Example 1.73 I

- Let's apply pumping lemma to prove that

$$B = \{0^n 1^n \mid n \geq 0\}$$

  is not regular

- Assume B is regular. From the lemma there is p such that $\forall s$ in the language with $|s| \geq p$ some properties hold

Example 1.73 II

- Now consider a particular s in the language

$$s = 0^p 1^p$$

We see that $|s| \geq p$. By the lemma, s can be split to

$$s = xyz$$

such that

$$xy^i z \in B, \forall i \geq 0, \quad |y| > 0, \quad \text{and } |xy| \leq p$$

- However, we will show that this is not possible

Example 1.73 III

1. If

$$y = 0 \cdots 0$$

then

$$xy = 0 \cdots 0 \text{ and } z = 0 \cdots 01 \cdots 1$$

Thus

$$xyyz : \#0 > \#1$$

Then $xy^2z \notin B$, a contradiction

Example 1.73 IV

2. If
$$y = 1 \cdots 1,$$
similarly
$$xy^2z \notin B \text{ as } \#0 < \#1$$
3. If
$$y = 0 \cdots 01 \cdots 1$$
then
$$xyyz \notin B \text{ as it is not in the form of } 0^?1^?$$

Example 1.73 V

- Therefore, we fail to find xyz with $|y| > 0$ such that

$$xy^i z \in B, \forall i \geq 0$$

  Thus we get a contradiction
- We see that the condition

$$|xy| \leq p$$

  is not used, but we already reach the contradiction
- For subsequent examples we will see that this condition is used

Example 1.39 I

- $C = \{w \mid \#0 = \#1\}$
- We follow the previous example to have

$$s = 0^p 1^p = xyz$$

- However, we cannot get the needed contradiction for the case of

$$y = 0 \cdots 0 1 \cdots 1$$

Example 1.39 II

- Earlier we said

$$xyyz \text{ not in the form of } 0^{?}1^{?}$$

  but now we only require

$$\#0 = \#1$$

- It is possible that

$$x = \epsilon, z = \epsilon, y = 0^p 1^p$$

  and then

$$|y| > 0 \text{ and } xy^i z \in C, \forall i$$

Example 1.39 III

- The 3rd condition should be applied

$$|xy| \leq p \Rightarrow y = 0 \cdots 0 \text{ in } s = 0^p 1^p$$

Then

$$xyyz \notin C$$

- Question: the pumping lemma says

$$\forall s \in A, \cdots$$

but why in the examples we analyzed a particular s?

Example 1.39 IV

- And it seems that the selection of s is important. Why?
- We will explain our use of the lemma in more detail