Chapter 4: Decidability I

- Now we have algorithms
- We want to check problems solvable or not by computers
- Need a TM to decide it

  i.e., accept/reject in a finite number of steps
- We will show some examples

Acceptance Problems for DFA I

$$A_{DFA} = \{\langle B, w\rangle \mid B \text{ is a DFA that accepts } w\}$$

- $\langle B, w\rangle$ is the input
  Note that a DFA can be represented as a string $(Q, \Sigma, \ldots)$
- Is A_{DFA} decidable?
- Idea: input $\langle B, w\rangle$
  1. simulate B on w
  2. ends in an accept state $\Rightarrow$ accept
     otherwise $\Rightarrow$ reject

Proof of A_{DFA} I

- Put
$$B = \langle Q, \Sigma, \delta, q_0, F \rangle$$
into a tape
- Check if $w \in \Sigma^*$ and B a valid DFA
- Simulate w according to δ
- After processing the last element of w, check if in a final state

A_{NFA} I

$$A_{NFA} = \{\langle B, w\rangle \mid B \text{ is an NFA that accepts } w\}$$

- We can convert B to a DFA and use the procedure for A_{DFA}
- It's like to use the procedure for A_{DFA} as a subroutine

A_{REX} I

$$A_{REX} = \{\langle R, w\rangle \mid R: \text{ regular expression generates } w\}$$

- It's similar
- We convert R to a DFA first
- Recall that we had a procedure to convert R to an NFA. Then we can convert the NFA to a DFA
- The key is that the conversion is a finite procedure

E_{DFA} I

$$E_{DFA} = \{\langle A \rangle \mid A : DFA, L(A) = \emptyset\}$$

- i.e. A accepts nothing
- Idea:

  DFA accepts something

  $\Leftrightarrow$ reaching a final state from q_0 after several links
- procedure
  1. mark q_0

E_{DFA} II

2. repeat until no new state marked
   mark all
   $$a \to b,$$
   where a has been marked
3. if no $q \in F$ marked, accept. otherwise, reject

- Example: a state diagram with 3 nodes and the following connections

$$1 \to 2, 3$$

E_{DFA} III

Marked states in running the procedure

$$1$$
$$12$$
$$12$$

- Each iteration: at least one new state marked
- At most n iterations: n: # states

EQ_{DFA} I

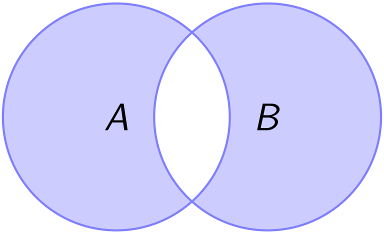
$$EQ_{DFA} = \{\langle A, B \rangle \mid A, B : DFAs, L(A) = L(B)\}$$

- EQ_{DFA} is decidable
- Idea for the proof:
- Let a DFA C be the exclusive or of A and B

  If
  $$L(A) = L(B)$$
  then
  $$L(C) = \emptyset$$

EQ_{DFA} II

Exclusive or of A and B



(latex source from `https://texample.net/tikz/examples/set-operations-illustrated-with-venn-diagrams/`)

EQ_{DFA} III

- Formally

$$L(C) = (L(A) \cap \overline{L(B)}) \cup (\overline{L(A)} \cap L(B))$$

- B DFA $\Rightarrow$ so is $\overline{B}$
- A, B DFA $\Rightarrow$ so is $A \cup B, A \cap B$
- We then use E_{DFA} to check if $L(C) = \emptyset$ or not