

- Please give details of your calculation. A direct answer without explanation is not counted.
- Your answers must be in English.
- Please carefully read problem statements.
- During the exam you are not allowed to borrow others' class notes.
- Try to work on easier questions first.

1. (20%) If we define a matrix norm by

$$\|A\| = \sqrt{\sum_{i=1}^n \sum_{j=1}^m a_{ij}^2}$$

does it satisfy the three conditions needed by a norm:

$$\begin{aligned} \|A\| &\geq 0 \\ \|A + B\| &\leq \|A\| + \|B\| \\ \|\alpha A\| &= |\alpha| \|A\|, \end{aligned}$$

where  $\alpha$  is a scalar. You need to check all three conditions.

2. (20%) Consider a matrix

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

- What is the 2-norm of this matrix? We mean the matrix norm defined in our lecture.
- What is the condition number of this matrix?

3. (10%)

- Give the compressed column format of the following matrix

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Give the compressed row format of the following matrix

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Assume matlab (Fortran) arrays are used so array index starts from 1.

4. (30%) Assume  $L \in R^{n \times n}$  is a lower-triangular sparse matrix with  $L_{ii} \neq 0$ , and  $x, b \in R^{n \times 1}$  are stored as regular arrays. We would like to solve the linear system

$$Lx = b.$$

- (a) Write the code if  $L$  is stored in the compressed column format
- (b) Write the code if  $L$  is stored in the compressed row format

We assume that in the column (row) format, row (column) indices are sorted in ascending order.

5. (20%) Let  $A$  be a  $2 \times 2$  matrix with nonzero diagonal entries and given a linear system  $Ax = b$ . Does the convergence of Jacobi method implies  $\rho(M^{-1}N) < 1$ ? Note that  $M$  is a diagonal matrix containing  $A$ 's diagonal elements and  $A = M - N$ .