Nonregular language I

We are interested in the limit of finite automata

- Some languages cannot be recognized

$$\{0^n 1^n \mid n \geq 0\}$$

- We might remember $\#0$ first
  But $\#$ of possible n's is ∞
- Thus we cannot recognize it by finite automata
- However, this is not a formal proof
- It may be difficult to quickly tell if a language is regular or not

Nonregular language II

- Consider two languages

$$C = \{w \mid \#0 = \#1\}$$
$$D = \{w \mid \#01 = \#10\}$$

  It seems that both are not regular

  Indeed, C is not regular but D is

  This is an exercise in the book, so we don't give details

- To formally prove a language is not regular, we will introduce the pumping lemma

Pumping lemma I

- Strategy: by contradiction
- We prove

$$\text{regular} \Rightarrow \text{some properties}$$

- If "some properties" cannot hold, then the language is not regular

Theorem 1.70 I

- If A regular $\Rightarrow \exists p$ (pumping length) such that $\forall s \in A, |s| \geq p$, $\exists x, y, z$ such that $s = xyz$ and
  1. $\forall i \geq 0, xy^i z \in A$
  2. $|y| > 0$
  3. $|xy| \leq p$

  Note that for y^i, we have $y^0 = \epsilon$

Proof of pumping lemma I

- Because A is regular, $\exists$ a DFA to recognize A
  Let $p = \#$ states of this DFA
- If no string s such that $|s| \geq p$, then the theorem statement is satisfied
- Now consider any s with $|s| \geq p$

$$s = s_1 \cdots s_n$$

Proof of pumping lemma II

- To process this string, assume the state sequence is

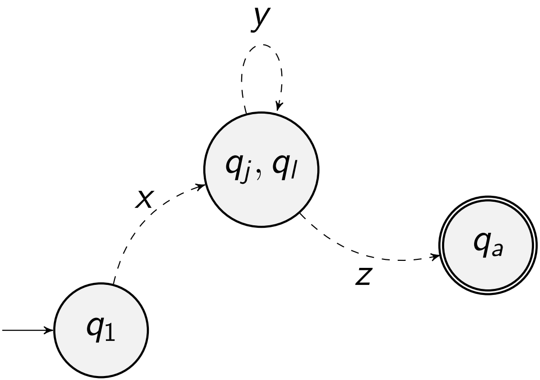
$$q_1 \cdots q_{n+1}$$

  Because $|s| \geq p$, we have

$$n + 1 > p$$

- In $1 \ldots p + 1$ two states must be the same (pigeonhole principle)
  Fig 1.72

Proof of pumping lemma III



Proof of pumping lemma IV

- Assume

$$q_j \text{ and } q_l \text{ with } j \leq p+1, l \leq p+1$$

are two same states. Then let

$$x = s_1 \cdots s_{j-1}, y = s_j, \cdots s_{l-1}, z = s_l \cdots s_n$$

We then have

$$\forall i \geq 0, xy^i z \in A$$

Proof of pumping lemma V

- Because $j \neq l$,
$$|y| > 0$$
From $l \leq p + 1$, we have
$$|xy| \leq p$$
Thus all conditions are satisfied