Computability theory vs. complexity theory I

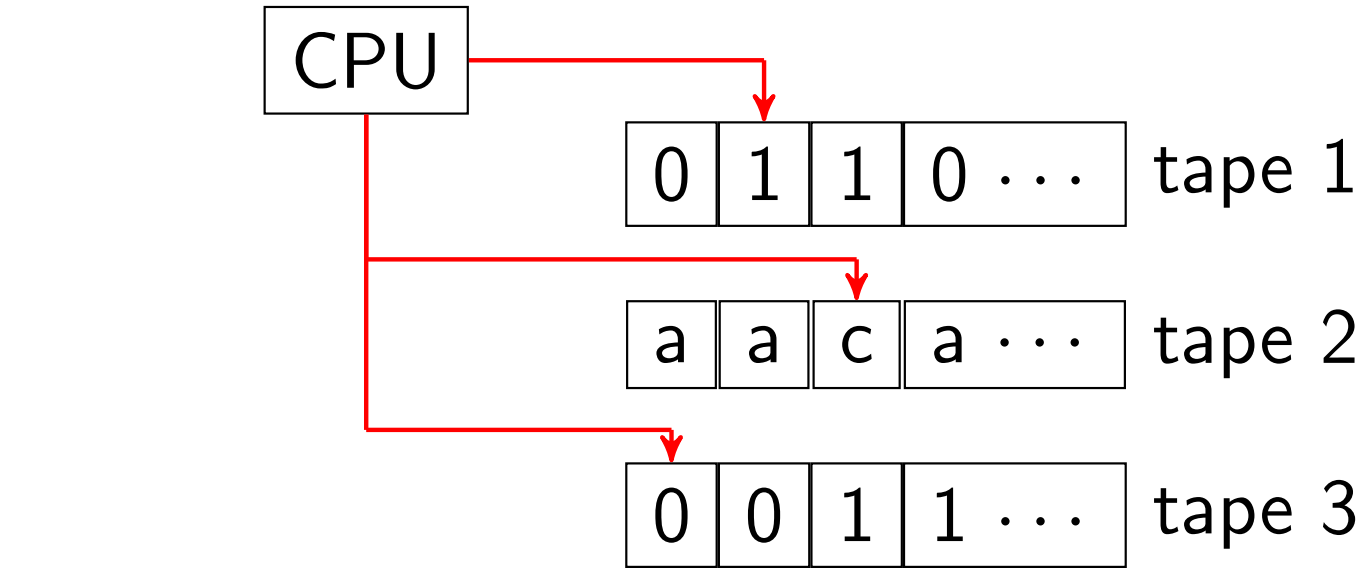
- In Chapter 3 we showed that various TMs are equivalent
- For example, single-tape and multi-tape TMs are equivalent
- However, their "time complexities" are different

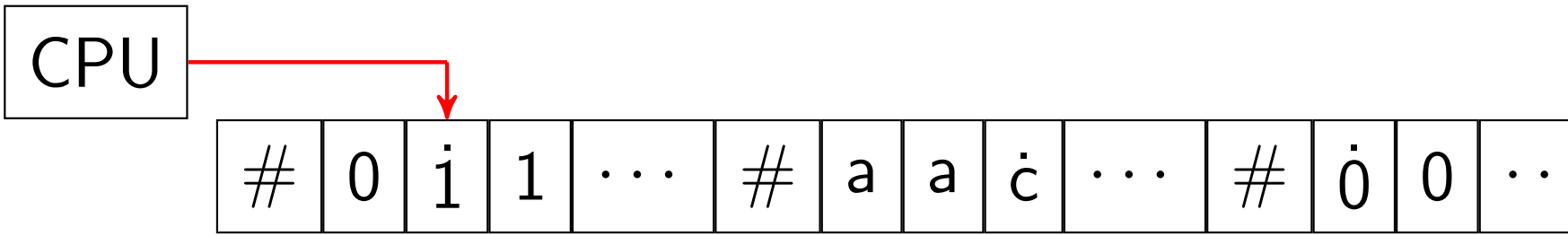
Complexity of Multi-tape TM I

- Theorem 7.8

  Let $t(n) \geq n$. For a $t(n)$ multi-tape TM
  $\Rightarrow \exists$ equivalent $O(t(n)^2)$ single-tape TM
- Idea for the proof: similar to how we proved their equivalence
- We will show that simulating each step of a multi-tape TM takes $O(t(n))$ on a single-tape TM
- Let k be the number of tapes
- How did we simulate a multi-tape TM?

Complexity of Multi-tape TM II





Complexity of Multi-tape TM III

- To simulate each step of multi-tape TM, we scan to know where heads point to and do the update
- However, we may have to right shift the tape
- So we need to know the tape length. It is

$$k \times O(t(n)) = O(t(n))$$

- Note that each tape of multi-tape TM has $O(t(n))$ length. Why?
- A $t(n)$ multi-tape TM generates $O(t(n))$ contents in $O(t(n))$ time

Complexity of Multi-tape TM IV

- Thus the cost of simulating each step of multi-tape TM on a single-tape TM is $O(t(n))$
- There are $O(t(n))$ multi-tape TM steps, so the total cost is

$$O(t(n)) \times O(t(n)) = O(t(n)^2)$$