Variants of TM I

- We will discuss some variants that have the same power
- The robustness of a type of machines means that its reasonable variants have the same power

  Not a strict definition though
- Example

$$\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R, S\}$$

S: stay at the same position

Variants of TM II

- It's equivalent to TM because S can be implemented by L & R moves:

$$q_1, a \to q_2, b, S$$

can be replaced by several rules

$$q_1, a \to q_3, b, R$$
$$q_3, ? \to q_2, ?, L, \forall ? \in \Gamma$$

Multi-tape TM I

- several tapes
- input: put into tape 1

  others: blank
- transition is applied on all tapes simultaneously

$$\delta : Q \times \Gamma^k \to Q \times \Gamma^k \times \{L, S, R\}^k$$
$$\delta(q_i, a_1, \ldots, a_k) = (q_j, b_1, \ldots, b_k, L, R, \ldots, L),$$

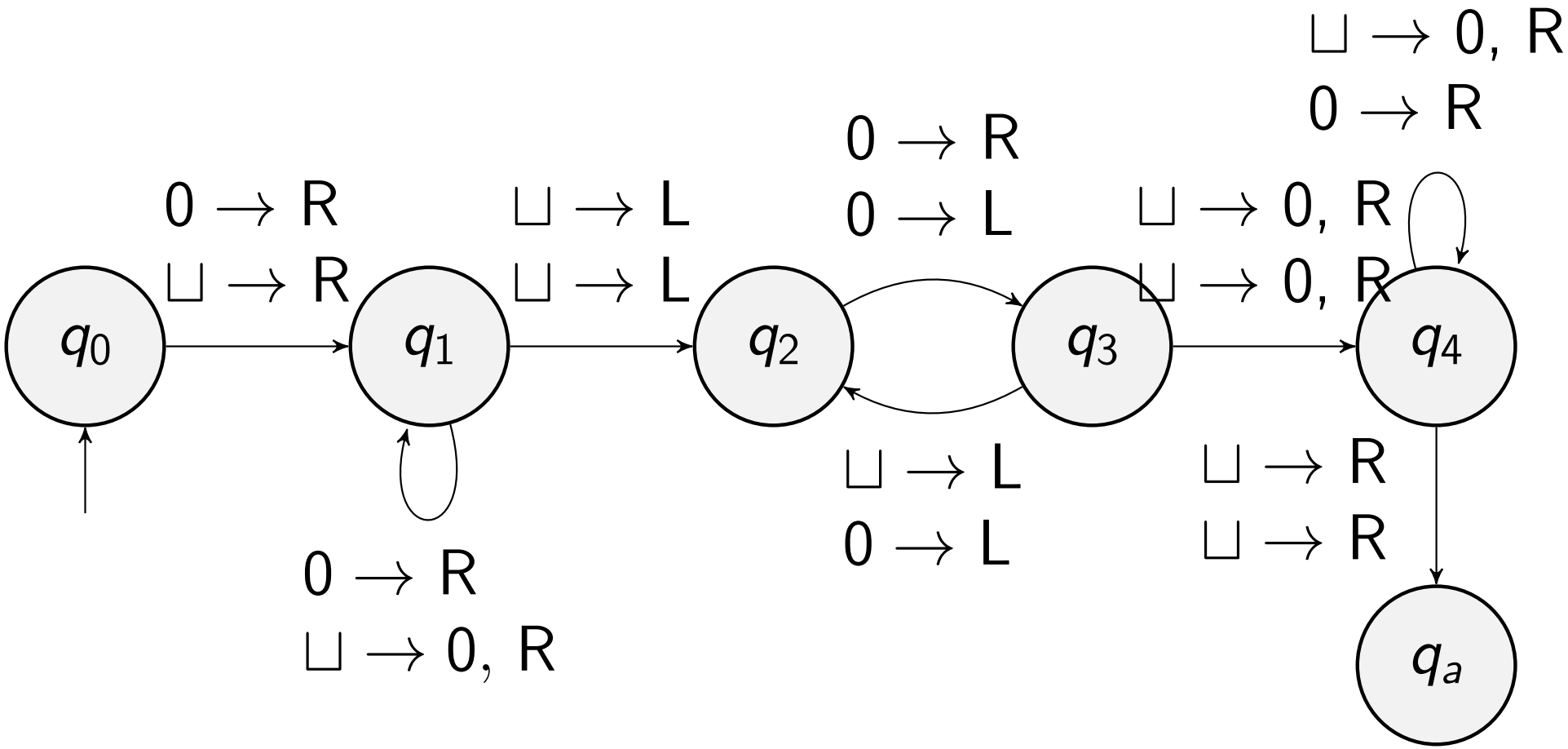
  where k is the number of tapes
- Looks more powerful but equivalent to single-tape TM (discussed later)
- Note that we allow S here though it can be removed

Example I

- Job: given $w = 0^{2n}, n \geq 1 \Rightarrow$ generate ww in the end

  Note that we also need to check if $|w|$ is even
- Idea
  - copy w to the second tape
  - check if $|w|$ is even
  - copy w from the second tape to the first
- State diagram

Example II



- q_0 to q_1:

Example III

let $\sqcup$ be used to indicate the beginning of the second tape

- loop at q_1:

  copy w to the second tape

- $q_2 \to q_3 \to q_2$:
  1. check if length is $2n$
  2. head of the first tape zig-zag between last 0 and the $\sqcup$ after
  3. head of the second tape moved to the beginning

Example IV

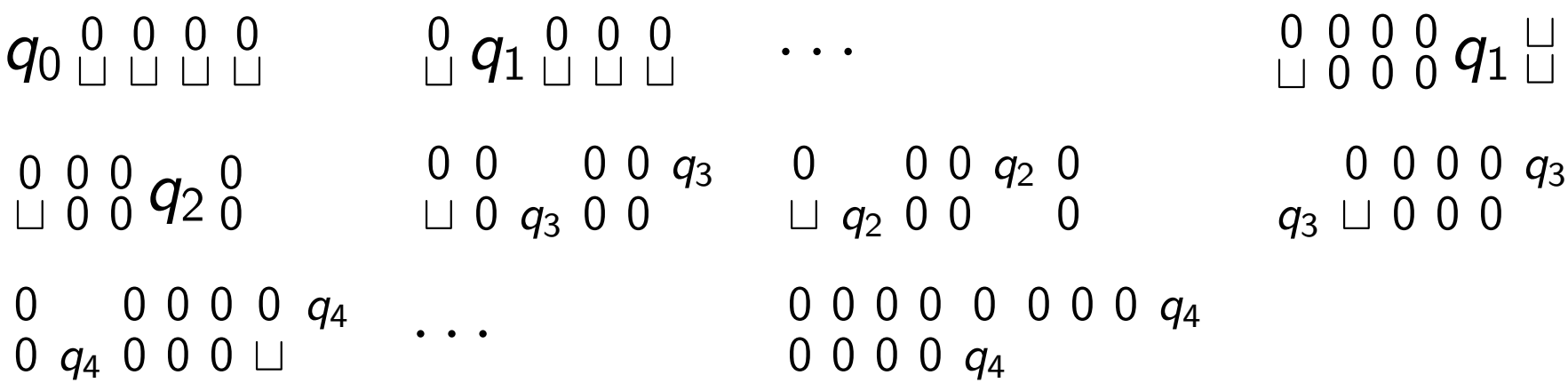
- If length $2n$, we should be at q_3 instead of q_2 when reaching the beginning of the second tape
- q_4: copy w from the second tape to the first
- Note that we have a deterministic machine. At q_0 we should have

$$\delta(q_0, 0, 0), \delta(q_0, 0, \sqcup)$$
$$\delta(q_0, \sqcup, 0), \delta(q_0, \sqcup, \sqcup)$$

Those not specified go to q_r

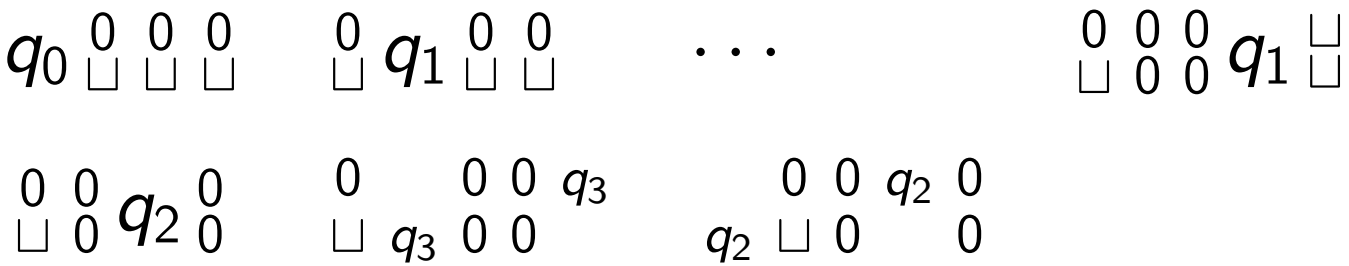
Example V

- Example: input 0000



accepted

- Example: input 000



Example VI

rejected