Let’s apply pumping lemma to prove that

\[ B = \{0^n1^n \mid n \geq 0\} \]

is not regular

Assume \( B \) is regular. From the lemma there is \( p \) such that \( \forall s \) in the language with \( |s| \geq p \) some properties hold
Example 1.73 II

Now consider a particular \( s \) in the language

\[ s = 0^p 1^p \]

We see that \( |s| \geq p \). By the lemma, \( s \) can be split to

\[ s = xyz \]

such that

\[ xy^i z \in B, \forall i \geq 0, \quad |y| > 0, \quad \text{and} \quad |xy| \leq p \]

However, we will show that this is not possible
Example 1.73 III

1. If

\[ y = 0 \cdots 0 \]

then

\[ xy = 0 \cdots 0 \text{ and } z = 0 \cdots 01 \cdots 1 \]

Thus

\[ xyyz : \#0 > \#1 \]

Then \[ xy^2z \notin B \], a contradiction.
Example 1.73 IV

2. If

\[ y = 1 \cdots 1, \]

similarly

\[ xy^2z \notin B \text{ as } \#0 < \#1 \]

3. If

\[ y = 0 \cdots 01 \cdots 1 \]

then

\[ xyyz \notin B \text{ as it is not in the form of } 0^?1^? \]
Example 1.73 V

Therefore, we fail to find \(xyz\) with \(|y| > 0\) such that

\[xy^iz \in B, \forall i \geq 0\]

Thus we get a contradiction

We see that the condition

\[|xy| \leq p\]

is not used, but we already reach the contradiction

For subsequent examples we will see that this condition is used
Example 1.39

- $C = \{ w \mid \#0 = \#1 \}$
- We follow the previous example to have $s = 0^p1^p = xyz$
- However, we cannot get the needed contradiction for the case of $y = 0 \cdots 01 \cdots 1$
Example 1.39 II

- Earlier we said

  \[ xyyz \text{ not in the form of } 0^?1^? \]

  but now we only require

  \[ \#0 = \#1 \]

- It is possible that

  \[ x = \epsilon, z = \epsilon, y = 0^p1^p \]

  and then

  \[ |y| > 0 \text{ and } xy^iz \in C, \forall i \]
Example 1.39 III

- The 3rd condition should be applied

\[ |xy| \leq p \Rightarrow y = 0 \cdots 0 \text{ in } s = 0^p 1^p \]

Then

\[ xyyz \notin C \]

- Question: the pumping lemma says

\[ \forall s \in A, \cdots \]

but why in the examples we analyzed a particular \( s \)?
And it seems that the selection of $s$ is important. Why?

We will explain our use of the lemma in more detail.