Reductions via Computation Histories I

- This is a technique to prove that A_{TM} is reducible to some languages
- Thus it can be used to prove that a language is undecidable
- We will show that the method is useful when the undecidable language involves testing for the existence of something (or say checking if a language is empty)
- For example, it is used to prove the undecidability of Hilbert's 10th problem, testing for the existence of integral roots in a polynomial (details not shown)

Reductions via Computation Histories II

- What is the computation history of a TM?
- It is the sequence of configurations the machine goes through
- Formal definition: M a TM and w an input string

  An accepting computation history for M and w is

  $$C_1, \ldots, C_l,$$

  where

  C_1 is the start configuration,
  C_l is an accepting configuration, and
  C_i follows from C_{i-1}

Reductions via Computation Histories III

- A rejecting computation history is similar, except that C_l is a rejecting configuration
- Computation histories are finite sequences

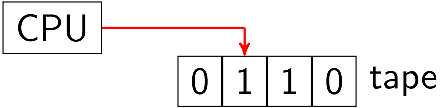
  If M does not halt on w, no accepting or rejecting computation history for M on w
- Deterministic TM has at most one history on an input

  It may have zero if a loop occurs
- NTM may have many histories

Linear Bounded Automaton (LBA) I

- We will use computation histories to show some related problems of LBA are undecidable
- A linear bounded automaton (LBA) is a special TM: head cannot move beyond the end of input
- If the head tries to move right at the end of input, the head stays
- It is similar to moving left at the beginning of the tape
- Therefore, we have

Linear Bounded Automaton (LBA) II



- An LBA is a TM with limited memory: the tape length is n, the input length
- This "limited memory" description seems to be strange as n is not a constant
- Let's use another (equivalent) way to define LBA as follows[1]
  1. Γ includes 2 special symbols: left and right end markers

Linear Bounded Automaton (LBA) III

2. For any input, let's have markers in the beginning and in the end
3. Transitions may not print other symbols over the endmarkers
4. Transitions may neither move to the left of the left endmarker nor to the right of the right endmarker.

- Then we still have a tape of infinite length, but impose restricted operations
- Despite memory or operation constraints, LBAs are quite powerful

Linear Bounded Automaton (LBA) IV

- A_{DFA}, A_{CFG}, E_{DFA}, E_{CFG} are all LBAs (details not discussed)

[1]Descriptions come from
https://en.wikipedia.org/wiki/Linear_bounded_automaton

LBA's # of Configurations I

- We prove that an LBA has a finite number of configurations
- For an LBA M, if

$$\begin{aligned} q&: \# \text{ states} = |Q|, \\ g&: \# \text{ symbols} = |\Gamma|, \end{aligned}$$

  then M has exactly qng^n distinct configurations for a tape of length n
- Proof: a configuration involves

  current state, head position, tape contents

LBA's # of Configurations II

- Immediately we see

$$q \qquad n \qquad g^n$$

  possibilities for each
- Thus the total number of possibilities is qng^n

A_{LBA} Decidable I

- Consider

$$A_{\text{LBA}} = \{\langle M, w\rangle \mid M \text{ is an LBA that accepts } w\}$$

- In applying M on w, the concern is that M loops on w
- Now tape length is fixed to $|w| = n$ (or say we will never use more than n positions)
- If a loop occurs, one configuration appears twice
- Thus we check if in qng^n steps, same configurations occur
- This is a finite procedure so A_{LBA} is decidable