If a wave function for two-particle system is factorable

$$y\ (r1, r2) = y_1(r1)y_2(r2)$$

we call these two particles are **<u>uncorrelated</u>**.

Most two-particle states are not the case!

For instance,

$$y = Ae^{-(x1-x2-a)^2}$$

is a eigenfunction of

$$H = -\frac{\hbar^2}{2m}\partial^2_{x1} - \frac{\hbar^2}{2m}\partial^2_{x2} + \frac{4\hbar^2}{m}(x1-x2-a)^2$$

with eigenvalue $\frac{2\hbar^2}{m}$, and y describes the **<u>correlation</u>** between particle 1 and particle 2.
(Once the position of one particle is measured, the other one' s position is **ascertained** without further measurement since x1-x2=a)

Another example is any function with the form $y\ (x1-x2)$. Note that

$$
\begin{aligned}
&(\widehat{p1} + \widehat{p2})y\ (x1 - x2) \\
&= \frac{\hbar}{i}(\partial_{x1} + \partial_{x2})y\ (x1 - x2) \\
&= \frac{\hbar}{i}(y\ '(x1 - x2) - y\ '(x1 - x2)) \\
&= 0y\ (x1 - x2)
\end{aligned}
$$

Hence, if the moment of particle 2 is measured and found to have the value p2, then particle 1 is **certain** to be found to have the sharp momentum value p1=-p2. (Corresponding to total momentum=0 in the center of mass frame)

If we assume the motion is constrained in one dimension and the momentum magnitude can only be |p|, the above state can be expressed as

$$|y\ > \equiv |\, p1, p2 >_{1,2}$$

$$= \frac{1}{\sqrt{2}}|1,-1> \pm \frac{1}{\sqrt{2}}|-1,1>$$

, an **<u>entanglement</u>**!

In fact, |P> are special cases of the so-called Bell or EPR states:

$$b1 = \frac{1}{\sqrt{2}}|1,1> + \frac{1}{\sqrt{2}}|-1,-1>$$

$$b2 = \frac{1}{\sqrt{2}}|1,1> - \frac{1}{\sqrt{2}}|-1,-1>$$

$$b3 = \frac{1}{\sqrt{2}}|1,-1> + \frac{1}{\sqrt{2}}|-1,1>$$

$$b4 = \frac{1}{\sqrt{2}}|1,-1> - \frac{1}{\sqrt{2}}|-1,1>$$

Their common feature is the inability to be factorized to the tensor product of two pure states.

For example,

If

$$b1 = (a|1> + b|-1>) \otimes (c|1> + d|-1>)$$

$$= ac|1,1> + ad|1,-1> + bc|-1,1> + bd|-1$$

$$= \frac{1}{\sqrt{2}}|1,1> + \frac{1}{\sqrt{2}}|-1,-1>$$

then

$$\begin{cases} ac = bd = \dfrac{1}{\sqrt{2}} \\ ad = bc = 0 \end{cases}$$

But the equations have no solution!

In summary, if a composite state cannot be separated as a tensor product of its component states, such composite state is then called **entangled**.