$$\frac{b}{\sqrt{2}}(|\downarrow_1\rangle \otimes |\uparrow_2\rangle \otimes |\downarrow_3\rangle. - |\downarrow_1\rangle \otimes |\downarrow_2\rangle \otimes |\uparrow_3\rangle)$$

Alice apply unitary operation U to the two particles she has.

Bell operator basis 1992

$$|\Psi^A\rangle = \frac{1}{\sqrt{2}}(|\uparrow_1\rangle|\downarrow_2\rangle. - |\downarrow_1\rangle|\uparrow_2\rangle)$$

$$|\Psi^B\rangle = \frac{1}{\sqrt{2}}(|\uparrow_1\rangle|\downarrow_2\rangle. + |\downarrow_1\rangle|\uparrow_2\rangle)$$

$$|\Psi^C\rangle = \frac{1}{\sqrt{2}}(|\uparrow_1\rangle|\uparrow_2\rangle. - |\downarrow_1\rangle|\downarrow_2\rangle)$$

$$|\Psi^D\rangle = \frac{1}{\sqrt{2}}(|\uparrow_1\rangle|\uparrow_2\rangle. + |\downarrow_1\rangle|\downarrow_2\rangle)$$

The total wave function for all three particles.

$$|\Psi\rangle = \frac{1}{2}[(|\Psi^A\rangle(-a|\uparrow_3\rangle - b|\downarrow_3\rangle) + |\Psi^B\rangle(-a|\uparrow_3\rangle + b|\downarrow_3\rangle) + |\Psi^C\rangle(a|\downarrow_3\rangle + b|\uparrow_3\rangle) + |\Psi^D\rangle(a|\downarrow_3\rangle - b|\uparrow_3\rangle)]$$

In the form of column vectors

$$|\Psi\rangle = \frac{1}{2}[(|\Psi^A\rangle\begin{pmatrix}-a\\-b\end{pmatrix} + |\Psi^B\rangle\begin{pmatrix}-a\\b\end{pmatrix} + |\Psi^C\rangle\begin{pmatrix}b\\a\end{pmatrix} + |\Psi^D\rangle\begin{pmatrix}-b\\a\end{pmatrix})]$$

.

After Alice makes her measurement on particles 1 and 2, they will be in a joint state described by one of the eigenvectors

$$|\Psi^A\rangle, |\Psi^B\rangle, |\Psi^C\rangle, \text{or } |\Psi^D\rangle$$

and particle 3 will be in one of the states described by

$$\begin{pmatrix}-a\\-b\end{pmatrix}_3, \begin{pmatrix}-a\\b\end{pmatrix}_3, \begin{pmatrix}b\\a\end{pmatrix}_3, \begin{pmatrix}-b\\a\end{pmatrix}_3$$

Each pair of outcome has a 1/4 chance of being the result of Alice's measurement.

By the use of an appropriate rotation

Alice's result for the state of particles 1 *and* 2	The rotation Bob must performe on particle 3
$\lvert\Psi^A\rangle$	$\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$
$\lvert\Psi^B\rangle$	$\begin{pmatrix}-1 & 0\\0 & 1\end{pmatrix}$
$\lvert\Psi^C\rangle$	$\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$
$\lvert\Psi^D\rangle$	$\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}$

Unitary rotation operators

$$L = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \quad \mathrm{R} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

The phase shift operators

$$\mathrm{S} = \begin{pmatrix} i & 0 \\ 0 & 1 \end{pmatrix} \quad \mathrm{T} = \begin{pmatrix} -1 & 0 \\ 0 & -\mathrm{i} \end{pmatrix}$$

$$L|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \qquad L|1\rangle = \frac{1}{\sqrt{2}}(-|0\rangle + |1\rangle)$$

$$R|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \qquad R|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$S|0\rangle = i|0\rangle \ :90^0 \text{ phase shift} \qquad S|1\rangle = |1\rangle \ :0^0 \text{ phase shift}$$

$$T|0\rangle = -|0\rangle \ :180^0 \text{ phase shift} \qquad \mathrm{T}|1\rangle = -i|1\rangle : -90^0 \text{ phase shift}$$

The XOR operator “controlled-NOT” gate

$$XOR = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$