Deterministic CFL I

- Recall that PDA is non-deterministic
- We can actually define deterministic PDA (DPDA)
- In Chapter 1,

  $$\text{DFA} \equiv \text{NFA}$$

  Both generate regular languages
- But

  $$\text{PDA} \neq \text{DPDA}$$

  and therefore

  $$\text{CFL} \neq \text{DCFL}$$

Deterministic CFL II

- DPDA was not discussed in earlier versions of the textbook
- As this topic is less important, we will only explain what DPDA is without getting into more details
- It's more complicated to define DPDA than PDA
- The reason is that in DPDA we must ensure the deterministic moves

Formal definition of DPDA I

- $(Q, \Sigma, \Gamma, \delta, q_0, F)$
  Q, Σ, Γ, F: finite sets
  1. Q: states
  2. Σ: alphabet
  3. Γ: stack alphabet
  4. δ:
     $$Q \times \Sigma_\epsilon \times \Gamma_\epsilon \rightarrow (Q \times \Gamma_\epsilon) \cup \{\emptyset\}$$
  5. $q_0 \in Q$: start state
  6. $F \subset Q$: set of accept states

Formal definition of DPDA II

- Note for PDA

$$\delta : Q \times \Sigma_\epsilon \times \Gamma_\epsilon \to P(Q \times \Gamma_\epsilon)$$

- Also δ satisfies $\forall q \in Q, a \in \Sigma, x \in \Gamma$, exactly one of

  $$\delta(q, a, x), \qquad \delta(q, a, \epsilon), \qquad \delta(q, \epsilon, x), \qquad \delta(q, \epsilon, \epsilon)$$

  is not $\emptyset$
- Reason: at q all four can be taken at PDA
- Rule: follow the one which is not $\emptyset$

Acceptance and rejection of DPDA I

- Acceptance: same as DFA.

  Reach an accept state after the last symbol

  Otherwise: reject
- Rejection: occurs if
  1. not at an accept state after the last symbol (same as DFA)
  2. DPDA fails to read the input
     1. pop an empty stack
     2. endless ϵ-input moves
- Example: pop an empty stack

Acceptance and rejection of DPDA II

	0		ϵ	
	0	ϵ	0	ϵ
q	$\emptyset$	$\emptyset$	(q, ϵ)	$\emptyset$

input $0,\emptyset$ is rejected: the only possible move is to pop up zero, but the stack is empty

- Example: fails to read the whole string

	0		ϵ	
	0	ϵ	0	ϵ
q	$\emptyset$	$\emptyset$	$\emptyset$	(q, 1)

input $0, \emptyset$ is rejected: endless ϵ-input

$$0, \emptyset \to 0, \{1\}, \to 0, \{1, 1\} \to \cdots$$