Homework 6

Problem 1

In lecture slides "approximation_spline2.pdf", we derived the procedure for solving cubic spline when boundary conditions $s''_0(x_0) = s''_{n-1}(x_n) = 0$ are used. In this problem, we explore the slope boundary condition:

$$s'_0(x_0) = f'(x_0) \text{ and } s'_{n-1}(x_n) = f'(x_n) \tag{1}$$

(a) Modify the procedure derived in lecture slide "approximation_spline2.pdf" so that the boundary condition given in (1) can be applied. You should not directly solve a system of linear equations of $4n$ variables. Instead, solve each group of variables sequentially, similar to the lecture slides.

(b) In MATLAB, it supports the cubic spline data interpolation function spline(x, y, xq) [1]. It supports both types of boundary conditions. When the slope boundary condition is used, the argument x specify inputs x_j, while y specifies both $f(x_j)$ and the boundary conditions $f'(x_0)$ and $f'(x_n)$. The arguments xq then specifies the points to be interpolated.

Based on your derivation in subproblem (a), write a MATLAB function that provides the same interface spline(x, y, xq) and solves cubic spline problem with the slope boundary conditions. Include you code in the report.

(c) Test your implementation by using some data and compare it with MATLAB's built-in spline. Plot the result from both implementations for comparison.

Problem 2

Consider a quadratic least sqaure

$$\min_f E = \sum_{n=1}^{m} (y_i - f(x_i))^2,$$

with

$$f(x) = ax^2 + bx + c.$$

(a) Write down the three equations of

$$\nabla E = 0 \tag{2}$$

and rearrange it into a system of linear equations:

$$A \begin{bmatrix} a \\ b \\ c \end{bmatrix} = z \tag{3}$$

That is, you need to give the matrix A and vector z.

(b) Write a MATLAB function that, given m pairs of (x_i, y_i), solves the quadratic least square by solving (3).

(c) Pick some function $f(x)$ and generate some points $\{(x_i, f(x_i))\}$. Then, draw a figure to show your generated points and the quadratic approximation. You should select one $f_1(x)$ that can be well approximated and another $f_2(x)$ where the quadratic least square approximates poorly.

[1] `https://www.mathworks.com/help/matlab/ref/spline.html`