

Homework 5

Problem 1

In the slides “condition of matrices”, we know that given two vector norms l_1 and l_2 , there exist c_1 and c_2 such that

$$c_1\|\mathbf{x}\|_{l_1} \leq \|\mathbf{x}\|_{l_2} \leq c_2\|\mathbf{x}\|_{l_1}, \forall \mathbf{x} \in \mathbb{R}^n.$$

That is, norm l_2 can be bounded by norm l_1 . Please prove the following inequalities in the slides for any given $\mathbf{x} \in \mathbb{R}^n$:

1. $\frac{1}{\sqrt{n}}\|\mathbf{x}\|_1 \leq \|\mathbf{x}\|_2 \leq \|\mathbf{x}\|_1$
2. $\|\mathbf{x}\|_\infty \leq \|\mathbf{x}\|_2 \leq \sqrt{n}\|\mathbf{x}\|_\infty$

Problem 2

Intel MKL supports the product of a sparse matrix and a dense matrix and stores the result as a dense matrix ¹. The function is

```
sparse_status_t mkl_sparse_d_mm (  
    const sparse_operation_t operation,  
    const float alpha,  
    const sparse_matrix_t A, const struct matrix_descr descr,  
    const sparse_layout_t layout,  
    const float *B, const MKL_INT columns, const MKL_INT ldb,  
    const float beta, float *C, const MKL_INT ldc);
```

In this problem, you are required to design a function in **C language** to support the multiplication

$$C = AB$$

where A, B and C are a sparse matrix, a dense matrix and the target dense matrix, respectively.

Requirements

- (1) **Implement the compressed column/row matrix format and the function for multiplication.** Include the code in your report. Your function should be able to support A with both CSC and CSR formats. Your storage scheme for the dense matrix can be either column or row-major layout.
- (2) Perform complexity analysis for your multiplication. Try to optimize your speed.
- (3) Benchmark your implementation and compare it with Intel MKL library. (You can generate random matrices by yourself and feed them to your function and Intel MKL library. Any reasonable size is ok.)

¹<https://software.intel.com/content/www/us/en/develop/documentation/onemkl-developer-reference-c/top/blas-and-sparse-blas-routines/inspector-executor-sparse-blas-routines/inspector-executor-sparse-blas-execution-routines/mkl-sparse-mm.html>