Decidability and CFL I

- Acceptance problem of CFG

$$A_{\text{CFG}} = \{\langle G, w\rangle \mid G : CFG, \text{ generates } w\}$$

- We prove that A_{CFG} is decidable
- But an issue is the ∞ possible derivations of a CFG
- For example,

$$A \rightarrow B, B \rightarrow A$$

- Chomsky normal form

$$A \rightarrow BC$$
$$A \rightarrow a$$

Decidability and CFL II

- Any $w, |w| = n$, derivation in exactly $2n - 1$ steps
- If q is the $\#$ rules, check all q^{2n-1} possibilities
- Proof
  1. Convert G to Chomsky
  2. Check all q^{2n-1} possibilities
- Results apply to PDA as well: for PDA we have a finite procedure to generate a CFG.

E_{CFG} I

$$E_{\mathsf{CFG}} = \{\langle G\rangle \mid G : CFG, L(G) = \emptyset\}$$

- idea: bottom up setting to see if any string can be generated from the start variable. From

$$A \rightarrow a$$

We search if there is a rule

$$B \rightarrow A$$

E_{CFG} II

- Proof:
  1. Mark all terminals
  2. Repeat until no new variables are marked

     if

     $$A \rightarrow U_1 \cdots U_k$$

     and

     $$\text{all } U_1, \ldots, U_k \text{ marked}$$

     $\Rightarrow$ mark A
  3. If start variable is not marked, accept

     Otherwise, reject

E_{CFG} III

- Number of iterations is finite: bounded by the number of variables
- Each iteration is a finite procedure: we check all rules

EQ_{CFG} I

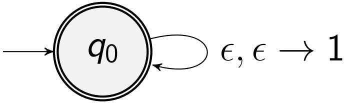
$$EQ_{\mathrm{CFG}} = \{\langle G, H\rangle \mid G, H : CFG, L(G) = L(H)\}$$

- Remember that EQ_{DFA} is decidable
- However, we cannot apply the same proof as CFL is not closed for $\cap$ and complementation
- It's proved in Chapter 5 that this language is not decidable
- We do not discuss details

CFL decidable I

- Let A be a CFL. The goal is to show that A is decidable
- How about converting PDA to a TM and use the TM to run any $w \in A$?
- But a difficulty is that our simulation of a PDA on w may not be a finite procedure
- Specifically, some branches of the PDA's computation may go on forever, reading and writing the stack without ever halting.
- For example, consider the following PDA

CFL decidable II



- By our way mentioned before for constructing a tree, at the first layer we have

$$q_0\emptyset \quad q_0\{1\} \quad q_0\{1,1\} \quad \cdots$$

- Then we may have troubles to go to the next layer for processing the first character
- So converting PDA to TM does not really work
- We need a different way

CFL decidable III

- Because we know A is a CFL, there is a corresponding grammar G
- Then we run TM for $\langle G, w \rangle$ by using A_{CFG}

Classes of languages I

- Fig 4.10