• Please give details of your calculation. A direct answer without explanation is not counted.

• Your answers must be in English.

• Please carefully read problem statements.

• During the exam you are not allowed to borrow others’ class notes.

• Try to work on easier questions first.

1. (15%) Consider the steepest descent method. Does it satisfy

\[ r_j^T r_{j-1} = 0 \]

If yes, prove the result. Otherwise, give a counter example.

2. (35%) Consider a twice continuously differentiable \( f(x), x \in \mathbb{R}^1 \). Assume \( f(x) \) has at least one root, \( f'(x) > 0 \) and \( f''(x) > 0, \forall x \), and \( f(x_0) \geq 0 \), where \( x_0 \) is the initial point of Newton methods.

   (a) Will \( \{x_n\} \) generated by Newton updates satisfy

\[ f(x_n) \geq 0, \forall n \]

(b) Will the sequence \( \{x_n\} \) converge to a root of \( f(x) \)? Theorems proved in our lectures can be considered as known results (though you may not need them). You need to show details of the proof.

3. (30%) Given three points \((0,1), (1,0), \) and \((2,2)\). Find the spline approximation. Draw a figure to show how \( s_j(x) \) looks like.

   (a) Consider the following boundary condition:

\[ s_0''(x_0) = 0 \text{ and } s_{n-1}''(x_n) = 0 \]

   (b) Consider the following boundary condition:

\[ s_0'(x_0) = -1 \text{ and } s_{n-1}'(x_n) = 1 \]
4. (20%) In regression we consider $a^T x + b$ as the approximate function. Instead we can use only $a^T x$ so that the function pass through the origin. Assume

\[
\begin{align*}
x_1 &= (1, 1, 0), \quad y_1 = -2 \\
x_2 &= (0, 0, 1), \quad y_2 = 2 \\
x_3 &= (0, 2, 0), \quad y_3 = 2 \\
x_4 &= (1, 1, 1), \quad y_4 = 0
\end{align*}
\]

Find the function $a^T x$. 