Grover's algorithm

Problem

Unsorted states: $S=\{S_1,S_2,\ldots S_N\}$

Condition function: $C(|S_i>)=d_{iu}$

Mission: find S_ν from S

Algorithm

(i) Initialization: $|y>=\frac{1}{\sqrt{N}}(|S_1>+|S_2>+\cdots+|S_N>)$

(ii) Repeat the following operations $O(\sqrt{N})$ times:

(a) Let the system be in any state S_i:

If $C(|S_i>)=1$, rotate the phase of by p radians;

If $C(|S_i>)=0$, leave the system unaltered.

(b) Apply the diffusion transform D=-I+2P

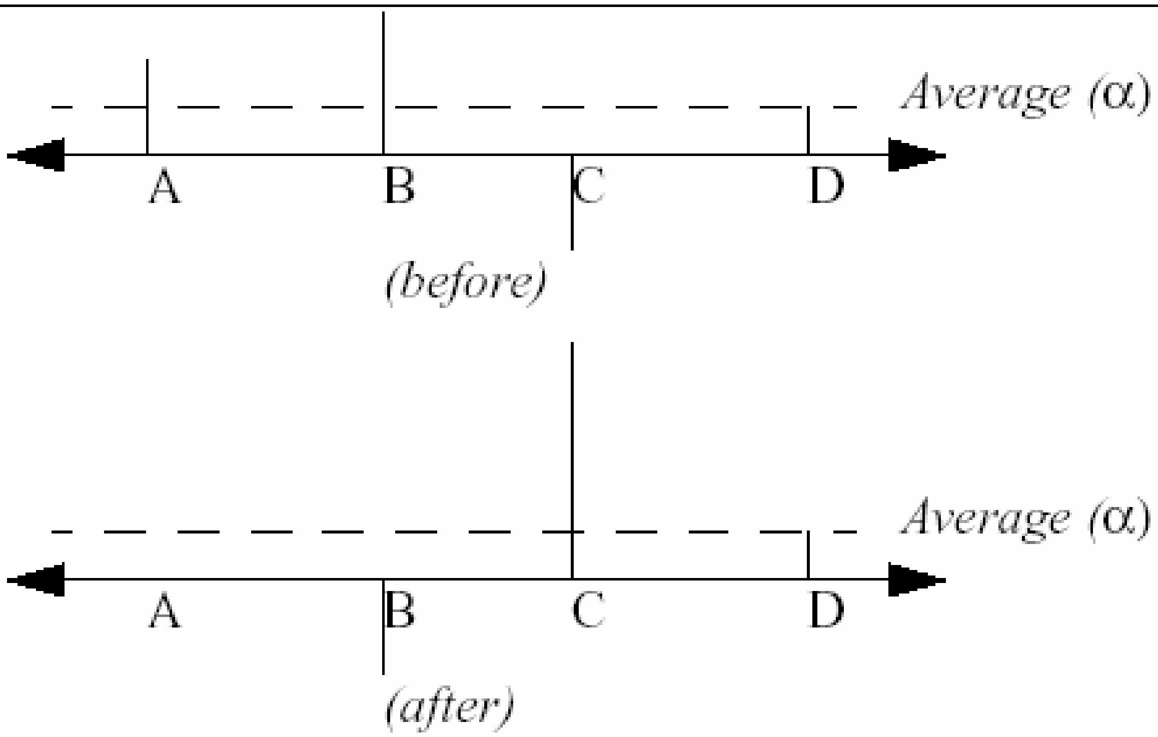
where $P_{ij}=\frac{1}{N}$ is the amplitude average operator.

(iii) Measure the resulting state. $S\nu$ appears with a probability of at least 0.5

Mechanism

$$D\overline{n} = (-I + 2P)\overline{n} = -\overline{n} + 2P\overline{n}$$

⇨ D is the inversion operation about average:

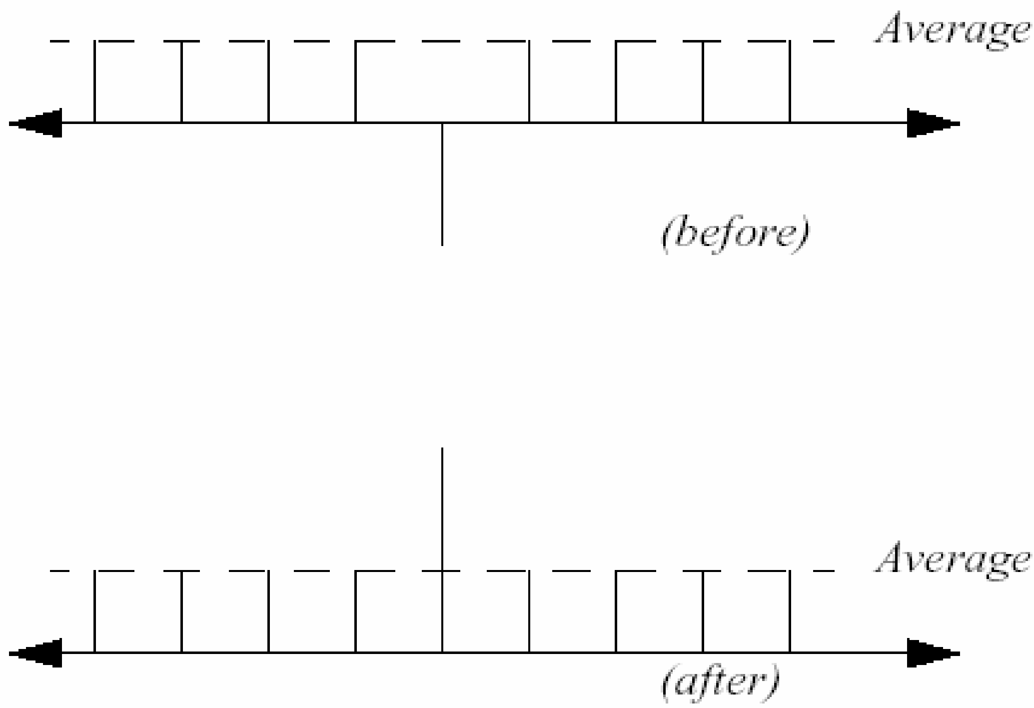


***Inversion about average* operation.**

For the case one component with amplitude= $-\sqrt{1-C^2}$ /others with amplitude= $C/\sqrt{N}$, step (ii) results in the magnitude increase $2C/\sqrt{N}$ (=distance from average +average-original value

$$=(\sqrt{1-C^2}+C/\sqrt{N})+C/\sqrt{N}-\sqrt{1-C^2}\ )$$

to this special component:



Inversion about average operation is applied to a superposition where all but one of the components are initially identical and of magnitude $O\left(\frac{1}{\sqrt{N}}\right)$.

Once its magnitude $\sqrt{1-C^2} < 1/\sqrt{2}$, the increase

$\frac{2C}{\sqrt{N}} > \frac{1}{\sqrt{2N}}$. Hence there exists a $M < \sqrt{N}$

such that perform step (ii) M times, the desired

state appears with a probability greater than 0.5.