

Data in Higher Dimensional Space I

- If $x \in R^m$, $m > 1$, then

$$f(x) = a^T x + b, a \in R^m$$

Let

$$E = \sum_{i=1}^n (y_i - (a^T x_i + b))^2$$

Data in Higher Dimensional Space II

Then

$$\begin{aligned} & \min_{a,b} \sum_{i=1}^n (y_i - (a^T x_i + b))^2 \\ \equiv & \min_{a,b} \sum_{i=1}^n y_i^2 - 2y_i(a^T x_i + b) + (a^T x_i + b)^2 \\ \equiv & \min_{a,b} \sum_{i=1}^n -2y_i(a^T x_i + b) + (a^T x_i)^2 + 2ba^T x_i + b^2 \\ \equiv & \min_{a,b} \left(\sum_{i=1}^n -2y_i x_i \right)^T a + \left(\sum_{i=1}^n -2y_i \right) b + \\ & \sum_{i=1}^n (a^T x_i)^2 + \left(\sum_{i=1}^n 2x_i \right)^T ab + nb^2 \end{aligned}$$

Data in Higher Dimensional Space III

- First derivative

$$\frac{\partial E}{\partial a} = 0 \Rightarrow$$
$$\left(\sum_{i=1}^n -2y_i x_i\right) + 2 \sum_{i=1}^n (a^T x_i) x_i + \left(\sum_{i=1}^n 2x_i\right) b = 0$$

$$\frac{\partial E}{\partial b} = 0 \Rightarrow$$
$$\left(\sum_{i=1}^n -2y_i\right) + \left(\sum_{i=1}^n 2x_i\right)^T a + 2nb = 0$$

Data in Higher Dimensional Space IV

- Note that

$$\sum_{i=1}^n (a^T x_i) x_i = \sum_{i=1}^n x_i x_i^T a,$$

where

$$x_i x_i^T$$

is an m by m matrix

Data in Higher Dimensional Space V

- Define

$$S_{xx} = \sum_{i=1}^n x_i x_i^T, S_x = \sum_{i=1}^n x_i$$

$$S_{xy} = \sum_{i=1}^n y_i x_i, S_y = \sum_{i=1}^n y_i$$

- Solve a linear system of a and b

$$\begin{bmatrix} S_{xx} & S_x \\ S_x^T & n \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} S_{xy} \\ S_y \end{bmatrix}$$

- This system has $m + 1$ variables