The Overall Procedure I

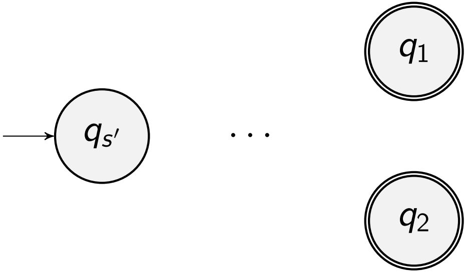
- Given
$$P = (Q, \Sigma, \Gamma, \delta, q_0, \{q_{accept}\})$$
- Construct a CFG G
$$\mathrm{var}(G) = \{A_{pq} \mid p, q \in Q\}$$
- Start variable:
$$A_{q_0, q_{accept}}$$
- Rules: see earlier slides

Needed modifications of PDA I

- Recall we need PDA to satisfy
  1. Single accept state
  2. Stack empty before accepting
  3. Each transition push or pop, but not both
- Let's handle the first two together: single accept and stack empty before accepting:
- A new start $q_s \to q_{s'}$ with $\epsilon, \epsilon \to \$$
- For any $q \in F$, we have $\epsilon, a \to \epsilon$ back to q, $\forall a$. This pops things out before accepting a string
- Then from any $q \in F$, we do $\epsilon, \$ \to \epsilon$ to q_a.

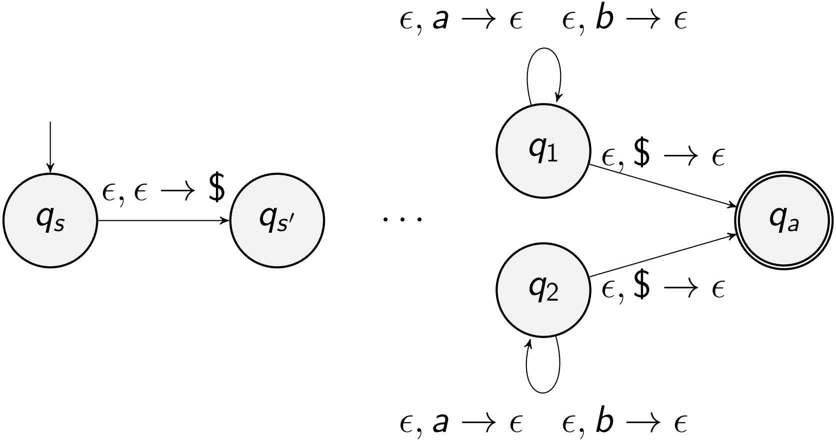
Needed modifications of PDA II

- $q \in F$ are no longer accept states
- See the illustration in the following figures
- Original PDA:



Needed modifications of PDA III

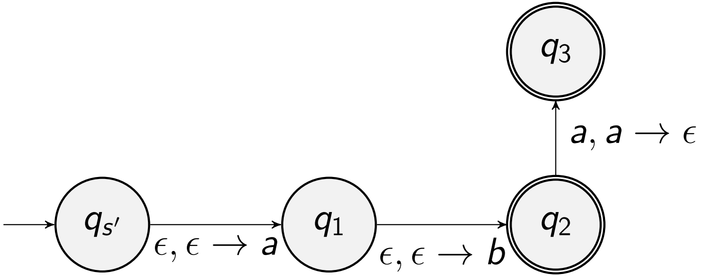
New:



Needed modifications of PDA IV

Is this correct? Let's check an example:

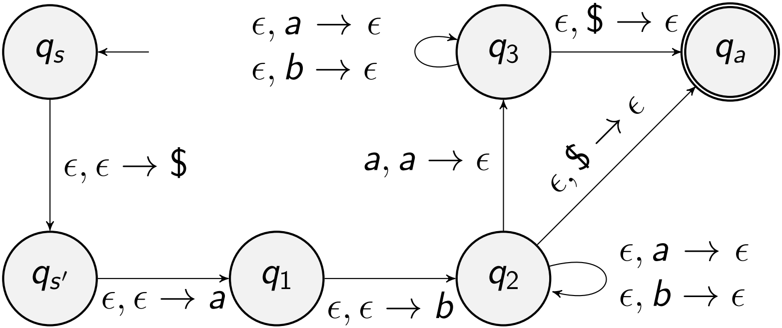
(Thank student 吳彥翔 for providing this example.)



- This machine would not accept a
- At q_2, stack is $\{b, a\}$. Then we cannot go to q_3 by processing a.

Needed modifications of PDA V

Applying the procedure described earlier:



- The machine now accepts $a \implies$ incorrect!

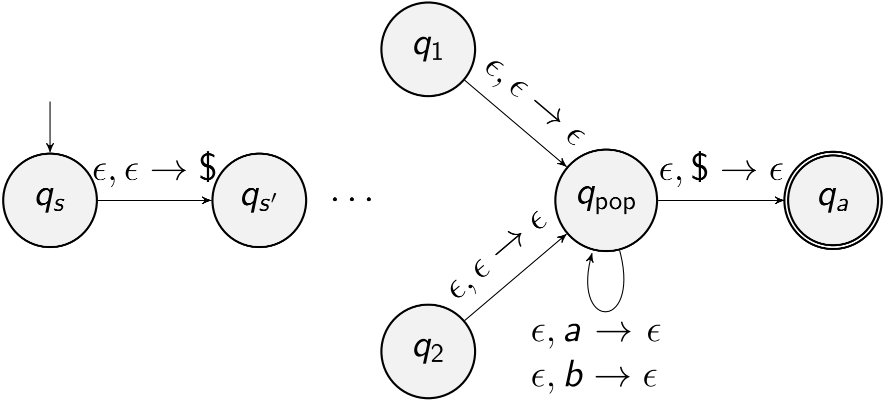
Needed modifications of PDA VI

We should only pop the stack at the end of input. Therefore, we should have:

- A new start $q_s \rightarrow q_{s'}$ with $\epsilon, \epsilon \rightarrow \$$
- A new state q_{pop} that have $\epsilon, a \rightarrow \epsilon$ back to q_{pop}, $\forall a$.
- For $q \in F$, add a transition $\epsilon, \epsilon \rightarrow \epsilon$ from q to q_{pop}
- Add a new accept state q_a and a transition $\epsilon, \$ \rightarrow \epsilon$ from q_{pop} to q_a

Needed modifications of PDA VII

A correct modification of the PDA:



Needed modifications of PDA VIII

- To have each transition push or pop, but not both, change

$$q_1 \rightarrow q_2 \text{ with } a, a \rightarrow b$$

to

$$\begin{aligned} q_1 &\rightarrow q_3, a, a \rightarrow \epsilon \\ q_3 &\rightarrow q_2, \epsilon, \epsilon \rightarrow b \end{aligned}$$

and change

$$q_1 \rightarrow q_2, a, \epsilon \rightarrow \epsilon$$

to

$$\begin{aligned} q_1 &\rightarrow q_3, a, \epsilon \rightarrow ? \\ q_3 &\rightarrow q_2, \epsilon, ? \rightarrow \epsilon \end{aligned}$$

Regular language is context Free I

- We roughly know this but didn't give a formal proof. Here are the steps
- Regular language $\Rightarrow$ recognized by DFA (in Chapter 1)
- DFA is a PDA
- Thus regular language recognized by PDA
- Then any regular language is context free (by the proof in this chapter)

Non-context free languages I

- There are such languages
- We omit the discussion