- **Please give details of your calculation. A direct answer without explanation is not counted.**
- **Your answers must be in English.**
- **Please carefully read problem statements.**
- **During the exam you are not allowed to borrow others' class notes.**
- **Try to work on easier questions first.**

Problem 1 (20 pts)

We say w_1 is w_2's subsequence if we can obtain w_1 by removing some symbols in w_2. For example, *ad* is a subsequence of *abcdc* by removing the second, the third, and the fifth symbols. As a result, ϵ is a subsequence of any string. Is

$$A = \{\langle C, w\rangle \mid C \text{ is a CFG which can generate at least one subsequence of } w^k \text{ for some } k\}$$

Turing decidable?

If you use any lemma which is not taught in the class, even if it is in the textbook, you need to prove it. This problem can be solved without using any lemma which is not taught.

Problem 2 (20 pts)

Assume

$$f_1(n) = O(2^n) \text{ and } f_2(n) = O(3^n).$$

Is

$$f_1(n) + f_2(n) = O(3^n)?$$

You need to formally prove your answer. You get 0 point if only answering yes/no.

Problem 3 (30 pts)

Assume

$$f_1(n) = O(2^n) \text{ and } f_2(n) = o(3^n).$$

Is

$$f_1(n) + f_2(n) = o(3^n)?$$

You need to formally prove your answer. You get 0 point if only answering yes/no.

Problem 4 (20 pts)

Prove whether the following two statements are true or false

(a) If A and B are countable, then $A \cup B$ is also countable

(b) All irrational numbers are not countable. (hint: use the first part of this question)

Problem 5 (5 pts or -10 pts)

In Chapter Seven (or in the lecture last week) we mentioned one of the greatest unsolved computer science problem. What is it?

If you correctly answer this question, you get 5 points. Otherwise, you get -10.

Problem 6 (5 or 25 pts)

(a) (5 pts) Calculate $48,023 \times 89,363$

(b) (Bonus 20 pts) Find $512,973,211 = p \times q$, where $p, q > 1$ and $p, q \in \mathbf{N}$.