Part II: computability

We would like to study problems that can and cannot be solved by computers.

We need a more powerful model:
- Finite automata: small memory (states)
- PDA: unlimited memory (stack) by push/pop
- Turing machine: unlimited and unrestricted memory

This is about everything a real computer can do.

Thus problems not solved by Turing machines
⇒ beyond the limit of computation.
A TM has a tape as the memory

Differences from finite automata
- write/read tape
- head moves left/right
- infinite space in the tape
- rejecting/accepting take immediate effect
- machine goes on forever, otherwise
Example

\[ B = \{ w\#w \mid w \in \{0, 1\}^* \} \]

We can prove that \( B \) is not CFL using pumping lemma for CFL (similar to example 2.38)

Running a sample input. Figure 3.2

\(|\): blank symbol

We assume infinite \(|\)'s after the input sequence

Strategy: zig-zag to the corresponding places on the two sides of the \(#\) and determine whether they match.
Algorithm:

1. scan to check \( \# \)
2. check \( w \) and \( \bar{w} \)
Formal definition of TM I

- It’s complicated and seldom used
- \( \delta: \)
  \[
  Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}
  \]
- Example:
  \[
  \delta(q, a) = (r, b, L)
  \]
  - \( q: \) current state
  - \( a: \) pointed in tape
  - \( r: \) next state
  - \( b: \) replace \( a \) with \( b \)
  - \( L: \) head then moved to the left
Formal definition of TM II

- \( (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}}) \)
  - \( Q \): states
  - \( \Sigma \): input alphabet (blank: \( \sqcup \notin \Sigma \))
  - \( \Gamma \): tape alphabet, \( \sqcup \in \Gamma, \Sigma \subset \Gamma \)
  - \( \delta \):
    \[
    Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}
    \]
  - \( q_0 \in Q \), start
  - \( q_{\text{accept}} \in Q \)
  - \( q_{\text{reject}} \in Q \), \( q_{\text{reject}} \neq q_{\text{accept}} \)
  - Single \( q_{\text{accept}}, q_{\text{reject}} \)
Formal definition of TM III

- The input
  \[ w_1 \cdots w_n \]
  is put in positions 1 \ldots, \ n of the tape in the beginning
  Assume $\square$ in all the rest of the tape
- If head points to first position and
  \[ \delta(q, ?) = (r, ?, L) \]
  then the head stays at the same position
Formal definition of TM IV