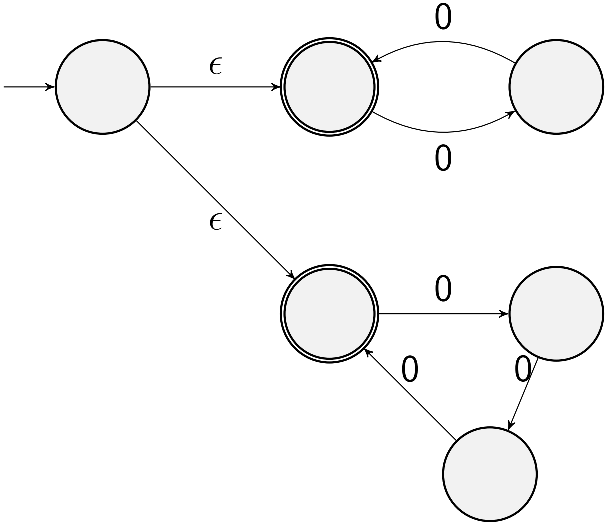
Example 1.33 I

- Consider the following figure

Example 1.33 II



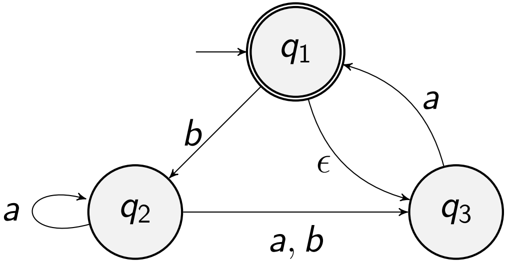
Example 1.33 III

- For this language, $\Sigma = \{0\}$. This is called unary alphabets
- The only non-deterministic place is at the start state
- What is the language?

$$\{0^k \mid k \text{ multiples of 2 or 3}\}$$

Example 1.35 I

- Fig 1.36



- Accept
  ϵ, a, $baba$, baa can be accepted
- But babba is rejected

Example 1.35 II

See the tree below

$$
\begin{array}{cccccc}
 & & & q_1 & & q_3 \\
b & & & \downarrow & & \\
 & & & q_2 & & \\
a & & \swarrow & & \searrow & \\
 & q_2 & & & & q_3 \\
b & \downarrow & & & & \\
 & q_3 & & & &
\end{array}
$$

- This example is later used to illustrate the procedure for converting NFA to DFA

Definition: NFA I

- $(Q, \Sigma, \delta, q_0, F)$
- δ: $Q \times \Sigma_\epsilon \to P(Q)$

  $P(Q)$: all possible subsets of Q
- $\Sigma_\epsilon = \Sigma \cup \{\epsilon\}$
- $P(Q)$: power set of Q

  "power": all $2^{|Q|}$ combinations; $|Q|$: size of Q

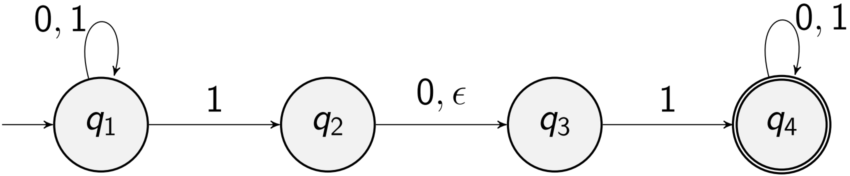
$$Q = \{1, 2, 3\}$$

$$P(Q) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$$

Definition: NFA II

- Thus $P(Q)$ is a set of sets

Example 1.38 I



- $Q = \{q_1, \ldots, q_4\}$
- $\Sigma = \{0, 1\}$
- Start state: q_1
- $F = \{q_4\}$
- δ:

Example 1.38 II

	0	1	ϵ
q_1	$\{q_1\}$	$\{q_1, q_2\}$	$\emptyset$
q_2	$\{q_3\}$	$\emptyset$	$\{q_3\}$
q_3	$\emptyset$	$\{q_4\}$	$\emptyset$
q_4	$\{q_4\}$	$\{q_4\}$	$\emptyset$

- Every element in the table is a set
- Note that DFA does not allow $\emptyset$. In NFA, "no link" means the output of δ is $\emptyset$
- So seriously speaking, a DFA is not an NFA. Need modifications to satisfy the NFA definition

N accepts w I

- First we have that w can be written as

$$w = y_1 \dots y_m$$

  where $y_i \in \Sigma_\epsilon$
- A sequence $r_0 \dots r_m$ such that
  1. $r_0 = q_0$
  2. $r_{i+1} \in \delta(r_i, y_{i+1})$
  3. $r_m \in F$
- So m may not be the original length (as y_i may be ϵ)