Homework #6

Due on December 19, 2022

4.7

Let $\mathcal{B}$ be the set of all infinite sequences over $\{0, 1\}$. Show that $\mathcal{B}$ is uncountable using a proof by diagonalization.

7.1 (e)

Prove or disprove $3^n = 2^{O(n)}$ using the definition of big-O notation.

7.2 (b)

Prove or disprove $2n = o(n^2)$ using the following definition of small-o notation: Let f and g be functons from $\mathbb{N} \to \mathbb{R}^+$. $f(n) = o(g(n))$ means that for any real number $c > 0$, there exists n_0 such that $f(n) < cg(n)$ for all $n \geq n_0$.

7.30

Let *SET-SPLITTING* $= \{\langle S, C\rangle \mid S$ is a finite set and $C = \{C_1, \ldots, C_k\}$ is a collection of subsets of S, for some $k > 0$, such that elements of S can be colored *red* or *blue* so that no C_i has all its elements colored with the same color $\}$. Show that *SET-SPLITTING* is NP by

(a) providing a polynomial time verifier.

(b) providing a nondeterministic polynomoal time Turing machine that decides *SET-SPLITTING*.