- **Please give details of your calculation. A direct answer without explanation is not counted.**
- **Your answers must be in English.**
- **Please carefully read problem statements.**
- **During the exam you are not allowed to borrow others' class notes.**
- **Try to work on easier questions first.**

1. (15%) Consider the steepest descent method. Does it satisfy

$$r_j^T r_{j-1} = 0$$

   If yes, prove the result. Otherwise, give a counter example.

2. (35%) Consider a twice continuously differentiable $f(x), x \in R^1$. Assume $f(x)$ has at least one root, $f'(x) > 0$ and $f''(x) > 0, \forall x$, and $f(x_0) \geq 0$, where x_0 is the initial point of Newton methods.

   (a) Will $\{x_n\}$ generated by Newton updates satisfy

$$f(x_n) \geq 0, \ \forall n$$

   (b) Will the sequence $\{x_n\}$ converge to a root of $f(x)$? Theorems proved in our lectures can be considered as known results (though you may not need them). You need to show details of the proof.

3. (30%) Given three points $(0, 1), (1, 0)$, and $(2, 2)$. Find the spline approximation. Draw a figure to show how $s_j(x)$ looks like.

   (a) Consider the following boundary condition:

$$s_0''(x_0) = 0 \text{ and } s_{n-1}''(x_n) = 0$$

   (b) Consider the following boundary condition:

$$s_0'(x_0) = -1 \text{ and } s_{n-1}'(x_n) = 1$$

4. (20%) In regression we consider $a^T x + b$ as the approximate function. Instead we can use only $a^T x$ so that the function pass through the origin. Assume

$$x_1 = (1, 1, 0),\ y_1 = -2$$
$$x_2 = (0, 0, 1),\ y_2 = 2$$
$$x_3 = (0, 2, 0),\ y_3 = 2$$
$$x_4 = (1, 1, 1),\ y_4 = 0$$

Find the function $a^T x$.