Homework 7

1 Linear independence of polynomials

Let $B = \{\phi_0, \phi_1, \phi_2\}$ be a set of polynomials, where

$$\phi_0 = 1$$
$$\phi_1 = x - 2$$
$$\phi_2 = x^2 + 2x + 3.$$

(a) Based on the definition of linear independence given in page 12 of lecture slide “FFT_basic1.pdf”, show that $\{\phi_0, \phi_1, \phi_2\}$ are linearly independent on the interval $[0, 2]$.

(b) Show that any polynomials in the form of

$$a_0 + a_1 x + a_2 x^2$$

can be represented as a linear combination of ϕ_0, ϕ_1 and ϕ_2:

$$c_0\phi_0 + c_1\phi_1 + c_2\phi_2$$

2 Discrete Fourier transform

In Section ”Discrete Analog to Fourier Series”, we discuss a way to approximate a function without expensive integral calculation. Consider a Fourier series

$$S_n(x) = \frac{a_0}{2} + a_n \cos nx + \sum_{k=1}^{n-1}(a_k \cos kx + b_k \sin kx)$$

with finite term n. To determine the coefficients (i.e. a_i and b_i), we sample $2m$ points from the original function and calculate each coefficients. In slide, we already discuss an example to approximate a function

$$f(x) = x^4 - 3x^3 + 2x^2 - \tan(x(x-2))$$

with $m = 5$ and $n = 3$.

In this problem, you are required to

(a) Choose a function $f(x)$ and m and write your program to calculate its coefficients a_i and b_i. Draw the approximation under different n and discuss what you have observed. You should pick a function so that both a and b have non-zero values and provide the calculated a_i and b_i.

(b) In slide “FFT_basic2.pdf”, an example of a function that has all b_i as zeros is given. Please find a function (that is not constant zero) so that all a_i coefficients are zeros and do the same thing as in (a).