- **Please give details of your calculation. A direct answer without explanation is not counted.**
- **Your answers must be in English.**
- **Please carefully read problem statements.**
- **During the exam you are not allowed to borrow others' class notes.**
- **Try to work on easier questions first.**

Problem 1 (15 pts)

Convert the following CFG into CNF with $\Sigma = \{a, b\}$.

$$S \rightarrow bS \mid E \mid \epsilon$$
$$E \rightarrow aEb \mid a$$

And please follow the formal procedure, i.e. Theorem 2.9 of the textbook.

Problem 2 (20 pts)

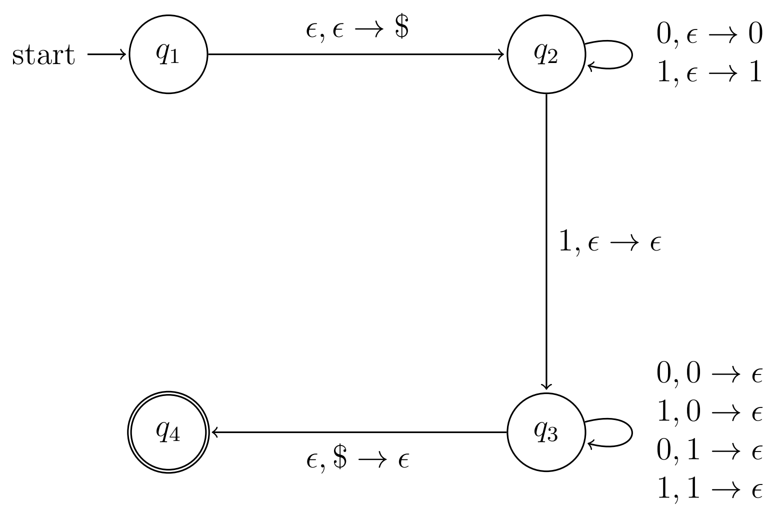
Consider the following language

$$\{w \mid 2n_1(w) \leq n_0(w) \leq 3n_1(w)\},$$

where $\Sigma = \{0, 1\}$ and $n_{0/1}(w)$ means the number of 0's (or 1's) in w. Construct a PDA with ≤ 5 states to recognize this language. Give the formal definition of your PDF.

Problem 3 (20 pts)

Consider the following PDA with $\Sigma = \{0, 1\}$



(a) What is the language?

(b) Draw the tree to process the string 011. Your tree must be complete. Note that we mean a tree to process an input string. We do not mean a parse tree of CFG.

(c) Find CFG of this PDA's language. You are required to follow the same procedure in lemma 2.27 to generate rules. You should **not** remove any redundant rules generated by the lemma.

Problem 4 (15 pts)

(a) Construct a Turing machine (i.e., showing the state diagram) for the language

$$\{0^n 1^n \mid n \geq 0\}.$$

Note that we use the standard Turing machine rather than extensions such as nondeterministic Turing machine. The number of states is ≤ 6, including q_a and q_r. You can assume $\Sigma = \{0, 1\}$.

(b) Give the formal definition.

Problem 5 (15 pts)

Consider the language

$$\{w\#w \mid w \in \{0,1\}^*\},$$

where $\Sigma = \{0, 1\}$.

(a) Construct a 2-tape Turing machine to recognize this language. We assume that

1. in the beginning, $\sqcup$(input) in the 1st tape.
2. we copy the second part to the 2nd tape and then compare strings in both tapes.
3. the number of states (including q_a and q_r) should be no more than 8.

No need to give the formal definition.

(b) Simulate two strings 01#01.

Problem 6 (15 pts)

Construct a nondeterministic Turing Machine with no more than 7 states (including q_a and q_r) to recognize the following language:

$$\{ww^R \mid w \in \{0, 1\}^*\},$$

where w^R is the reverse of a string. No need to give the formal definition.