Support Vector Machines and Kernel Methods

Chih-Jen Lin
Department of Computer Science
National Taiwan University

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Outline

- Basic concepts: SVM and kernels
- Training SVM
- Practical use of SVM
- Research directions: large-scale training
- Research directions: linear SVM
- Research directions: others
- Conclusions

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Data Classification

- Given training data in different classes (labels known)

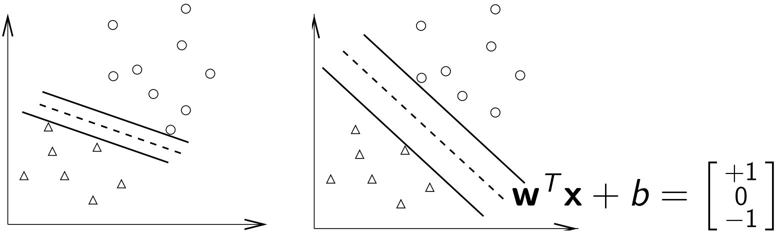
  Predict test data (labels unknown)
- Training and testing

Support Vector Classification

- Training vectors : $\mathbf{x}_i, i = 1, \ldots, l$
- Feature vectors. For example,
  A patient = [height, weight, . . .]^T
- Consider a simple case with two classes:
  Define an indicator vector $\mathbf{y}$

$$y_i = \begin{cases} 1 & \text{if } \mathbf{x}_i \text{ in class 1} \\ -1 & \text{if } \mathbf{x}_i \text{ in class 2} \end{cases}$$

- A hyperplane which separates all data



- A separating hyperplane: $\mathbf{w}^T\mathbf{x} + b = 0$

$$\begin{aligned} (\mathbf{w}^T\mathbf{x}_i) + b \geq 1 \quad & \text{if } y_i = 1 \\ (\mathbf{w}^T\mathbf{x}_i) + b \leq -1 \quad & \text{if } y_i = -1 \end{aligned}$$

- Decision function $f(\mathbf{x}) = \text{sgn}(\mathbf{w}^T\mathbf{x} + b)$, $\mathbf{x}$: test data

  Many possible choices of $\mathbf{w}$ and b

Maximal Margin

- Distance between $\mathbf{w}^T\mathbf{x} + b = 1$ and -1:

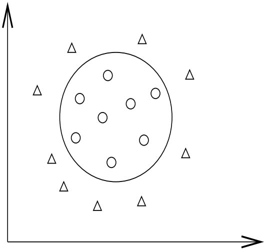
$$2/\|\mathbf{w}\| = 2/\sqrt{\mathbf{w}^T\mathbf{w}}$$

- A quadratic programming problem (Boser et al., 1992)

$$\begin{aligned} \min_{\mathbf{w},b} \quad & \frac{1}{2}\mathbf{w}^T\mathbf{w} \\ \text{subject to} \quad & y_i(\mathbf{w}^T\mathbf{x}_i + b) \geq 1, \\ & i = 1, \ldots, l. \end{aligned}$$

Data May Not Be Linearly Separable

- An example:



- Allow training errors
- Higher dimensional (maybe infinite) feature space

$$\phi(\mathbf{x}) = [\phi_1(\mathbf{x}), \phi_2(\mathbf{x}), \ldots]^T.$$

- Standard SVM (Boser et al., 1992; Cortes and Vapnik, 1995)

$$
\begin{aligned}
\min_{\mathbf{w},b,\boldsymbol{\xi}} \quad & \frac{1}{2}\mathbf{w}^T\mathbf{w} + C\sum_{i=1}^{l}\xi_i \\
\text{subject to} \quad & y_i(\mathbf{w}^T\phi(\mathbf{x}_i) + b) \geq 1 - \xi_i, \\
& \xi_i \geq 0,\ i = 1, \ldots, l.
\end{aligned}
$$

- Example: $\mathbf{x} \in R^3, \phi(\mathbf{x}) \in R^{10}$

$$
\begin{aligned}
\phi(\mathbf{x}) = \; & [1, \sqrt{2}x_1, \sqrt{2}x_2, \sqrt{2}x_3, x_1^2, \\
& x_2^2, x_3^2, \sqrt{2}x_1x_2, \sqrt{2}x_1x_3, \sqrt{2}x_2x_3]^T
\end{aligned}
$$

Finding the Decision Function

- **w**: maybe infinite variables
- The dual problem: finite number of variables

$$\begin{aligned} \min_{\boldsymbol{\alpha}} \quad & \frac{1}{2}\boldsymbol{\alpha}^T Q \boldsymbol{\alpha} - \mathbf{e}^T \boldsymbol{\alpha} \\ \text{subject to} \quad & 0 \leq \alpha_i \leq C, i = 1, \ldots, l \\ & \mathbf{y}^T \boldsymbol{\alpha} = 0, \end{aligned}$$

where $Q_{ij} = y_i y_j \phi(\mathbf{x}_i)^T \phi(\mathbf{x}_j)$ and $\mathbf{e} = [1, \ldots, 1]^T$

- At optimum

$$\mathbf{w} = \sum_{i=1}^{l} \alpha_i y_i \phi(\mathbf{x}_i)$$

- A finite problem: #variables = #training data

Kernel Tricks

- $Q_{ij} = y_i y_j \phi(\mathbf{x}_i)^T \phi(\mathbf{x}_j)$ needs a closed form
- Example: $\mathbf{x}_i \in R^3, \phi(\mathbf{x}_i) \in R^{10}$

$$\phi(\mathbf{x}_i) = [1, \sqrt{2}(x_i)_1, \sqrt{2}(x_i)_2, \sqrt{2}(x_i)_3, (x_i)_1^2,$$
$$(x_i)_2^2, (x_i)_3^2, \sqrt{2}(x_i)_1(x_i)_2, \sqrt{2}(x_i)_1(x_i)_3, \sqrt{2}(x_i)_2(x_i)_3]^T$$

Then $\phi(\mathbf{x}_i)^T \phi(\mathbf{x}_j) = (1 + \mathbf{x}_i^T \mathbf{x}_j)^2$.

- Kernel: $K(\mathbf{x}, \mathbf{y}) = \phi(\mathbf{x})^T \phi(\mathbf{y})$; common kernels:

$$e^{-\gamma\|\mathbf{x}_i - \mathbf{x}_j\|^2}, \text{ (Radial Basis Function)}$$
$$(\mathbf{x}_i^T \mathbf{x}_j / a + b)^d \text{ (Polynomial kernel)}$$

Can be inner product in infinite dimensional space

Assume $x \in R^1$ and $\gamma > 0$.

$$e^{-\gamma\|x_i-x_j\|^2} = e^{-\gamma(x_i-x_j)^2} = e^{-\gamma x_i^2+2\gamma x_i x_j-\gamma x_j^2}$$

$$=e^{-\gamma x_i^2-\gamma x_j^2}\left(1 + \frac{2\gamma x_i x_j}{1!} + \frac{(2\gamma x_i x_j)^2}{2!} + \frac{(2\gamma x_i x_j)^3}{3!} + \cdots\right)$$

$$=e^{-\gamma x_i^2-\gamma x_j^2}\left(1\cdot 1+\sqrt{\frac{2\gamma}{1!}}x_i\cdot\sqrt{\frac{2\gamma}{1!}}x_j+\sqrt{\frac{(2\gamma)^2}{2!}}x_i^2\cdot\sqrt{\frac{(2\gamma)^2}{2!}}x_j^2\right.$$

$$\left.+\sqrt{\frac{(2\gamma)^3}{3!}}x_i^3\cdot\sqrt{\frac{(2\gamma)^3}{3!}}x_j^3+\cdots\right) = \phi(x_i)^T\phi(x_j),$$

where

$$\phi(x) = e^{-\gamma x^2}\left[1, \sqrt{\frac{2\gamma}{1!}}x, \sqrt{\frac{(2\gamma)^2}{2!}}x^2, \sqrt{\frac{(2\gamma)^3}{3!}}x^3, \cdots\right]^T.$$

Issues

- So what kind of kernel should I use?
- What kind of functions are valid kernels?
- How to decide kernel parameters?
- Some of these issues will be discussed later

Decision function

- At optimum

$$\mathbf{w} = \sum_{i=1}^{l} \alpha_i y_i \phi(\mathbf{x}_i)$$

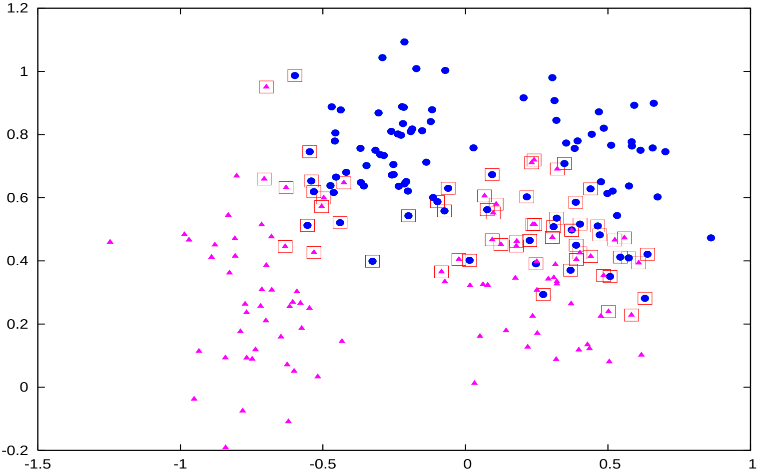
- Decision function

$$\begin{aligned} & \mathbf{w}^T \phi(\mathbf{x}) + b \\ = & \sum_{i=1}^{l} \alpha_i y_i \phi(\mathbf{x}_i)^T \phi(\mathbf{x}) + b \\ = & \sum_{i=1}^{l} \alpha_i y_i K(\mathbf{x}_i, \mathbf{x}) + b \end{aligned}$$

- Only $\phi(\mathbf{x}_i)$ of $\alpha_i > 0$ used $\Rightarrow$ support vectors

Support Vectors: More Important Data

Only $\phi(\mathbf{x}_i)$ of $\alpha_i > 0$ used $\Rightarrow$ support vectors



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Large Dense Quadratic Programming

$$\begin{aligned} \min_{\boldsymbol{\alpha}} \quad & \frac{1}{2}\boldsymbol{\alpha}^T Q \boldsymbol{\alpha} - \mathbf{e}^T \boldsymbol{\alpha} \\ \text{subject to} \quad & 0 \le \alpha_i \le C, i = 1, \ldots, l \\ & \mathbf{y}^T \boldsymbol{\alpha} = 0 \end{aligned}$$

- $Q_{ij} \neq 0$, Q : an l by l fully dense matrix
- 50,000 training points: 50,000 variables:
  $(50,000^2 \times 8/2)$ bytes $=$ 10GB RAM to store Q
- Traditional optimization methods:
  Newton, quasi Newton cannot be directly applied

Decomposition Methods

- Working on some variables each time (e.g., Osuna et al., 1997; Joachims, 1998; Platt, 1998)
- Similar to coordinate-wise minimization
- Working set B, $N = \{1, \ldots, l\} \backslash B$ fixed
- Sub-problem at the kth iteration:

$$\min_{\boldsymbol{\alpha}_B} \quad \frac{1}{2} \begin{bmatrix} \boldsymbol{\alpha}_B^T & (\boldsymbol{\alpha}_N^k)^T \end{bmatrix} \begin{bmatrix} Q_{BB} & Q_{BN} \\ Q_{NB} & Q_{NN} \end{bmatrix} \begin{bmatrix} \boldsymbol{\alpha}_B \\ \boldsymbol{\alpha}_N^k \end{bmatrix} - \begin{bmatrix} \mathbf{e}_B^T & (\mathbf{e}_N^k)^T \end{bmatrix} \begin{bmatrix} \boldsymbol{\alpha}_B \\ \boldsymbol{\alpha}_N^k \end{bmatrix}$$

$$\text{subject to} \quad 0 \le \alpha_t \le C, t \in B, \ \mathbf{y}_B^T \boldsymbol{\alpha}_B = -\mathbf{y}_N^T \boldsymbol{\alpha}_N^k$$

Avoid Memory Problems

- The new objective function

$$\frac{1}{2}\boldsymbol{\alpha}_B^T Q_{BB} \boldsymbol{\alpha}_B + (-\mathbf{e}_B + Q_{BN} \boldsymbol{\alpha}_N^k)^T \boldsymbol{\alpha}_B + \text{ constant}$$

- Only B columns of Q needed ($|B| \geq 2$)
- Calculated when used

  Trade time for space

How Decomposition Methods Perform?

- Convergence not very fast
- But, no need to have very accurate α

  Prediction not affected much
- In some situations, # support vectors $\ll$ # training points

  Initial $\alpha^1 = 0$, some instances never used

- An example of training 50,000 instances using LIBSVM

  ```
  $svm-train -c 16 -g 4 -m 400 22features
  Total nSV = 3370
  Time 79.524s
  ```

- On a Xeon 2.0G machine
- Calculating the whole Q takes more time
- $\#\text{SVs} = 3{,}370 \ll 50{,}000$

  A good case where some remain at zero all the time

Issues of Decomposition Methods

Techniques for faster decomposition methods

- store recently used kernel elements
- working set size/selection
- theoretical issues: convergence
- and others (details not discussed here)

Major software:

- LIBSVM

  `http://www.csie.ntu.edu.tw/~cjlin/libsvm`
- *SVM^{light}*

  `http://svmlight.joachims.org`

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Let's Try a Practical Example

A problem from astroparticle physics

```
1 1:2.61e+01 2:5.88e+01 3:-1.89e-01 4:1.25e+02
1 1:5.70e+01 2:2.21e+02 3:8.60e-02  4:1.22e+02
1 1:1.72e+01 2:1.73e+02 3:-1.29e-01 4:1.25e+02
1 1:2.17e+01 2:1.24e+02 3:1.53e-01  4:1.52e+02
1 1:9.13e+01 2:2.93e+02 3:1.42e-01  4:1.60e+02
1 1:5.53e+01 2:1.79e+02 3:1.65e-01  4:1.11e+02
1 1:2.95e+01 2:1.91e+02 3:9.90e-02  4:1.03e+02
```

Training and testing sets available: 3,089 and 4,000

Sparse format: zero values not stored

Poor Results from Direct Training/Testing

Training

```
$./svm-train train.1
optimization finished, #iter = 6131
nSV = 3053, nBSV = 724
Total nSV = 3053
```

Testing

```
$./svm-predict test.1 train.1.model test.1.out
Accuracy = 66.925% (2677/4000)
```

nSV and nBSV: number of SVs and bounded SVs ($\alpha_i = C$).

Why this Fails

- After training, nearly 100% support vectors
- Training and testing accuracy different

```
$./svm-predict train.1 train.1.model o
Accuracy = 99.7734% (3082/3089)
```

- Most kernel elements:

$$K_{ij} = e^{-\|\mathbf{x}_i - \mathbf{x}_j\|^2/4} \begin{cases} = 1 & \text{if } i = j, \\ \to 0 & \text{if } i \neq j. \end{cases}$$

- Some features in rather large ranges

Data Scaling

- Without scaling

  Attributes in greater numeric ranges may dominate
- Linearly scale the first to $[0, 1]$ by:

$$\frac{\text{feature value} - \text{min}}{\text{max} - \text{min}},$$

  There are other ways
- Scaling generally helps, but not always

Data Scaling: Same Factors

A common mistake

```
$./svm-scale -l -1 -u 1 train.1 > train.1.scale
$./svm-scale -l -1 -u 1 test.1 > test.1.scale
```

Same factor on training and testing

```
$./svm-scale -s range1 train.1 > train.1.scale
$./svm-scale -r range1 test.1 > test.1.scale
```

After Data Scaling

Train scaled data and then predict

```
$./svm-train train.1.scale
$./svm-predict test.1.scale train.1.scale.model
  test.1.predict
Accuracy = 96.15%
```

Training accuracy now is

```
$./svm-predict train.1.scale train.1.scale.model
Accuracy = 96.439%
```

Default parameter: $C = 1, \gamma = 0.25$

Different Parameters

- If we use $C = 20, \gamma = 400$

```
$./svm-train -c 20 -g 400 train.1.scale
$./svm-predict train.1.scale train.1.scale.m
Accuracy = 100% (3089/3089)
```

- 100% training accuracy but

```
$./svm-predict test.1.scale train.1.scale.mo
Accuracy = 82.7% (3308/4000)
```

- Very bad test accuracy
- Overfitting happens

Overfitting

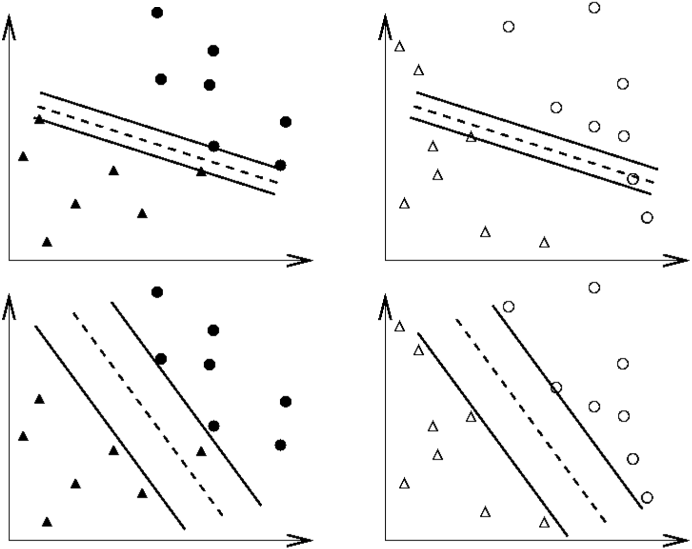
- In theory

  You can easily achieve 100% training accuracy
- This is useless
- When training and predicting a data, we should

  Avoid underfitting: small training error

  Avoid overfitting: small testing error

● and ▲: training; ○ and △: testing



Parameter Selection

- Need to select suitable parameters
- C and kernel parameters
- Example:

$$\gamma \text{ of } e^{-\gamma\|\mathbf{x}_i-\mathbf{x}_j\|^2}$$

$$a, b, d \text{ of } (\mathbf{x}_i^T\mathbf{x}_j/a + b)^d$$

- How to select them?
  So performance better?

Performance Evaluation

- Available data $\Rightarrow$ training and validation
- Train the training; test the validation
- k-fold cross validation (CV):

  Data randomly separated to k groups

  Each time $k-1$ as training and one as testing
- Select parameters/kernels with best CV result

Selecting Kernels

- RBF, polynomial, or others?
- For beginners, use RBF first
- Linear kernel: special case of RBF

  Performance of linear the same as RBF under certain parameters (Keerthi and Lin, 2003)
- Polynomial: numerical difficulties

  $(< 1)^d \rightarrow 0, (> 1)^d \rightarrow \infty$

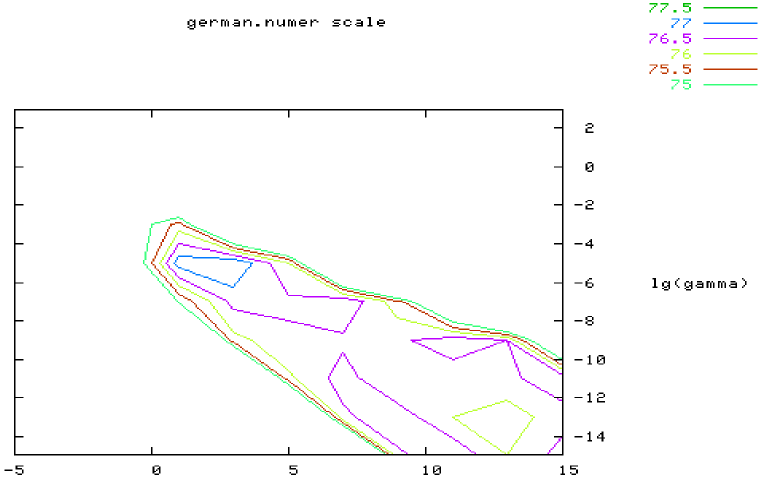
  More parameters than RBF

A Simple Procedure

1. Conduct simple scaling on the data
2. Consider RBF kernel $K(\mathbf{x}, \mathbf{y}) = e^{-\gamma \|\mathbf{x}-\mathbf{y}\|^2}$
3. Use cross-validation to find the best parameter C and γ
4. Use the best C and γ to train the whole training set
5. Test

For beginners only, you can do a lot more

Contour of Parameter Selection



- The good region of parameters is quite large
- SVM is sensitive to parameters, but not that sensitive
- Sometimes default parameters work

  but it's good to select them if time is allowed

Outline

SVM doesn't Scale Up

Yes, if using kernels

- Training millions of data is time consuming
- Cases with many support vectors: quadratic time bottleneck on

$$Q_{\text{SV, SV}}$$

- For noisy data: # SVs increases linearly in data size (Steinwart, 2003)

Some solutions

- Parallelization
- Approximation

Parallelization

Multi-core/Shared Memory/GPU

- One line change of LIBSVM

Multicore		Shared-memory	
1	80	1	100
2	48	2	57
4	32	4	36
8	27	8	28

  50,000 data (kernel evaluations: 80% time)
- GPU (Catanzaro et al., 2008)

Distributed Environments

- Chang et al. (2007); Zanni et al. (2006); Zhu et al. (2009).

Approximately Training SVM

- Can be done in many aspects
- Data level: sub-sampling
- Optimization level:

  Approximately solve the quadratic program
- Other non-intuitive but effective ways

  I will show one today
- Many papers have addressed this issue

Approximately Training SVM (Cont'd)

Subsampling

- Simple and often effective

More advanced techniques

- Incremental training: (e.g., Syed et al., 1999))
  Data $\Rightarrow$ 10 parts
  train 1st part $\Rightarrow$ SVs, train SVs + 2nd part, ...
- Select and train good points: KNN or heuristics
  For example, Bakır et al. (2005)

Approximately Training SVM (Cont'd)

- Approximate the kernel; e.g., Fine and Scheinberg (2001); Williams and Seeger (2001)
- Use part of the kernel; e.g., Lee and Mangasarian (2001); Keerthi et al. (2006)
- Early stopping of optimization algorithms

  Tsang et al. (2005) and others
- And many more

  Some simple but some sophisticated

Approximately Training SVM (Cont'd)

- Sophisticated techniques may not be always useful
- Sometimes slower than sub-sampling
- covtype: 500k training and 80k testing
  rcv1: 550k training and 14k testing

covtype		rcv1	
Training size	Accuracy	Training size	Accuracy
50k	92.5%	50k	97.2%
100k	95.3%	100k	97.4%
500k	98.2%	550k	97.8%

Approximately Training SVM (Cont'd)

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Discussion: Large-scale Training

- We don't have many large and well labeled sets
  Expensive to obtain true labels
- Specific properties of data should be considered
  We will illustrate this point using linear SVM
- The design of software for very large data sets should be application different

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Linear SVM

- Data not mapped to another space

$$\begin{aligned} \min_{\mathbf{w},b,\boldsymbol{\xi}} \quad & \frac{1}{2}\mathbf{w}^T\mathbf{w} + C\sum_{i=1}^{l}\xi_i \\ \text{subject to} \quad & y_i(\mathbf{w}^T\mathbf{x}_i + b) \geq 1 - \xi_i, \\ & \xi_i \geq 0,\ i = 1, \ldots, l. \end{aligned}$$

- In theory, RBF kernel with certain parameters ⇒ as good as linear (Keerthi and Lin, 2003):

  Test accuracy of linear ≤ Test accuracy of RBF
- But can be an approximation to nonlinear

  Recently linear SVM an important research topic

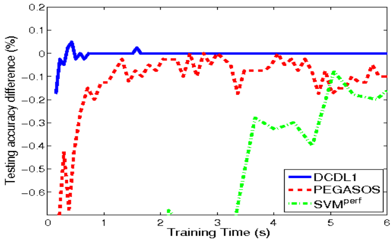
Linear SVM for Large Document Sets

- Bag of words model (TF-IDF or others)
  A large # of features
- Accuracy similar with/without mapping vectors
- What if training is much faster?
  A very effective approximation to nonlinear SVM

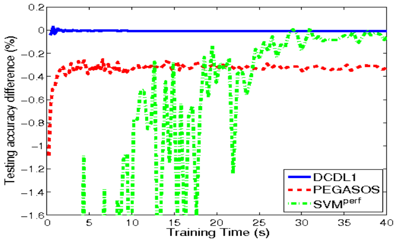
A Comparison: LIBSVM and LIBLINEAR

- rcv1: # data: > 600k, # features: > 40k
- Using LIBSVM (linear kernel): > 10 hours
- Using LIBLINEAR (same stopping condition)
  Computation: < 5 seconds; I/O: 60 seconds
- Accuracy similar to nonlinear; more than 100x speedup
- Training millions of data in a few seconds
- See some results in Hsieh et al. (2008) by running LIBLINEAR
  `http://www.csie.ntu.edu.tw/~cjlin/liblinear`

Testing Accuracy versus Training Time



news20



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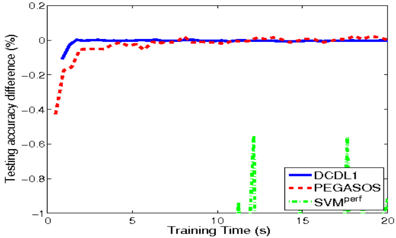
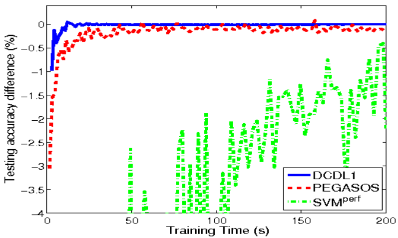


rcv1



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Why Training Linear SVM Is Faster?

- In optimization, each iteration often needs

$$\nabla_i f(\boldsymbol{\alpha}) = (Q\boldsymbol{\alpha})_i - 1$$

- Nonlinear SVM

$$\nabla_i f(\boldsymbol{\alpha}) = \sum_{j=1}^{l} y_i y_j K(\mathbf{x}_i, \mathbf{x}_j)\alpha_j - 1$$

  cost: $O(nl)$; n: # features, l: # data
- Linear: use

$$\mathbf{w} \equiv \sum_{j=1}^{l} y_j \alpha_j \mathbf{x}_j \text{ and } \nabla_i f(\boldsymbol{\alpha}) = y_i \mathbf{w}^T \mathbf{x}_i - 1$$

- Only $O(n)$ cost if $\mathbf{w}$ is maintained

Extension: Training Explicit Form of Nonlinear Mappings

Linear-SVM method to train $\phi(\mathbf{x}_1), \ldots, \phi(\mathbf{x}_l)$

- Kernel not used
- Applicable only if dimension of $\phi(\mathbf{x})$ not too large

Low-degree Polynomial Mappings

$$K(\mathbf{x}_i, \mathbf{x}_j) = (\mathbf{x}_i^T \mathbf{x}_j + 1)^2 = \phi(\mathbf{x}_i)^T \phi(\mathbf{x}_j)$$

$$\phi(\mathbf{x}) = [1, \sqrt{2}x_1, \ldots, \sqrt{2}x_n, x_1^2, \ldots, x_n^2, \sqrt{2}x_1x_2, \ldots, \sqrt{2}x_{n-1}x_n]^T$$

- When degree is small, train the explicit form of $\phi(\mathbf{x})$

Testing Accuracy and Training Time

Data set	Degree-2 Polynomial			Accuracy diff.	
	Training time (s)		Accuracy	Linear	RBF
	LIBLINEAR	LIBSVM			
a9a	1.6	89.8	85.06	0.07	0.02
real-sim	59.8	1,220.5	98.00	0.49	0.10
ijcnn1	10.7	64.2	97.84	5.63	−0.85
MNIST38	8.6	18.4	99.29	2.47	−0.40
covtype	5,211.9	NA	80.09	3.74	−15.98
webspam	3,228.1	NA	98.44	5.29	−0.76

Training $\phi(\mathbf{x}_i)$ by linear: faster than kernel, but sometimes competitive accuracy

Discussion: Directly Train $\phi(\mathbf{x}_i), \forall i$

- See details in our work (Chang et al., 2010)
- A related development: Sonnenburg and Franc (2010)
- Useful for certain applications

Linear Classification: Data Larger than Memory

- Existing methods cannot easily handle this situation
- See our recent KDD work (Yu et al., 2010)
  KDD 2010 best paper award
- Training several million data (or more) on your laptop

Linear Classification: Online Learning

For extremely large data, cannot keep all data

- After using new data to update the model; may not need them any more

Online learning instead of offline learning

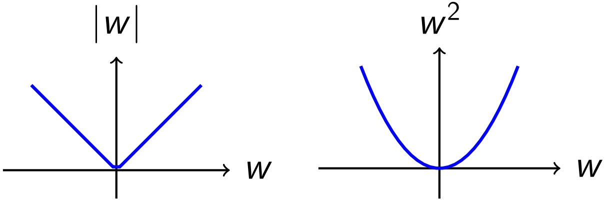
- Training often by stochastic gradient descent methods

  They use only a subset of data at each step
- Now an important research topic (e.g., Shalev-Shwartz et al., 2007; Langford et al., 2009; Bordes et al., 2009)

Linear Classification: L1 Regularization

- 1-norm versus 2-norm

$$\|\mathbf{w}\|_1 = |w_1| + \cdots + |w_n|, \quad \|\mathbf{w}\|_2^2 = w_1^2 + \cdots + w_n^2$$



- 2-norm: all w_i are non-zeros; 1-norm: some w_i may be zeros; useful for feature selection
- Recently a hot topic; see our survey (Yuan et al., 2010)

Outline

Multiple Kernel Learning (MKL)

- How about using

$$t_1 K_1 + t_2 K_2 + \cdots + t_r K_r, \text{ where } t_1 + \cdots + t_r = 1$$

  as the kernel
- Related to parameter/kernel selection
  If K_1 better $\Rightarrow$ t_1 close to 1, others close to 0
- Earlier development (Lanckriet et al., 2004): high computational cost
- Many subsequent works (e.g., Rakotomamonjy et al., 2008).
- Still ongoing; so far MKL has not been a practical tool yet

Ranking

- Labels become ranking information
  e.g., $\mathbf{x}_1$ ranks higher than $\mathbf{x}_2$
- RankSVM (Joachims, 2002): add constraint

$$\mathbf{w}^T\mathbf{x}_i \geq \mathbf{w}^T\mathbf{x}_j + \xi_{ij} \text{ if } \mathbf{x}_i \text{ ranks better than } \mathbf{x}_j$$

- Many subsequent works
- However, whether SVM is the most suitable method for ranking is an issue

Other Directions

- Semi-supervised learning
  Use information from unlabeled data
- Active learning
  Needs cost to obtain labels of data
- Cost sensitive learning
  For unbalanced data
- Structured Learning
  Data instance not an Euclidean vector
  Maybe a parse tree of a sentence
- Feature selection

Outline

- Basic concepts: SVM and kernels
- Training SVM
- Practical use of SVM
- Research directions: large-scale training
- Research directions: linear SVM
- Research directions: others
- Conclusions

Discussion and Conclusions

- SVM and kernel methods are rather mature areas
- But still quite a few interesting research issues
- Many are extensions of standard classification (e.g., semi-supervised learning)
- It is possible to identify more extensions through real applications

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