The current configuration means
  current state, tape contents, head location

\( uqv \):
  \( q \): current state
  \( uv \): current tape content
  \( u \): left, \( v \): right
head: first of \( v \)
Example of configuration 1

- $a, b, c \in \Gamma$, $u, v \in \Gamma^*$ (i.e., strings from $\Gamma$)
- $q_i, q_j$: states
- if $\delta(q_i, b) = (q_j, c, L)$
  
  $u{aq_i}{bv}$ yields $u{q_j}{acv}$

- if $\delta(q_i, b) = (q_j, c, R)$
  
  $u{aq_i}{bv}$ yields $u{acq_j}{v}$
More about Configurations I

- start configuration: $q_0w$
- accepting configuration: $q_{accept}$
- rejecting configuration: $q_{reject}$

A TM accepts $w$ if configurations $c_1 \cdots c_k$

1. $c_1$: start configuration
2. $c_i$ yields $c_{i+1}$
3. $c_k$ accepting configuration

Language: $L(M)$: strings accepted by $M$
A language is Turing-recognizable if it is recognized by a TM.

For a Turing machine, there are three possible outcomes:

- accept
- reject
- loop

If an input fails: reject or loop.

This is difficult to decide.

We prefer a TM that never loops.

Deciders: only accept or reject.
A language is Turing-decidable if some TM decides it.

In Chapter 4 we will discuss decidable languages.
Example 3.9

- Consider the following language

\[ \{ w \# w \mid w \in \{0, 1\}^* \} \]

- Fig 3.10
Example 3.9 II
Example 3.9 III

- Links to $q_r$ are not shown
- Simulate $01 \# 01$

\[
\begin{align*}
q_1 & \quad 01 \# 01 \quad \text{xq}21 & \# 01 & \quad \text{x}1q_2 & \# 01 & \quad \text{x}1 & \# q_401 \\
\text{x}1q_6 & \# \text{x}1 & \quad \text{xq}71 & \# \text{x}1 & \quad q_7 & \text{x}1 & \# \text{x}1 & \quad \text{xq}11 & \# \text{x}1 \\
\text{xxq}3 & \# \text{x}1 & \quad \text{xx} & \# \text{q}5 & \text{x}1 & \quad \text{xx} & \# \text{xq}51 & \quad \text{xx} & \# \text{q}6 & \text{xx} \\
\text{xxq}6 & \# \text{xx} & \quad \text{xq}7 & \text{x} \# \text{xx} & \quad \text{xxq}1 & \# \text{xx} & \quad \text{xx} & \# \text{q}8 & \text{xx} \\
\text{xx} & \# \text{xxq}8 & \sqcup & \quad \text{xx} & \# \text{xx} & \sqcup & \quad q_a
\end{align*}
\]
Example 3.9 IV

- Idea of the diagram:

\[ q_1 \rightarrow q_2 \rightarrow q_4 \rightarrow q_6 \]

check 0 at the same position of the two strings

\[ q_1 \rightarrow q_3 \rightarrow q_5 \rightarrow q_6 \]

check 1 at the same position of the two strings

- \( q_6 \): move left to the beginning of the second string
Example 3.9 V

- \( q_7 \): move left by

\[
q_7 \xrightarrow{0,1 \rightarrow L} q_7
\]

until finding the first 0, 1 not handled yet:

\[
q_7 \xrightarrow{x \rightarrow R} q_1
\]

- Thus \( q_6 \) and \( q_7 \) cannot be combined. At \( q_6 \),

\[
x \rightarrow L
\]

but at \( q_7 \)

\[
x \rightarrow R
\]
Example 3.11

\[ C = \{a^i b^j c^k \mid i \times j = k, i, j, k \geq 1\} \]

Procedure

1. check if the input is \( a^+ b^+ c^+ \)
2. back to start
3. fix \( a \), for each \( b \), cancel \( c \)
4. store \( b \) back, cancel one \( a \), go to step 3

Too complicated to draw state diagram

But one may wonder if TM can really do the above procedure

Here are more details
Example 3.11 II

- Step 1 can be done by a DFA (as DFA is a special case of TM)
- Step 2 can be done by using a special symbol in the beginning
- Step 3 is similar to the procedure of handling $w\#w$

Now we see the concept of subroutines
Example 3.12

- \( E = \{\#x_1\#x_2\cdots\#x_l \mid x_i \in \{0, 1\}^*, x_i \neq x_j\} \)
- Idea: sequentially compare every pair

\[
x_1x_2, x_1x_3, \ldots, x_1x_l \\
x_2x_3, \ldots, x_2x_l \\
x_{l-1}x_l 
\]

- This description is rough. Let’s check more details
  - For \( x_i, x_j \) mark \#’s of both strings by \#
  - \#x_1\#x_2\#x_3: \( x_1 \) and \( x_3 \) being compared
Example 3.12 II

- Compare $x_i$ and $x_j$:
  - Can use a TM similar to that for $w \neq w$
  - We can copy $x_i, x_j$ to the end and do the comparison there