

Nondeterministic TM \equiv deterministic TM

- A language recognized by TM \Rightarrow recognized by NTM
 - A deterministic TM is a nondeterministic TM
- A language recognized by NTM \Rightarrow recognized by TM
 - more difficult
- We must simulate NTM by TM
- How did we run NTM?

Nondeterministic TM \equiv deterministic TM

II

Like NFA we use a tree for processing the input (# branches finite)

- To traverse a tree we can do

depth-first search

or

breadth-first

- If using depth-first search, one branch may lead to ∞ steps

Nondeterministic TM \equiv deterministic TM

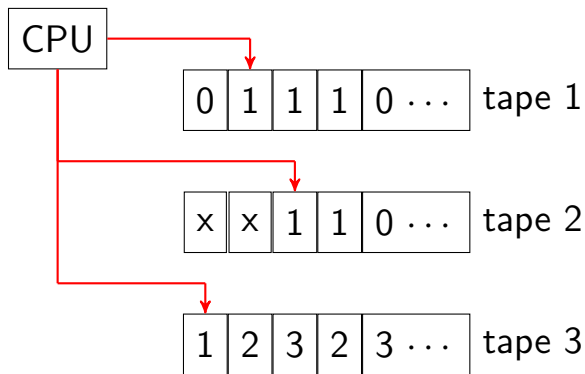
III

Then we cannot consider other branches even if the input is accepted

- Thus we should consider breadth-first
- Fig 3.17: a deterministic TM to simulate a nondeterministic TM

Nondeterministic TM \equiv deterministic TM

IV



- Tape 1: input, never altered

Nondeterministic TM \equiv deterministic TM

V

- Tape 2: copy input from tape 1 and run one branch up to certain layer
- Tape 3: store a path to a node
- The key is the 3rd tape
- Suppose max # branches 3
- If contents of 3rd tape are

231

it means

root \rightarrow 2nd child \rightarrow 3rd child \rightarrow 1st child

Nondeterministic TM \equiv deterministic TM

VI

- Thus tape 3 contents in the procedure can be like

1

2

3

11

...

33

111

...

333

Nondeterministic TM \equiv deterministic TM

VII

- What if say 111 is not a valid configuration? For example, after 11, there is no link to go to the 1st child
- That is fine. We can still check such a path as long as it is finite
- Therefore, an NTM can be simulated by a three-tape TM
- We have shown that a multi-tape TM can be simulated by a single-tape TM
- Thus the proof is completed

Corollary 3.19 I

- Definition: NTM is a decider if all branches halt on all inputs
- Language decidable \Leftrightarrow some NTM decides it
- \Rightarrow easy, one TM decides it and a TM is an NTM
This TM halts on all inputs (one branch)
- \Leftarrow :
Now NTM terminates on all branches
We can construct a TM to decide the language
 - each branch is finite
every input halts \exists a finite max length

Corollary 3.19 II

- # branches finite at each node
The tree to process this input is finite
- Thus the three-tape TM used earlier can accept/reject the input in a finite number of steps