DFA $\equiv$ NFA I

- DFA $\Rightarrow$ NFA
- Formally, a language recognized by a DFA $\rightarrow$ recognized by an NFA
- The proof is easy because a DFA is an NFA
- However, formally DFA is not an NFA because DFA uses Σ but not Σ_ϵ

  Can easily handle this by adding

$$q_i, \epsilon \rightarrow \emptyset$$

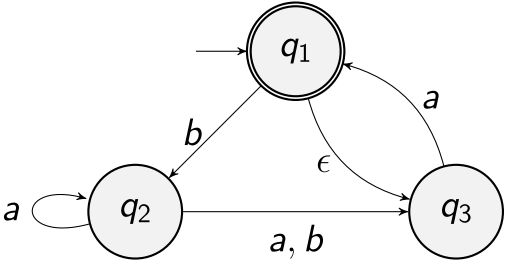
- The other direction: NFA $\Rightarrow$ DFA

DFA $\equiv$ NFA II

- Need to convert NFA to an equivalent DFA
  That is, they recognize the same language
- We do the proof by an example

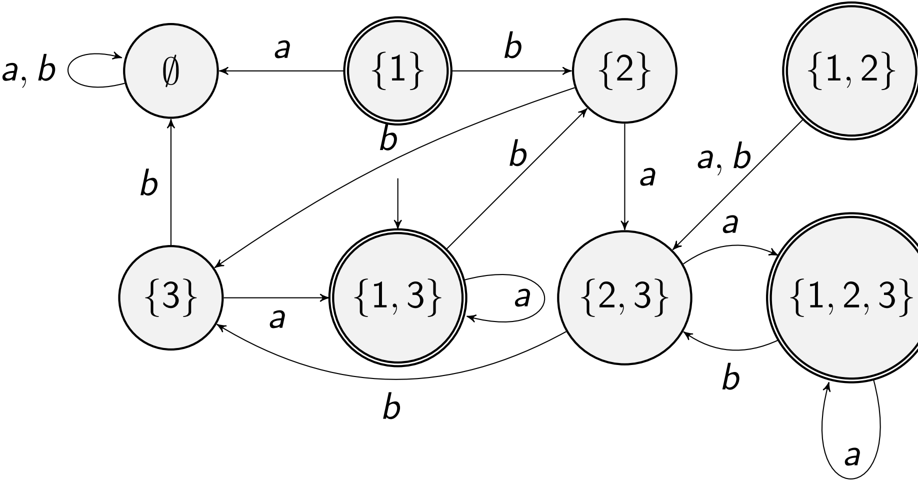
Example 1.41 I

- Consider the following NFA (we discussed this NFA before)



- The resulting DFA diagram

Example 1.41 II



Explanation of the Procedure I

- Each state is a subset of $\{1, 2, 3\}$. So each state is an element of $P(Q)$
- Let's check details of

$$\{1, 2\} \xrightarrow{a} \{2, 3\}$$

Explanation of the Procedure II

- We see

$$q_1 \xrightarrow{a} \emptyset$$
$$q_2 \xrightarrow{a} \{q_2, q_3\}$$

Thus

$$\emptyset \cup \{2,3\} = \{2,3\}$$

- Details of

$$\{3\} \xrightarrow{a} \{1,3\}$$

Explanation of the Procedure III

We must take care of ϵ

- Start state:

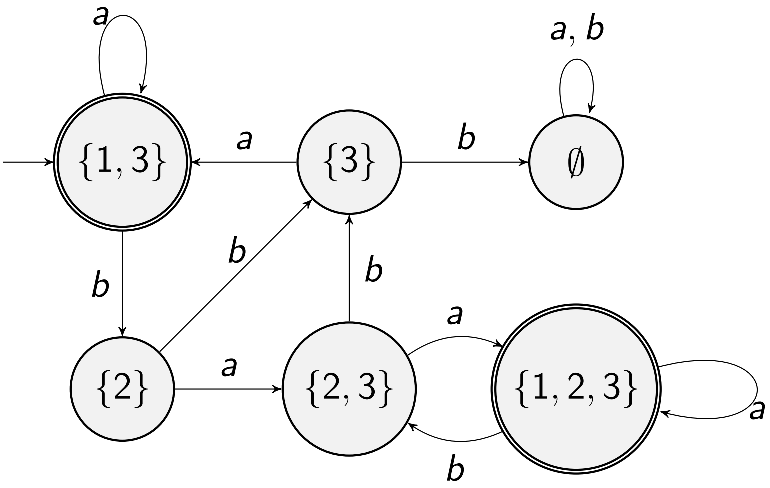
$$\{1,3\} \text{ but not } \{1\}$$

  The reason is that in the beginning, even without any input, we can already reach q_3
- Accept states: any state including q_1 is an accept state

Removing unused states I

- Some states can never be reached
- We can remove them to simplify the diagram
- Turns out any state having 1 but without 3 can never be reached

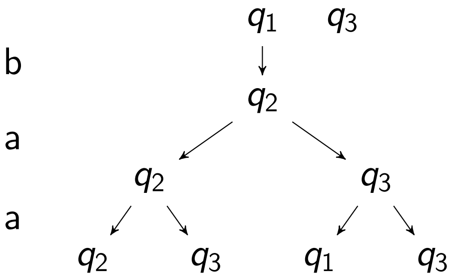
Removing unused states II



More explanation of example 1.41 I

- Idea:

  Each node includes all states at the current layer
- Example: *baa*



More explanation of example 1.41 II

We see

$$\{1,3\} \xrightarrow{b} \{2\} \xrightarrow{a} \{2,3\} \xrightarrow{a} \{1,2,3\}$$

- Formal description of the procedure
- Given NFA

$$(Q, \Sigma, \delta, q_0, F)$$

We would like to convert it to a DFA

$$(Q', \Sigma, \delta', q_0', F')$$

- Details of this DFA:

More explanation of example 1.41 III

- $Q' = P(Q)$
- $q_0' \in P(Q)$ includes

  $$\{q_0\} \cup \{\text{states reached by } \epsilon \text{ from } q_0\}$$

  We call such a set $E(\{q_0\})$
- $F' = \{R \mid R \in Q', R \cap F \neq \emptyset\}$
- δ':

  $$\delta'(R, a) = \cup_{r \in R} E(\delta(r, a))$$

  Now r is a state of NFA and we use δ to see where to go by taking the input a

More explanation of example 1.41 IV

- Note that we cannot just do

$$\cup_{r \in R} \delta(r, a)$$

ϵ must be handled