

## Randomness and Quantum Computers

$$U(\mathbf{q}) = \begin{pmatrix} \cos \mathbf{q} & -\sin \mathbf{q} \\ \sin \mathbf{q} & \cos \mathbf{q} \end{pmatrix}$$

$$U(\mathbf{q}) \begin{pmatrix} \mathbf{w}_0 \\ \mathbf{w}_1 \end{pmatrix} = \begin{pmatrix} \cos \mathbf{q} & -\sin \mathbf{q} \\ \sin \mathbf{q} & \cos \mathbf{q} \end{pmatrix} \begin{pmatrix} \mathbf{w}_0 \\ \mathbf{w}_1 \end{pmatrix} = \begin{pmatrix} \mathbf{w}_0 \cos \mathbf{q} - \mathbf{w}_1 \sin \mathbf{q} \\ \mathbf{w}_0 \sin \mathbf{q} + \mathbf{w}_1 \cos \mathbf{q} \end{pmatrix}$$

In particular,

$$\begin{aligned} U\left(\frac{\mathbf{p}}{4}\right) |0\rangle &= U\left(\frac{\mathbf{p}}{4}\right) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} \\ &= \frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle \end{aligned}$$