- **Please give details of your calculation. A direct answer without explanation is not counted.**
- **Your answers must be in English.**
- **Please carefully read problem statements.**
- **During the exam you are not allowed to borrow others' class notes.**
- **Try to work on easier questions first.**

Problem 1 (15 pts)

1. Consider the following lanagage

   $$A = \{0^n 1 \mid n \geq 0\}$$

   Give a DFA state diagram for this language (no more than 3 states)

   Give the formal definition of your DFA

2. Consider the following lanagage

   $$B = \{(10)^n \mid n \geq 0\}$$

   Give a DFA state diagram for this language (no more than 3 states)

   No need to give the formal definition of your DFA

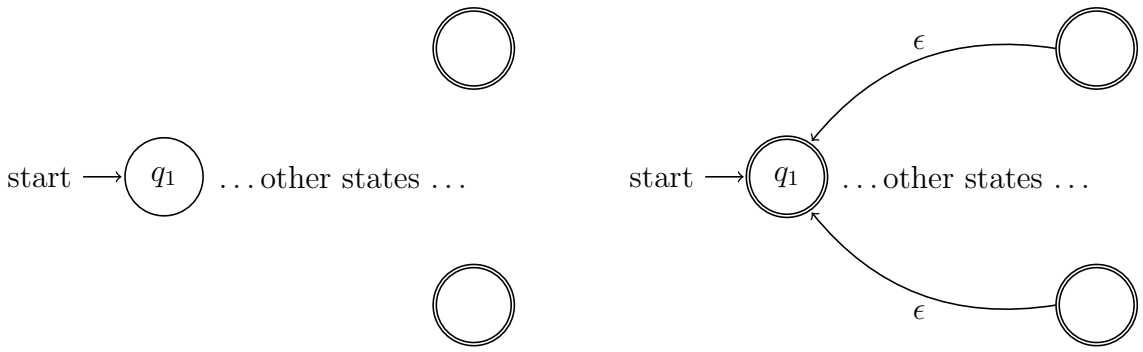
3. Construct the state diagram of $A \cup B$ using the method used in Theorem 1.25 (i.e., the method before we introduced NFA). Please remove unnecessary states in final answer.

   No need to give the formal definition of your DFA

Problem 2 (15 pts)

1. For any regular language A, consider the following way to generate language A^*.

   1. Find a DFA M (Figure 1a) which recognizes language A.
   2. Add an edge from each accepting state to the initial state with empty character ϵ.
   3. Since we need to accept empty string, the initial state should also be an accepting state.
   4. So we get a new NFA (Figure 1b).



(a) original DFA M (b) new NFA

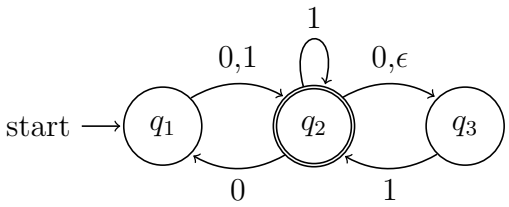
Figure 1: A NFA generated from DFA M by the procedure in Problem 2.

Give a DFA M with number of states = 2, such that the language generated by the procedure above is **not** A^*. You only need to give a state diagram, and you can assume $\Sigma = \{0, 1\}$.

2. For a general DFA $M = (Q_1, \Sigma, \delta_1, q_0, F_1)$, what is the formal definition of the NFA in Figure 1?

Problem 3 (15 pts)

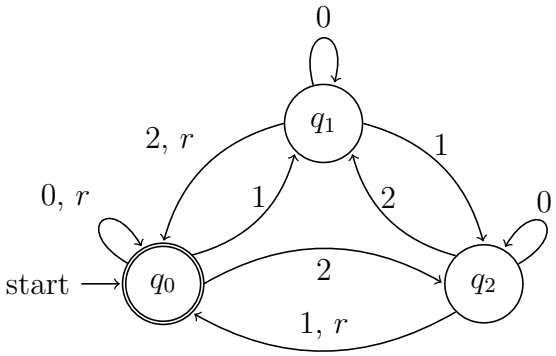
Consider following NFA:



Please convert this NFA to DFA and remove unnecessary state in final answer.

Problem 4 (20 pts)

Given DFA in Figure 1.14.



$\Sigma = \{r, 0, 1, 2\}$ and we treat r as a single symbol. Transform it to a GNFA and then obtain a regular expression by sequentially removing state q_2, q_1, and q_0.

Problem 5 (15 pts)

Is the following language regular?

$$L = \{0^i 1^j \mid \gcd(i, j) = 1\},$$

where $\gcd(i, j)$ means the greatest common divider of i and j. Explain your answer clearly.

Problem 6 (20 pts)

Is the following language regular?

$$L = \{uww^R v \mid u, v, w \in \{0, 1\}^+, |u| \geq |v|\},$$

where $\{0, 1\}^+$ means the set of strings which are composed of 0 and 1 (ϵ is not included). And w^R means the reverse of the string w. Explain your answer clearly.