Turing Machines: I

- Part II: computability

  We would like to study problems that can and cannot be solved by computers
- We need a more powerful model

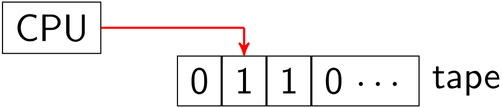
  Finite automata: small memory (states)

  PDA: unlimited memory (stack) by push/pop
- Turing machine: unlimited and unrestricted memory
- This is about everything a real computer can do
- Thus problems not solved by Turing machines

  $\Rightarrow$ beyond the limit of computation

Turing Machines: II

- A TM has a tape as the memory



- Differences from finite automata
  - write/read tape
  - head moves left/right
  - infinite space in the tape
  - rejecting/accepting take immediate effect
  - machine goes on forever, otherwise

Turing Machines: III

- Example

$$B = \{w\#w \mid w \in \{0,1\}^*\}$$

- We can prove that B is not CFL using pumping lemma for CFL (similar to example 2.38)
- Running a sample input. Figure 3.2
- $\sqcup$: blank symbol

  We assume infinite $\sqcup$'s after the input sequence
- Strategy: zig-zag to the corresponding places on the two sides of the $\#$ and determine whether they match.

Turing Machines: IV

$$\overset{\downarrow}{0}\ 1\ 1\ 0\ 0\ 0\ \#\ 0\ 1\ 1\ 0\ 0\ 0\ \sqcup$$

$$\mathrm{x}\ \overset{\downarrow}{1}\ 1\ 0\ 0\ 0\ \#\ 0\ 1\ 1\ 0\ 0\ 0\ \sqcup$$

$$\mathrm{x}\ 1\ 1\ 0\ 0\ 0\ \#\ \overset{\downarrow}{\mathrm{x}}\ 1\ 1\ 0\ 0\ 0\ \sqcup$$

- Algorithm:
    1. scan to check $\#$
    2. check w and w

Formal definition of TM I

- It's complicated and seldom used
- δ:

$$Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$$

- Example:

$$\delta(q, a) = (r, b, L)$$

q: current state

a: pointed in tape

r: next state

b: replace a with b

L: head then moved to the left

Formal definition of TM II

- $(Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$
  Q: states
  Σ: input alphabet (blank: $\sqcup \notin \Sigma$)
  Γ: tape alphabet, $\sqcup \in \Gamma, \Sigma \subset \Gamma$
  δ:
  $$Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$$
  $q_0 \in Q$, start
  $q_{accept} \in Q$
  $q_{reject} \in Q, q_{reject} \neq q_{accept}$
  Single q_{accept}, q_{reject}

Formal definition of TM III

- The input

$$w_1 \cdots w_n$$

is put in positions $1 \ldots, n$ of the tape in the beginning

Assume $\sqcup$ in all the rest of the tape

- If head points to first position and

$$\delta(q, ?) = (r, ?, L)$$

then the head stays at the same position

Formal definition of TM IV