The Overall Procedure I

- Given

\[ P = (Q, \Sigma, \Gamma, \delta, q_0, \{ q_{accept} \} ) \]

- Construct a CFG \( G \)

\[ \text{var}(G) = \{ A_{pq} \mid p, q \in Q \} \]

- Start variable:

\[ A_{q_0, q_{accept}} \]

- Rules: see earlier slides
Needed modifications of PDA I

- Recall we need PDA to satisfy:
  1. Single accept state
  2. Stack empty before accepting
  3. Each transition push or pop, but not both

- Let’s handle the first two together: single accept and stack empty before accepting:
  - A new start \( q_s \rightarrow q_s' \) with \( \epsilon, \epsilon \rightarrow \$ \)
  - For any \( q \in F \), we have \( \epsilon, a \rightarrow \epsilon \) back to \( q \), \( \forall a \).
    This pops things out before accepting a string
  - Then from any \( q \in F \), we do \( \epsilon, $ \rightarrow \epsilon \) to \( q_a \).
Needed modifications of PDA II

- $q \in F$ are no longer accept states
- See the illustration in the following figures
- Original PDA:
Needed modifications of PDA III

New:

\[ \epsilon, a \rightarrow \epsilon \quad \epsilon, b \rightarrow \epsilon \]

\[ \epsilon, \$ \rightarrow \epsilon \]

\[ \epsilon, a \rightarrow \epsilon \quad \epsilon, b \rightarrow \epsilon \]
needed modifications of PDA IV

Is this correct? Let’s check an example:
(Thank student 吳彥翔 for providing this example.)

- This machine would not accept $a$
- At $q_2$, stack is $\{b, a\}$. Then we cannot go to $q_3$ by processing $a$. 
Needed modifications of PDA V

Applying the procedure described earlier:

The machine now accepts $a \rightarrow \epsilon$  incorrect!
Needed modifications of PDA VI

We should only pop the stack at the end of input. Therefore, we should have:

- A new start $q_s \rightarrow q_{s'}$ with $\epsilon, \epsilon \rightarrow \$ $
- A new state $q_{\text{pop}}$ that have $\epsilon, a \rightarrow \epsilon$ back to $q_{\text{pop}}$, $\forall a$.
- For $q \in F$, add a transition $\epsilon, \epsilon \rightarrow \epsilon$ from $q$ to $q_{\text{pop}}$
- Add a new accept state $q_a$ and a transition $\epsilon, \$ \rightarrow \epsilon$ from $q_{\text{pop}}$ to $q_a$
A correct modification of the PDA:
Needed modifications of PDA VIII

- To have each transition push or pop, but not both, change

  \[ q_1 \rightarrow q_2 \text{ with } a, a \rightarrow b \]

  to

  \[ q_1 \rightarrow q_3, a, a \rightarrow \epsilon \]
  \[ q_3 \rightarrow q_2, \epsilon, \epsilon \rightarrow b \]

  and change

  \[ q_1 \rightarrow q_2, a, \epsilon \rightarrow \epsilon \]

  to

  \[ q_1 \rightarrow q_3, a, \epsilon \rightarrow ? \]
  \[ q_3 \rightarrow q_2, \epsilon, ? \rightarrow \epsilon \]
Regular language is context Free I

- We roughly know this but didn’t give a formal proof. Here are the steps
- Regular language $\Rightarrow$ recognized by DFA (in Chapter 1)
- DFA is a PDA
- Thus regular language recognized by PDA
- Then any regular language is context free (by the proof in this chapter)
Non-context free languages I

- There are such languages
- We omit the discussion