A language recognized by TM $\implies$ recognized by NTM
A deterministic TM is a nondeterministic TM
A language recognized by NTM $\implies$ recognized by TM
more difficult
We must simulate NTM by TM
How did we run NTM?
Nondeterministic TM \equiv \text{deterministic TM}\\

Like NFA we use a tree for processing the input (\# branches finite)

- To traverse a tree we can do
  
  depth-first search

  or

  breadth-first

- If using depth-first search, one branch may lead to $\infty$ steps
Nondeterministic TM ≡ deterministic TM

Then we cannot consider other branches even if the input is accepted

- Thus we should consider breadth-first
- Fig 3.17: a deterministic TM to simulate a nondeterministic TM
Nondeterministic TM $\equiv$ deterministic TM

- Tape 1: input, never altered
Nondeterministic TM $\equiv$ deterministic TM

- Tape 2: copy input from tape 1 and run one branch up to certain layer
- Tape 3: store a path to a node
- The key is the 3rd tape
- Suppose max # branches 3
- If contents of 3rd tape are 231
  - it means root $\rightarrow$ 2nd child $\rightarrow$ 3rd child $\rightarrow$ 1st child
Thus tape 3 contents in the procedure can be like

1
2
3
11
... 
33
111
... 
333
What if say 111 is not a valid configuration? For example, after 11, there is no link to go to the 1st child.

That is fine. We can still check such a path as long as it is finite.

Therefore, an NTM can be simulated by a three-tape TM.

We have shown that a multi-tape TM can be simulated by a single-tape TM.

Thus the proof is completed.
Corollary 3.19 I

- Definition: NTM is a decider if all branches halt on all inputs
- Language decidable $\iff$ some NTM decides it
- $\Rightarrow$ easy, one TM decides it and a TM is an NTM
  This TM halts on all inputs (one branch)
- $\Leftarrow$
  Now NTM terminates on all branches
  We can construct a TM to decide the language
    - each branch is finite
    - every input halts $\exists$ a finite max length
Corollary 3.19 II

- # branches finite at each node
  The tree to process this input is finite
- Thus the three-tape TM used earlier can accept/reject the input in a finite number of steps