- **Please give details of your calculation. A direct answer without explanation is not counted.**
- **Your answers must be in English.**
- **Please carefully read problem statements.**
- **During the exam you are not allowed to borrow others' class notes.**
- **Try to work on easier questions first.**

Problem 1 (15%)

Write the binary format of 0.134765625 as a double floating point number.

Problem 2 (20%)

Use the outer product form to find Cholesky factorization of the following matrix:

$$\begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 4 & 2 & 2 \\ -1 & 2 & 6 & 1 \\ 2 & 2 & 1 & 7 \end{bmatrix}$$

The outer product form means that

$$A = \begin{bmatrix} \sqrt{\alpha} & 0 \\ v/\sqrt{\alpha} & I \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & B - \frac{vv^T}{\alpha} \end{bmatrix} \begin{bmatrix} \sqrt{\alpha} & v^T/\sqrt{\alpha} \\ 0 & I \end{bmatrix}$$

Problem 3 (20%)

Consider a symmetric positive definite matrix A. Is the matrix L with positive diagonal entries such that $A = LL^T$ unique? In other words, is it possible to find L_1 and L_2 with positive diagonal entries such that $L_1 \neq L_2$ and $A = L_1L_1^T = L_2L_2^T$.

Problem 4 (25%)

In the lecture we define l-norm as

$$\|v\|_l = \sqrt[l]{|v_1|^l + \cdots + |v_n|^l}, \tag{1}$$

Moreover, we say a norm should satisfy three basic properties:

$$\begin{aligned} \|v\| &\geq 0 \\ \|v+u\| &\leq \|v\| + \|u\| \\ \|\alpha v\| &= |\alpha|\|v\|, \end{aligned}$$

where α is a scalar. In the lecture we stated that if l is a positive integer, then (1) defines a valid norm.

a. If we define 0-norm as $\lim_{p\to 0} \|v\|_p^p$, where $\|v\|_p$ is defined in (1). Does this 0-norm definition still lead to a valid norm? You need to check all three properties.

b. If $l = \frac{1}{2}$, does the definition in (1) still lead to a valid norm? You need to check all three properties.

Problem 5 (20%)

In the lecture we discuss the following theorem:

Theorem 1 *Let $x_0 = x, x_1 = (x_0 \ominus y) \oplus y, \ldots, x_n = (x_{n-1} \ominus y) \oplus y$, if $\oplus$ and $\ominus$ are exactly rounded using rounded to even, then $x_n = x, \forall n$ or $x_n = x_1, \forall n \geq 1$.*

We say that if using "rounding up" for the digit 5, then the result is increased by 0.01 until $x_n = 9.45$. Explain the situation when reaching 9.45 and explain why the value does not increase.